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TWO-COLUMN DEMONSTRATIONS IN MATH OLYMPIAD GEOMETRY PROBLEMS: THE CASE OF HONDURAS

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Abstract

In elementary plane geometry courses since the beginning of the 20th century, two-column demonstrations have been taught as a form of formal proof. Nevertheless, there is not much research on the use of this type of demonstrations in Mathematical Olympiad Problems. As a result, this study arises in which we analyzed Mathematics mistakes that students make when attempting to solve geometry problems in the Mathematical Olympiad before and after the implementation of two-column demonstrations in the problem-solving process. A total of 32 students from different departments of Honduras participated in the study. The place of study was the Virtual Math Competition (CVM) and the analysis is based on a method called Newman Error Analysis. In the pre-exam, the results show that students do not use two-column demonstrations for their answers, and the most common mistakes analyzed using the Newman Error are transformation, process skill, and encoding errors. On the other hand, in the final test, the use of two-column demonstrations by the students confirmed that this writing technique helps to order the information given in the problem.

Keywords: *competition, mathematics education, problem-solving, high school students, teaching methods*

Demostraciones de Doble Columna en Problemas de Geometría de las Olimpiadas de Matemáticas: El Caso de Honduras

Resumen

En los cursos elementales de geometría plana desde principios del siglo XX, las demostraciones a dos columnas se han enseñado como una forma de demostración formal. Sin embargo, no existen muchas investigaciones sobre el uso de este tipo de demostraciones en los problemas de la Olimpiada Matemática. Como resultado, surge este estudio en el que analizamos los errores matemáticos que cometen los estudiantes al intentar resolver problemas de geometría en la Olimpiada Matemática antes y después de la implementación de demostraciones a dos columnas en el proceso de resolución de problemas. Un total de 32 estudiantes de diferentes departamentos de

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Honduras participaron en el estudio. El lugar de estudio fue la Competencia Virtual de Matemáticas (CVM) y el análisis se basa en un método denominado Análisis de Errores de Newman. En el pre-examen, los resultados muestran que los estudiantes no utilizan demostraciones a dos columnas para sus respuestas, y los errores más comunes analizados utilizando el Error de Newman son los errores de transformación, habilidad de proceso y codificación. Por otro lado, en el examen final, el uso de demostraciones a dos columnas por parte de los alumnos confirmó que esta técnica de escritura ayuda a ordenar la información dada en el problema.

***Keywords**: competición, educación matemática, resolución de problemas, estudiantes de secundaria, métodos de enseñanza*

INTRODUCTION

One of the ways to detect young people with talent for mathematics is the preparation of mathematics competitions at local, departmental, national and international levels (Ramos, 2006). However, the main problem of Math Olympiad Students in Honduras is writing solutions of problems. This issue stems from varying logical approaches and diverse conceptual understandings. Each student possesses unique levels of comprehension, knowledge, and contextual factors that impact their problem-solving abilities (Bellanca J, 2011).

According to other researchers, aspects that needed in solving National Olympiad problems are the maturity of mathematics with advanced levels such as concepts, comprehension, accuracy, foresight, ingenuity, ways of thinking and mathematical experience (Idris R, 2017). As a result, when students are trying to solve a geometry problem begins by sketching out the geometric figure, but difficulties arise when they struggle to provide justifications for seemingly evident aspects.

The context of contemporary education also plays a role. The shift towards the “Era of Texts” has transformed the way mathematical proofs are conveyed. However, research by (Herbst, 2002) suggests that textual explanations lack the clarity and methodological structure needed for effective understanding. Conversely, the traditional approach of Two-Column demonstrations, prevalent in high school geometry courses, is acknowledged as a valuable tool for helping students develop proof-solving skills. These demonstrations provide a clear, connected sequence of assertions supported by corresponding reasons (Hung. W, 1996).

