Adrián Jiménez, C., Jiménez-Fanjul, N., Madrid, M.J. y Pedrosa-Jesús, C. (2022). Use of mathematical manipulatives and development of number sense in first grade primary-students. *Matemáticas, Educación y Sociedad*, *5*(3), 10-21

ISSN: 2603-9982

USE OF MATHEMATICAL MANIPULATIVES AND DEVELOPMENT OF NUMBER SENSE IN FIRST GRADE PRIMARY-STUDENTS

Cristina Adrián-Jiménez, Universidad de Córdoba, España

Noelia Jiménez-Fanjul, Universidad de Córdoba, España

María José Madrid, Universidad Pontificia de Salamanca, España

Cristina Pedrosa-Jesús, Universidad de Córdoba, España

Abstract

Different studies consider the possibilities of the use of manipulatives for mathematics teaching. It is with this in mind that this papers analyses whether the utilisation of manipulatives fosters development of the number sense in first grade Primary School students of a public Early Years and Primary Education school in the region of Córdoba (Spain). In order to detect the possible differences between the students using them and those who do not, quantitative data was collected from a test at the end of the school year. Such test is known as TEMA-3 (test of early mathematics ability). The test looks into two aspects of mathematics: one, based on formal mathematics; the other, on informal mathematics. However, no significant statistical differences were found associated with the utilisation of manipulatives.

Keywords: *manipulatives; mathematical competence; number sense; primary education.*

Recibido: 02/10/2022; Aceptado:21/12/2022

INTRODUCTION

From their early learning years and on, boys and girls tend to use naturally their thinking skills to bring order into their world, applying mathematics and logic to such end. For this reason, the use of an appropriate methodology is essential at the beginning of the long and complex process of construction of the mathematical thinking.

The number sense is not the kind of knowledge that is usually taught. Therefore, it is not easy to define in a precise manner the meaning carried by the expression number sense. In general terms, it has to do with some important abilities of a person, "including flexible mental calculation, numerical estimation and quantitative reasoning" (Greeno, 1991, p. 170). Schneider y Thompson add that if a student has good number sense, he will be able to think flexible about numbers, understand their meaning and the relationships between them.

Consequently, it refers to the general understandings that one has about numbers and operations, together with the ability to make use of such understandings in a flexible way so as to make mathematical judgements and develop strategies that are useful in solving complex problems.

The National Council of Teachers of Mathematics (NCTM, 2000) identified five elements that characterise number sense: meaning of numbers, number relationships, size of numbers, operations with numbers, and referents for numbers and quantities. To develop a good number sense, it is necessary to gain sufficient skills in mental calculation, estimation of the relative size of numbers and the result of operations with them, recognition of part-whole relationships, place value and problem solving. Expressions like number sense, number awareness or numerical thought are increasingly prevailing in modern studies about mathematical knowledge.

Number sense evolves and improves as students understand the size of numbers, develop multiple ways of thinking about and representing numbers, use numbers as referents, and develop accurate perceptions about the effects of operations on numbers (Sowder, 1992).

In the view of McIntosh, Reys and Reys (1992), numerical thought encompasses each individual's understanding, in general terms, of numbers and operations, together with the ability and tendency to apply such understanding with flexibility in making mathematical judgements and developing useful strategies for handling numbers and operations.

Moreover, Spanish curriculum for mathematics in Primary Education search for an effective numerical literacy, this is defined as the ability to deal successfully with situations that involve numbers and their relationships, obtaining effective information directly from them, or through comparison, estimation and mental or written calculation. It states that in order to achieve true numerical literacy, it is not enough to master the algorithms of written calculation, but it is necessary to act confidently facing numbers and quantities, using them whenever necessary and identifying the basic relationships that exist between them ("Real Decreto 126/2014", 2014).

Real Decreto 157/2022 (2022) characterises number sense as the development of skills and ways of thinking based on the understanding, representation and flexible use of numbers and operations, for example, to guide decision-making.

The standards established by the NCTM (2000) emphasise that students learn mathematics through the experiences they are provided with by their teachers.

Accordingly, their knowledge and ability to apply them to problem solving, as well as their confidence in so doing, are determined by the instruction they receive at school.

Different aspects may play a role in children's numerical knowledge, for example Ramani and Siegler (2014) analyse the impact of the early home environment and the children's experiences with informal learning activities, like games, prior to children beginning school.

Experiences related with concrete objects are carried out using manipulatives (manipulative didactic materials). These are physical objects used in teaching and learning mathematics.