In light of these challenges, the research aims to investigate whether adopting the Two-Column demonstration approach can alleviate the difficulties faced by Geometry Olympiad participants in expressing their problem-solving processes. The objective is to determine if this method, known for its clarity and organization, can provide students with a structured format for communicating their mathematical reasoning effectively. By exploring the potential impact of Two-Column demonstrations on enhancing students' ability to write comprehensive solutions, the study seeks to contribute to addressing the prevalent issue of articulation barriers faced by Math Olympiad participants.

Furthermore, this study will encompass a dedicated section outlining the historical evolution of geometry education in Honduras. This section aims to provide insight into the trajectory and emphasis of geometry pedagogy within the country. Beyond its descriptive value, this historical perspective serves the purpose of archiving pertinent information within a tangible form, such as a research paper.

History of the Teaching and Learning of Geometry in Honduras

In 1731, bishop Fray Antonio López, founded the Colegio Tridentino de San Agustin de Comayagua. It opened two years later to teach Latin language, culture, religion and mathematics. This institution became the first instructional center where mathematics was taught in Honduras. However, geometry education does not begin at this time. It is between 1847 and 1879 when the teaching of geometry appeared in the country.

To begin with, the foundation of higher education centers in the country made geometry education possible. As mentioned (Valencia, M., 2014) “with the creation of the first faculty of the National Autonomous University of Honduras (UNAH) in 1847, mathematics, geometry, physics and other subjects of humanities and arts are included in their curriculums”. In the same way, in the next few years other institutions of higher education began to appear in the country and to include geometry in their curricula as (Valencia, M., 2014; & Pérez, 1996, 1997) says

> —In 1874 the "Instituto Científico de San Carlos de Occidente" was created to attend the secondary level, and in 1879 it was transformed into the Universidad Nacional de Occidente, which was closed in 1884. The University offers courses in Algebra, Geometry and Trigonometry, Geometry in Space, Plane Geometry, Logic, Arithmetic and Notions of Surveying. — (p. 272).

At the beginning of the 20th century, the National Library of Honduras published a book called "Education, Work and Science" in which some pedagogical strategies for teaching geometry were mentioned. In the text, the author (Moncada, J. M., 1904) mentions that

> —when children go about drawing with chalk, sticks or charcoal, they first draw whimsical shapes, and then, little by little, lines, angles, triangles, quadrilaterals, circumferences follow. They need not enter into definitions, but only by checking the direction of the lines, the opening of the angles, the dimensions of the triangles, etc., will they be able to understand their properties, conditions and relationships. — (p.72).

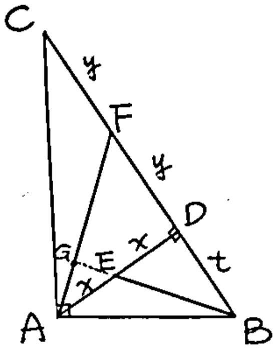
It was not until the 1980s that the Department of Mathematics of the National Autonomous University of Honduras established more concrete guidelines for geometry education. According to (Portillo, 2003), it was suggested to teach positive numbers arithmetic and geometry intuitively from the primary school level.

Lastly, in 2010, the project for improving technical education in the area of mathematics (PROMETAM) included two-column demonstrations in the Honduran curriculum during Phase I, supported by the Ministry of Education with the technical assistance of the Universidad Pedagógica Nacional Francisco Morazán (UPNFM) and the Japan International Cooperation Agency (JICA).

Geometry in Mathematical Olympiads

According to Chen, E. (2021) “the only way to learn mathematics is by doing”. Essentially, problem solving is the only way to learn mathematics. Moreover, Chen. E., (2021) states that the ideas and techniques for solving geometry problems that he knows come from countless resources - lectures on the MOP∗ resources found online, discussions on the Art of Problem Solving, or even late-night chats with friends. This indicates that the geometry used in Math Olympiads aims to develop students' problem-solving skills.

The next question is how mathematical Olympiad solutions are usually written. In Barbeau (2000), he states that writing a solution is an act of communication between two people. As a result, Mathematical Olympiad solutions have always been written in prose.



$$\triangle ABD \sim \triangle CAD \Rightarrow \frac{\overline{BD}}{\overline{AD}} = \frac{\overline{AD}}{\overline{CD}}$$
$$\Rightarrow \frac{t}{2x} = \frac{2x}{2y}$$
$$\Rightarrow \frac{t}{x} = \frac{2x}{y}$$

In $\triangle BDE, \triangle ADF$, $\frac{\overline{BD}}{\overline{ED}} = \frac{t}{x} = \frac{2x}{y} = \frac{\overline{AD}}{\overline{FD}}$

$\therefore \triangle BDE \sim \triangle ADF$ $(\angle BDE = 90^\circ = \angle ADF)$

$\angle FAD + \angle FEG = \angle EBD + \angle DEB = 90^\circ$

$\therefore \angle AGE = 90^\circ$ #

Figure 1. Solution to a geometry problem written in prose style. Source: Romantics of Geometry, Problem 11708

Problem Solving Strategies used in Mathematical Olympiads Geometry Problems

There are many definitions for the word strategy in Mathematics Education. However, in the context of International Mathematical Olympiads Engel, A. (2008) defines them as "information acquired by massive problem solving". In fact, he mentions that "At first the problems are far below a hard competitive level. But if you do most of the problems you are fit for any competition"

Outlined are several strategies employed in solving geometry problems within the context of Mathematical Olympiads. These strategies have been identified and included based on insights derived from participant comments during the International Mathematical Olympiad (IMO 2022). Initially, students were individually interviewed, with a specific focus on the prevalent geometry problems encountered in various Olympiads, including the Central American Mathematical Olympiad and the Iberoamerican Mathematical Olympiad.

These discussions yielded valuable observations, forming a foundation for the compilation of strategies. Furthermore, an additional criterion for the inclusion of strategies was their feasibility for implementation as instruments in Mathematical Olympiad exams for participating students in this study

Methods to Find an Angle Equality

Equality of angles problems in Mathematics Olympiads are a type of problem where one is usually asked to prove that two angles are equal. An example problem is provided below

Let M be the midpoint of the lateral side AB of trapezoid $ABCD$, O be intersection point of its diagonals, and $AO = BO$. The point P was marked on the ray OM such that that $\angle PAC = 90^o$. Prove that $\angle AMD = \angle APC$.

Figure 2. An Olympiad problem asking for proof that two angles are equal

Mathematics Olympiads are commonly plagued by problems of this type. Their prevalence is so significant that they emerge not only within the confines of competition settings but also in educational textbooks, as substantiated by notable works (see Djukić, D. et al., 2011; Hang, K. et al., 2017 & Chen, E., 2021). Subsequently, the ensuing section outlines a spectrum of strategies employed to effectively navigate and address challenges of this specific category.

- Let B and D be points on a circle Ω it holds that angles ABC and ADC are equal in the case that AC is an arc of Ω.

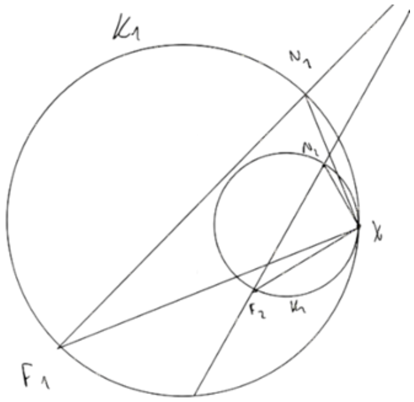


Figure 3. It can be seen that $\measuredangle N_1XF_1 = \measuredangle N_2XF_2 = 90°$

- Identifying isosceles or equilateral triangles can also assist in determining equal angles, which may be the ones required by the problem.
- A transversal line is one that intersects two or more lines. When it intersects perpendicular lines, then several congruent angles are created.
- The angles of two similar or congruent triangles are the same

- Finding the measures of certain segments helps find isosceles triangles or similar triangles.
- Determine two equal angles, even if they are not those requested by the problem. As a general rule, two equal angles can aid in the determination of the requested angles.