The history of manipulatives for teaching mathematics is not recent, Alsina and Martínez (2016) says that since the beginning of the 20th century, the use of manipulatives as a tool to develop mathematical and scientifical knowledge has been highly investigated by authors such as Montessori, Piaget, Decroly, Freinet, Dienes and Mialaret.

Different authors have defined, classified, examined or considered their use in mathematics classroom as shown in Flores, Lupiáñez, Berenguer, Marín and Molina (2011).

In particular, didactic materials are normally used as curriculum organisers in two ways (Coriat, 2001):

- On the basis of resources, the teacher wonders what kind of activities are more suitable for enhancing mathematics learning in making use of them.
- On the basis of planned activities, the teacher wonders which manipulatives are optimal for improving learning.

It is evident that the best way of encouraging and strengthening mathematics learning is action or physical experimentation. Nonetheless, the fact that this experimentation is understood as using manipulatives in class, it is something with no agreement among mathematics education researchers (Ball, 1992; Hoong, Kin, and Pien, 2015). Moreover, the debate about the use of manipulatives in math classes is currently on the table. Physical experimentation plays a crucial role during the early years in global development, and especially in the development of logical-mathematical thought, understanding the latter as a personal, active, and reflective construction that is based on the relationships that students establish with the objects and situations they face in their environment.

This reality carries an important methodological implication for classroom practice: it is essential that we support verbal and graphical information with concrete materials (manipulatives) that students can actually see, manipulate and which may provide the grounds for students to initiate and deploy the processes of reasoning.

As Arrieta (1998) states, manipulatives can help to understand and to communicate mathematics, they allow us to refer to a physical support, they facilitate visualization, and they favour motivation and a positive attitude towards mathematics; that makes their use the starting point for the construction of knowledge. Along the same lines, Maz-Machado et al. (2019) concluded that the use of manipulatives, as well as the resolution of practical tasks, is considered by students as a means to connect theoretical mathematical knowledge with real problems they may encounter in everyday life, leading them to reflect on the usefulness of mathematics and even enabling an improvement in their attitude towards the subject.

However, the approach of Nührenbörger and Steinbrig (2008, p. 179) require that: "the conception that manipulatives […] are not spontaneously working methods as means of help in order to directly understand abstract mathematics, but that they become, in the course of mathematical learning processes, quasi- symbolical representatives for mathematical operations, structure and concepts".

Castro and Palop (2019) indicate that manipulatives can improve the teaching and learning of mathematics, but they also consider that the effectiveness of manipulatives depends on the type of task.

As well, Carbonneau, Marley and Selig (2013) analysed 55 studies that compared mathematics instruction with manipulatives to mathematics instruction with only abstract mathematics symbols. They identified statistically significant results with small to moderate effect sizes.

Considering that during the last years, different studies about the number sense have been carried out; for example, the impact of aging on basic non-symbolic and symbolic numerical skills was studied by Norris, McGeown, Guerrini and Castronovo (2015) or the influence of high level math education on two mechanisms in adult number processing: the approximate number sense and the exact number system (Castronovo & Göbel, 2012); which show the interest on this topic.

It is on this basis that we centered our interest on the extent to which the use of manipulatives assists the development of number sense in first graders. Therefore, this paper aims to identify the impact of manipulatives in mathematical competence, regarding number sense.

MATERIALS AND METHODS

The research methodology will be eminently quantitative. A quasi-experiment was carried out with a pretest-posttest research design with control group (Bisquerra, 2004). The study was carried out with a group of Year 1 Primary School students using the manipulatives, the experimental group, and another group of students from the same school and grade acting as the control group.

The main objective established for this study was to identify the possible differences in mathematical competence, regarding number sense, among students who used manipulatives during maths lessons and those who did not.

The sample used in this research is composed of the students in two Year-1 groups of a public Primary Education School in the region of Córdoba (Spain).

The one assigned to be the control group (1A) comprises 27 subjects and has not used continuously nor systematically any manipulatives during the in-class explanations. The other group (1B), taken as experimental group, has 25 subjects and has, on the contrary, used the referred kind of materials.

Out of the 27 subjects in the control group, 14 are 6-year-old students and 13 are 7. Similarly, the 25 subjects that make up the experimental group are 13 age-6 and 12 age-7 students. As far as gender is concerned, the control group is made of 12 girls and 15 boys while the experimental group has 12 girls and 13 boys.

The participant teachers attended a series of periodical seminars and workshops where they received orientation and advice about the knowledge and use of the classroom manipulatives.

The didactic materials used in the lessons were: a number tape, hundred boards, bundles of sticks and holding boxes, addition wheels, and dot cards. The characteristics of these manipulatives have already been described in literature on the topic (Bracho, Maz-Machado, Jiménez-Fanjul and García, 2011).