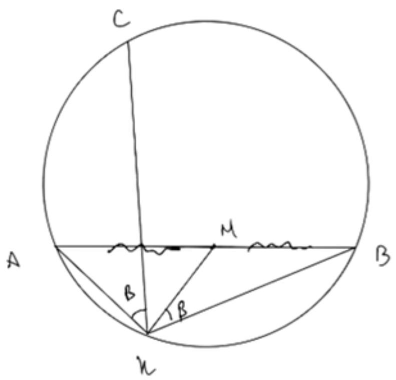


Figure 4. In the graph $\measuredangle CKA = \measuredangle BKM$

- An incenter and excentre can be useful since they are the points of convergence of bisectors, and bisectors divide an angle into two equal portions.

- An isosceles triangle is formed by any point on the perpendicular bisector of *AE*.

- In any triangle *ABC,* if *D* is a point such that the symmedian with respect to *B* is *BD,* and *M* is the midpoint of *AC,* then angles *ABD* and *MBC* are equal.

Techniques to Finding Equal Sides

This type of problem involves students demonstrating the equality of two sides of a figure that is constructed by them. In the following, it will be finding an example of a problem in this style that appeared in the Central American and Caribbean Mathematics Olympiad.

Figure 5. The first equal segment wanted problem appeared in the Central

Let ABC be an acute-angled triangle. C_1 and C_2 are two circles of diameters AB and AC, respectively. C_2 and AB intersect again at F, and C_1 and AC intersect again at E. Also, BE meets C_2 at P and CF meets C_1 at Q. Prove that $AP = AQ$. (Central American and Caribbean Mathematics Olympiad, 2000)

American and Caribbean Mathematics Olympiad. Source: Central American and Caribbean Mathematics Olympiad, 2000)

- Find a special side or measure and show that each of the sides is equal to this special side.
- Analyze the position of the sides because if they share a point or are the sides of a triangle you should focus on those points.
- Try to apply angle chasing, it depends a lot on the position of the sides, but it is useful.
- It is very useful to look at congruences with the sides.
- If there are no congruences create them by moving triangles or doing translations.
- Create parallelograms
- In the case where there is similarity between triangles formed by the ends of the sides, it is advisable to use the center of similarity by selecting one of each side.
- If there are moving dots then apply animation
- Use radical axis in point circles if two sides have a common vertex.

Patterns for Demonstrating Collinearity

These problems require students to demonstrate that three or more points are collinear. Here is an example of a problem of this type.

Consider ABC as an acute triangle, and G as the intersection of the medians of triangle ABC. Let D be the foot of the height measured from A to BC. Draw a line parallel to BC and touching point A. Suppose that the point S is the intersection of the parallel line that passes through A and the excircle of the triangle ABC. Show that S, G, D are collinear (Azerbaijan Junior National Olympiad, 2022).

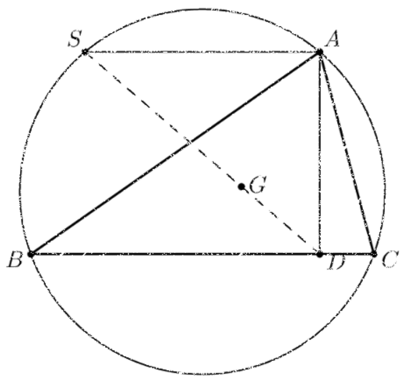


Figure 6. Mathematical Olympiad problem requiring proof that three points are collinear. Fuente: Azerbaijan Junior National Olympiad, 2022

- The points *A*, *B* and *C* are taken. Then, a line is drawn between *A* and *B*. In this instance, the strategy is to provide facts supporting the assertion that point *C* must be located on the line connecting *A* and *B*.
- Create a point *C'* such that *A*, *B* and *C'* are collinear and satisfy the requirements of the problem and then determine the conditions for point *C* to be the same as point *C'*.
- Identify properties related to collinearity that can assist in demonstrating the result, for example, in comparing the collinearity of circumcenter, orthocenter, and centroid.