The experimental group was presented daily, by the teacher, with exercises and problems in which the proposed materials were used. These materials were always in the classroom so that students could choose from and use them as an aid or support in solving each corresponding activity (exercises, problems, activities). Each student was encouraged to make use of the manipulatives at least three times per week but had freedom to use them more frequently according to their needs. This is carried out in a systematic way, fostering the use of manipulatives among students by providing them models of how activities can be done using the different manipulatives.

To collect data for the study, the so-called TEMA-3 test (Test of early mathematics ability, third edition) by Ginsburg and Baroody (2007), adapted to the Spanish context, was used at the end of the school year. This test is applied individually to children of 3 to 8 years of age. It takes around 30 to 45 minutes.

The TEMA-3 test consists of 72 items that evaluate the basic mathematical competence and provides separate specific information about formal mathematics (31 items) and informal mathematics (41), which are both separated into components. Within the area of informal mathematics, the items concerning numbering are well represented, given the importance of the processes of counting in this area. In the area of formal mathematics, numerical facts and calculation skills have greater representation, reflecting their relevance for teaching basic mathematics. Firstly, the items that measure informal knowledge will be listed, followed by those evaluating formal knowledge (Table 1).

Table 1. *Items that evaluate informal and formal mathematics.*

COMPONENTS	ITEMS
Informal mathematics	
Numbering	2, 3, 4, 5, 6, 9, 10, 12, 13, 20, 21, 22, 25, 27, 29, 32, 33, 37, 38, 40, 41, 45 and 66
(Quantity) comparison	1, 16, 17, 26, 35 and 60
Informal calculation	8, 19, 23, 24, 34, 62, 65 and 72
Concepts	7, 11, 39 and 46
Formal mathematics	
Conventionalisms	14, 18, 28, 30, 31, 42, 43 and 55
Numerical facts	36, 47, 48, 50, 51, 52, 61, 67 and 68
Formal calculation	44, 49, 54, 57, 58, 59, 63, 69 and 70
Concepts	15, 53, 56, 64 and 71

For each item of TEMA-3 test, if the answer given by the student is correct, a score of 1 is assigned to that item; if it is incorrect, 0 is assigned. The mathematical performance in each component of both formal and informal mathematics is analysed from the average scores of each group. This analysis has been done considering age and gender intra-group (between experimental and control group) and intergroup.

To determine the Mathematical Competence Index (MCI), the direct scoring achieved by each subject is put in relation with their age, differentiating years and months (Ginsburg & Baroody, 2003).

Finally, we analyse whether there are significant differences between the MCI of the groups, which, since the sample is not random and each group has a different size, is done through a nonparametric test (U Mann-Whitney's test) for independent samples. To that end, two hypotheses are established.

H0: Both groups mark no statistically significant differences in the index of mathematical competence (MCI) because of the use of manipulatives: $\mu A = \mu B$

H1: Both groups mark a statistically significant different index of mathematical competence (MCI) because of the use of manipulatives: $\mu A \neq \mu B$

RESULTS

When we segregate the groups according to age (Figure 1), it is revealed that the case of formal competence presents differences between the control group and the experimental group: the latter shows greater development of all the components, independently of age; the most remarkable differences appeared in number facts and formal concepts.

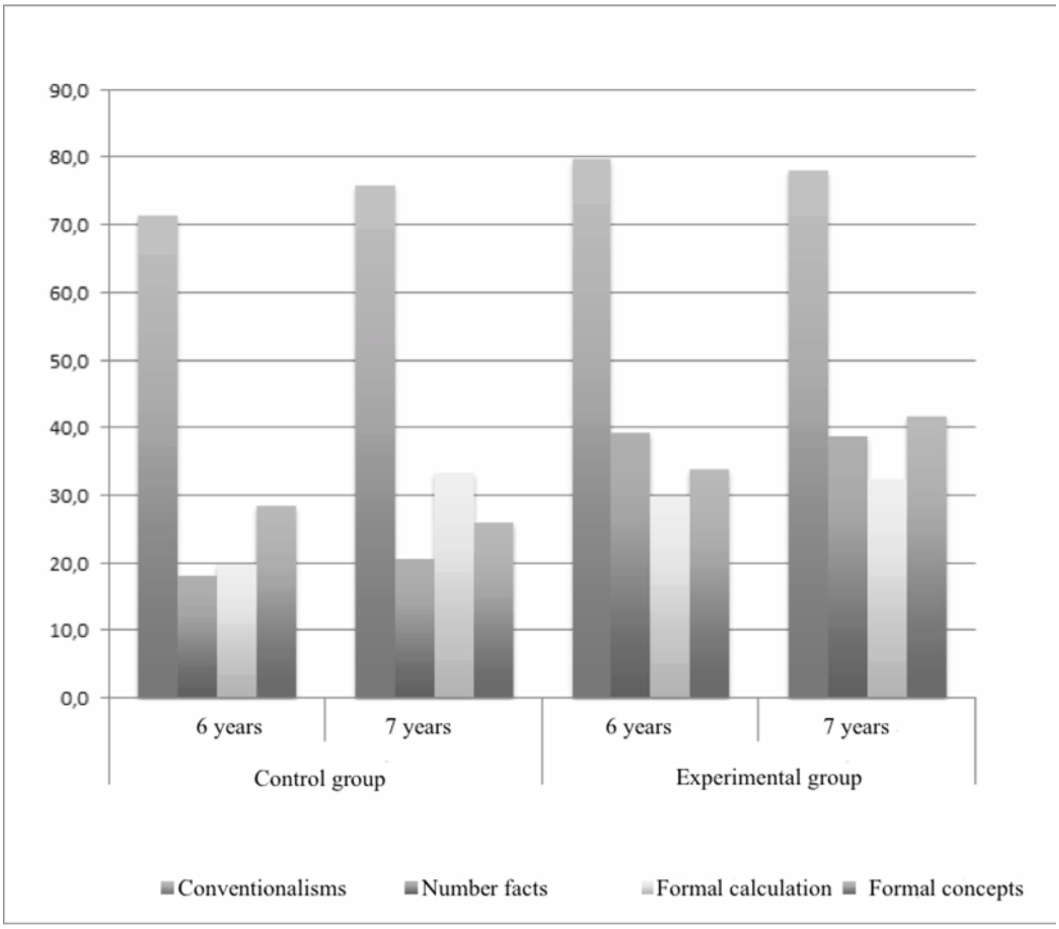

Figure 1. Comparison of the competence in formal mathematics, by age, in the control group vs. experimental group.

Moreover, if we consider the differences in formal mathematics between 6-year-old students and 7-year-old students in each group, the only significant difference is that in the control group the latter show greater development of formal calculation.

In the case of informal mathematics, it is revealed that the average percentage generally increases for 6-year-old students in the experimental group, being the differences more significant in informal concepts and without increase in informal calculation (Figure 2). However, in the case of informal mathematical competence barely differences between the control group and the experimental group appeared in 7-year-old students.

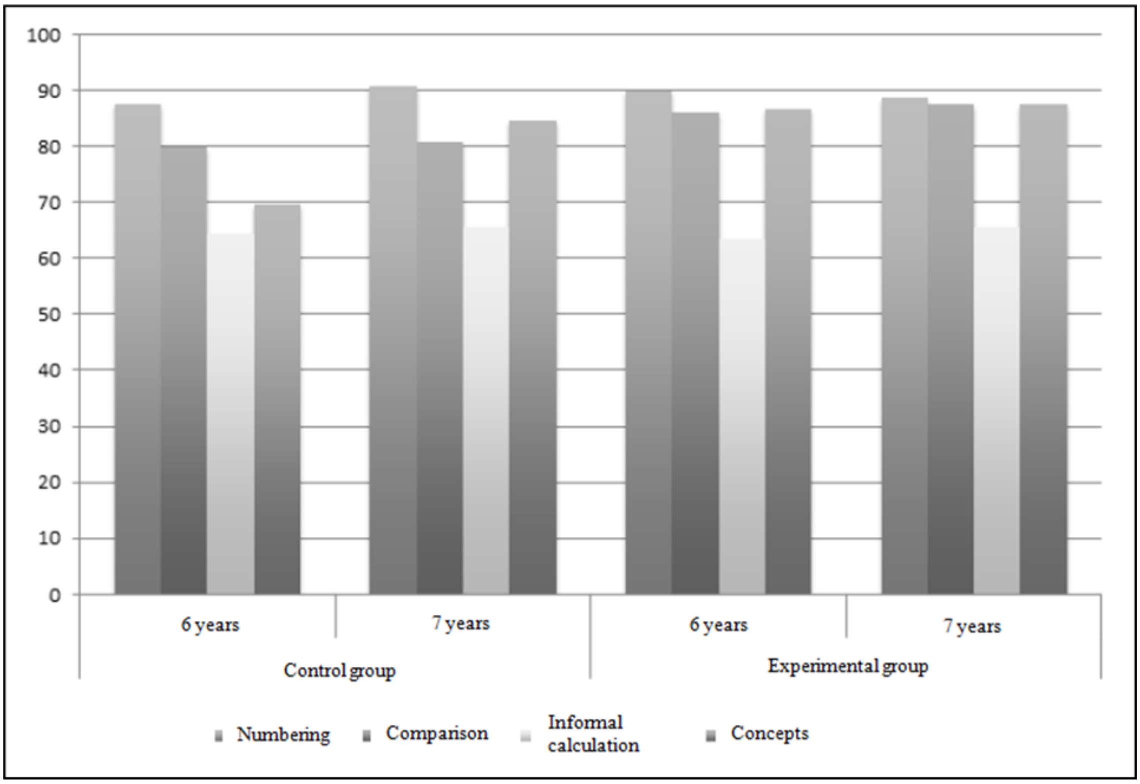

Figure 2. Comparison of the competence in informal mathematics, by age, in the control group vs. experimental group.

Furthermore, if we consider the differences in informal mathematics between 6-year-old students and 7-year-old students in each group, the only significant difference is that in the control group the latter show greater development of informal concepts.

Regarding gender, it appears that, in the control group, the average percentages of all the aspects of informal mathematics are higher for boys than for girls (Figure 3). This difference is even greater in some components of the formal mathematics competence, for example Number facts or Formal calculation (Figure 4).

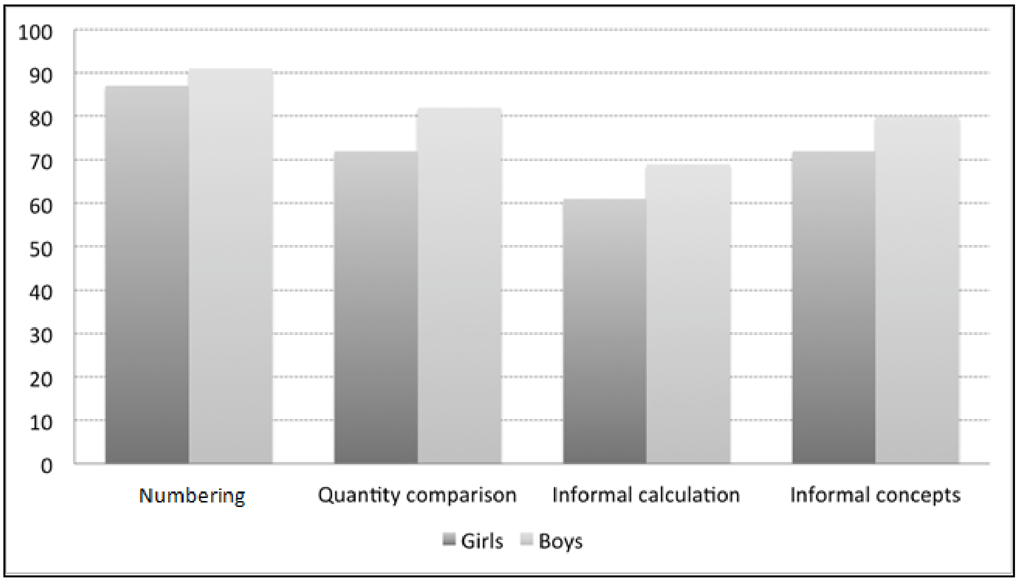

Figure 3. Comparison, by gender, of performance in informal mathematics within the control group.

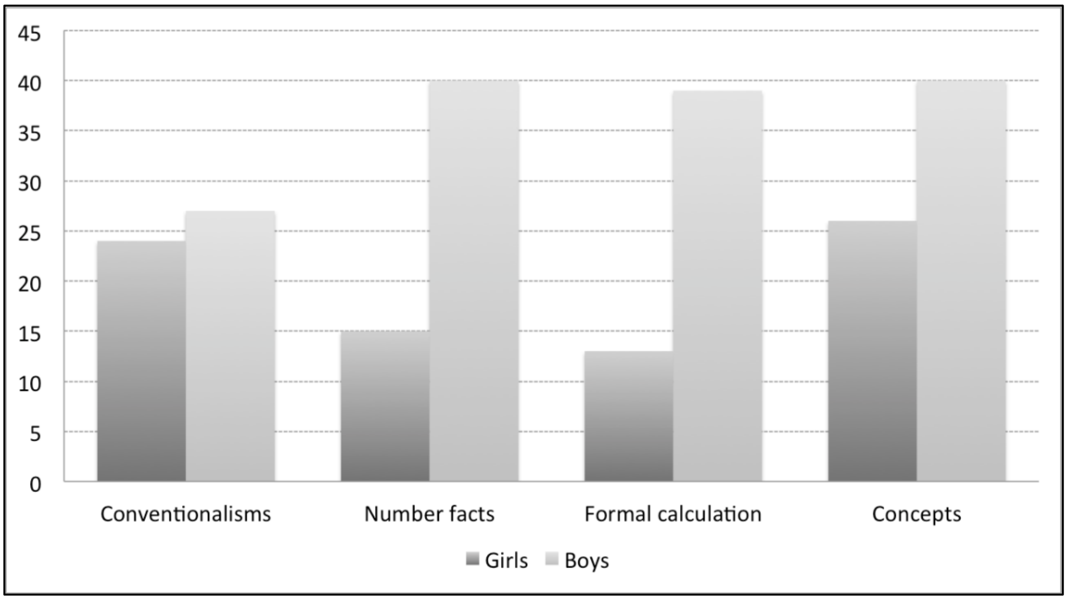

Figure 4. Comparison, by gender, of performance in formal mathematics within the control group.

In the experimental group, the average percentages of all the aspects of informal mathematics are higher for boys than for girls (Figure 5), while the average percentages of all the aspects of formal mathematics are pretty similar for boys and girls (Figure 6).

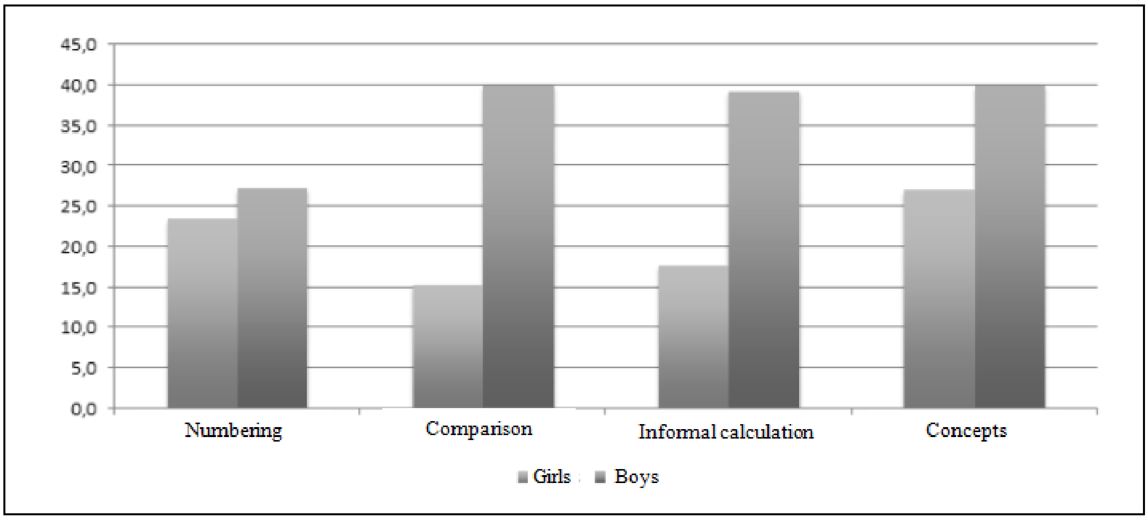

Figure 5. Comparison, by gender, of performance in informal mathematics within the experimental group.

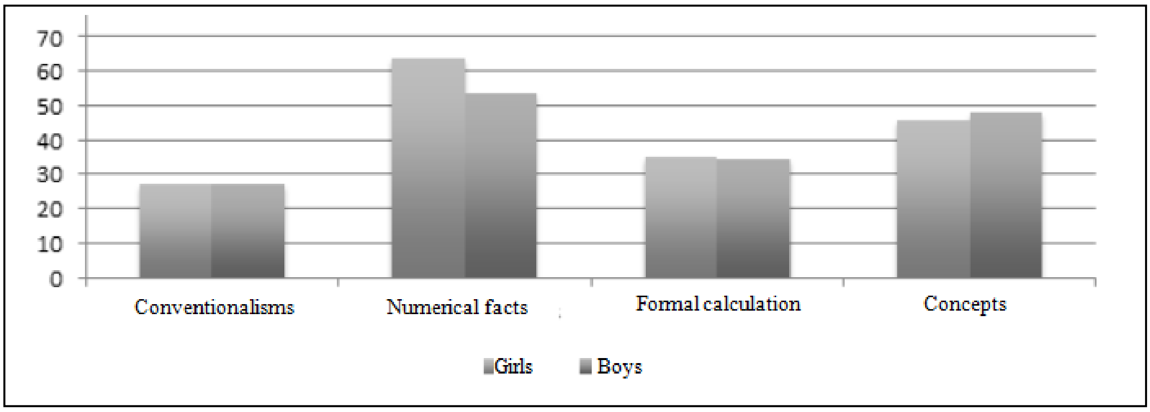

Figure 6. Comparison, by gender, of performance in formal mathematics within the experimental group.

Table 2. *Mann-Witney's test results.*

Group		**Ranges**		
		N	**Average range**	**Sum of ranges**
MCI	Experimental group	25	29.16	729.00
	Control group	27	24.04	649.00
	Total	52		

Contrast stats[a] *a. Grouping variable: Group*

Group	**MCI**
Mann-Whitney's U	271.000
Wilcoxon's W	649.000
Z	-1.219
(bilateral) asymptotic significance	.223

When we segregate the groups according to gender: girls in the experimental group show greater development of all the components (both formal and informal mathematics) than girls in the control group. For the boys, the average percentage in the experimental group is not always higher as in the control group, although it is in comparison and concepts (informal mathematics) and numerical facts and concepts (formal mathematics).

At this point, the Mann-Whitney's U test for independent samples is applied, with a confidence level of α=0.05. We compare the MCI value of both groups to determine whether to confirm or reject the null hypothesis (H0) that we formulated.

As can be seen (Table 2), the value of bilateral asymptotic significance (p-value) is higher than 0.05 and, accordingly, the null hypothesis is accepted. Since the data are non-significant, we cannot assert that the differences in MCI are due to the different use of manipulatives in both groups.

DISCUSSION AND CONCLUSIONS

It can be concluded that major differences between 6 and 7-year-old students' informal and formal mathematics competence from the experimental group are not noticeable. On the other hand, within the control group greater differences can be appreciated in formal calculation and informal concepts, balanced favourably toward 7-year-old students.

Regarding gender, it appears that in the control group the average percentages of all the aspects of informal and formal mathematics are higher for boys than for girls. In the experimental group, the average percentages of all aspects of informal mathematics are higher for boys than for girls, but this does not happen in some components of formal mathematics.

In comparing formal mathematical competence between the control group and the experimental group, the latter shows greater development of all the components. However, no clear differences appear in informal mathematical between the control group and the experimental group. Therefore, it is not possible to determine if the manipulatives used in the classroom, helped to developed number sense in the students. This was confirmed after Mann-Whitney's U test; both groups mark no statistically significant differences in the index of mathematical competence (MCI) because of the use of manipulatives.

Our findings are consistent with Uttal, Scudder and DeLoache's (1997) observations in scientific literature on the matter insofar as they indicate that the use of manipulatives in mathematics gives ambiguous results. They argued that research on the efficacy of the manipulative materials has not proven any clear nor consistent advantage of teaching using manipulatives over other more traditional teaching methods.

Also, Marshall and Swan (2008) say that manipulatives on their own do does not teach, children can look active while they use manipulatives but that does not necessarily mean that they are learning.

In this regard pointed the words of Ball (1992, p. 18): "My main concern about the enormous faith in the power of manipulatives, in their almost magical ability to enlighten, is that we will be misled into thinking that mathematical knowledge will automatically arise from their use".

Therefore, this paper is a first step to understand the effectiveness of manipulatives in the numerical sense development of Primary-Students; other complementary quantitative or qualitative studies may be done in the future to analyse whether different variables like age, kind of task, kind of manipulatives, influence the process of teaching and learning mathematics with manipulatives.

REFERENCIAS

Alsina, Á. & Martínez, M. (2016). La adquisición de conocimientos matemáticos intuitivos e informales en la Escuela Infantil: el papel de los materiales manipulativos. *RELAdEI. Revista Latinoamericana de Educación Infantil*, 5(2), 127-136.

Arrieta, M. (1998). Medios materiales en la enseñanza de las matemáticas. *Revista de Psicodidáctica*, (5), 107-114.

Ball, D. L. (1992). Magical hopes: Manipulatives and the reform of math education. *American Educator: the professional journal of the American Federation of Teachers*, *16*(2).

Bisquerra, R. (coord.) (2004). *Metodología de la investigación educativa* (1st ed.). Editorial La Muralla.

Bracho, R., Maz-Machado, A., Jiménez-Fanjul, N. & García, T. (2011). Formación del profesorado en el uso de materiales manipulativos para el desarrollo del sentido numérico. *UNIÓN. Revista Iberomaericana de Educación Matemática*, 28, 41-60.

Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology*, 105(2), 380–400. https://doi.org/10.1037/a0031084

Castronovo, J. & Göbel, S. M. (2012). Impact of high mathematics education on the number sense. *PloS one*, 7(4), e33832. https://doi.org/10.1371/journal.pone.0033832

Coriat, M. (2002). Jornadas sobre tutoría y orientación. *Universidad de Granada*.

De Castro, C. & Palop, B. (2019). ¿Ayudan los materiales manipulativos a resolver tareas matemáticas? Sí, pero... In J. M. Marbán, M. Arce, A. Maroto, J. M. Muñoz-Escolano & A. Alsina (Eds.), Investigación en Educación Matemática XXIII (pp. 243-252). Universidad de Valladolid.

Flores, P., Lupiáñez, J. L., Berenguer, L., Marín, A. & Molina, M. (2011). Materiales y recursos en el aula de matemáticas. Departamento de Didáctica de la Matemática de la Universidad de Granada.

Ginsburg, H. & Baroody, A. J. (2007). Tema-3: test de competencia matemática básica (M. C. Núñez del Río y I. Lozano Guerra, Trads.). Madrid, Spain: TEA Ediciones.

Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for research in mathematics education*, 22(3), 170-218.

Hoong, L. Y., Kin, H. W., & Pien, C. L. (2015). Concrete-Pictorial-Abstract: Surveying its origins and charting its future. *The Mathematics Educator*, 16(1), 1-19.

Marshall, L. & Swan, P. (2008). Exploring the use of mathematics manipulative materials: Is it what we think it is? In Proceedings of the EDU-COM 2008 International Conference. Sustainability in Higher Education: Directions for Change (pp. 338-350). Perth, Australia: Edith Cowan University.

Maz-Machado, A., Madrid, M. J., León-Mantero, C., & Jiménez-Fanjul, N. (2019). Mathematical practical sessions with manipulatives: Trainee teachers' perceptions of their utility. *South African Journal of Education, 39*.

McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining basic number sense. *For the learning of mathematics*, 12(3), 2-44.

NCTM (2000). Principles and Standards for School Mathematics.

Norris, J. E., McGeown, W.J., Guerrini, C. & Castronovo, J. (2015). Aging and the number sense: preserved basic non-symbolic numerical processing and enhanced basic symbolic processing. *Frontiers in Psychology*, 6, 999. https://doi.org/10.3389/fpsyg.2015.00999

Nührenbörger, M. & Steinbring, H. (2008). Manipulatives as tools in mathematics teacher education. In D. Tirosh & T. Woods (eds.), *International handbook of mathematics teacher education. Vol. 2: Tools and processes in mathematics teacher education* (pp. 157-182). Rotterdam, Netherlands: Sense Publishers.

Ramani, G. B. & Siegler, R. S. (2014). How informal learning activities can promote children's numerical knowledge. In R. Cohen & A. Dowker (eds.), *The Oxford handbook of numerical cognition* (pp. 1135-1154). Oxford University Press.

Real Decreto 126/2014, de 28 de febrero, por el que se establece el currículo básico de la Educación Primaria. (2014, March 1). Boletín Oficial Estado, 2014(52), 19349–19420. Retrieved from https://www.boe.es/eli/es/rd/2014/02/28/126

Real Decreto 157/2022, de 1 de marzo, por el que se establecen la ordenación y las enseñanzas mínimas de la Educación Primaria. (2022, March 2). Boletín Oficial Estado, 2022(52), 24386- 24504. Retrieved from https://www.boe.es/buscar/doc.php?id=BOE-A-2022-3296

Schneider, S. B., & Thompson, C. S. (2000). Incredible equations develop incredible number sense. *Teaching Children Mathematics*, *7*(3), 146-168.

Sowder, J. T. (1992). Making Sense of Numbers in School Mathematics. In G. Leinhardt, R. Putman y R. A. Hattrup (Eds.). *Analysis of Arithmetic for Mathematics Teaching* (pp. 1-51). Lawrence Erlbaum Associates.

Uttal, D., Scudder, K. & DeLoache, J. (1997). Manipulatives as symbols: A new perspective on the use of concrete objects to teach mathematics. *Journal of Applied Developmental Psychology*, 18(1), 37-54. https://doi.org/10.1016/S0193-3973(97)90013-7

Cristina Adrián-Jiménez
Universidad de Córdoba, España
s0pejec@uco.es

Noelia Jiménez-Fanjul
Universidad de Córdoba, España
noelia.jimenez@uco.es

María José Madrid Martín
Universidad Pontificia de Salamanca, España
mjmadridma@upsa.es

Cristina Pedrosa-Jesús
Universidad de Córdoba, España
s0pejec@uco.es