Procedures for Proving Concyclic Points

Four or more points must be shown to be collinear in this type of problem. There is also a simpler case that involves proving four collinear points by demonstrating that the quadrilateral is cyclic. As an example, the following is a type of problem.

In a scalene triangle ABC, let K be the intersection of the angle bisector of $\angle A$ and the perpendicular bisector of BC. Prove that the points A,B,C,K are concyclic.

Figure 7. Mathematical Olympiad problem requiring proof of concyclic points.
Source: AoPS Blogroll Collection

- Observe those angles that are repeated in the use of angle chasing to form cyclic quadrilaterals.
- Instead of proving all points at once, it might be best to prove 4 points first. For example, if it is required to demonstrate that 5 points are concyclic, attempt to demonstrate 4 of them first. To put it another way, consider points *A*, *B*, *C*, and *D*, and then 4 other points *A*, *B*, *C*, and *E*; perhaps proving that the four points above are very similar to one another.
- Using known circles to demonstrate collinearity is recommended because some circles pass through known points.
- Those points forming 90° with the same hypotenuse or diameter are concyclic.
- In order to prove that a set of points is concyclic, the circumcenter must be determined by building triangles. If, for instance, *A*, *B*, *C*, and *D* are points and it is desired to determine whether they are concyclic, two triangles must be constructed with the points. Those points are concyclic if the circumcenters of both triangles are the same.

Based on observations of the strategies used to solve some geometry problems in the Math Olympiads, and emphasizing the mathematics education of geometry in secondary schools in Honduras, the following research question emerges and will be the main focus of this study.

> How would the implementation of Two-column demonstrations in Math Olympiad Geometry Problems impact students?

OBJECTIVES

The key objectives of the study are summarized as follows

1. Assess the baseline knowledge of students before the intervention.
2. Analyze the academic development of students when applying Two-column demonstrations to solve problems.

3. Examine the functionality of two-column demonstrations among Math Olympiad students.

METHODS

According to the topic to be investigated and the stated objectives, the following research has a mixed approach, since, Hernandez (2018) states that “a mixed approach is a set of processes of collection, analysis and linking of quantitative and qualitative data in the same study.” (p. 534). Similarly, the scope that it carries out is experimental, since the experiment to be carried out has a start test and a final test.

For this part Hernandez (2018) states that “In pre-experiments, a group is given a pre-stimulus test or experimental treatment, then the treatment is administered and finally a post-stimulus test is applied.” (p. 136). Finally, this study employs a descriptive approach, aiming to illustrate and comprehend the posed problem, taking into consideration certain students’ solutions, rather than encompassing the perspective of every participant.

Population and Sample

The study it was carried out on students of the Mathematics Olympiads from secondary schools in Honduras located in the departments of Atlántida, Choluteca, Copán, El Paraiso, Francisco Morazán, Gracias a Dios, Lempira, Santa Bárbara and Valle. At the beginning of the research, 32 students participated (6 females and 26 males). However, in that test some students were eliminated so that 13 students (6 females and 7 males) participated in the final test and in the intervention period.

Instrument/Sampling

Two instruments have been used in this research, firstly a common survey to identify students with basic geometry skills and a pre-experiment with identified students consisting of two tests analyzed using Newman's error method (see Newman A., 1983). In the following table, Newman's error indicators are listed.

Table 1. *Indicator of student’s error.*

No.	Newman Procedure	Indicator
1.	Reading the problem (reading)	a. Students can read or recognize symbols or keywords in question b. Students interpret the meaning of every word, term or symbols in the matter
2.	Comprehend the problem (Comprehension)	a. Students understand what is known b. Students understand what is being asked
3.	Transformation of the problem (Transformation)	a. Students know what formulas will be used to solve the problem b. Students know the counting operation that will be used c. Students can create a mathematical model of the problem presented
	Process Skill	a. Student know the procedure or steps that will be used to solve the problem

		b. Students can explain the procedure or steps used to solve problem c. Students can find the final result according to the procedure or the steps used to solve the problem
4.	Writing of the final Answer (Encoding)	a. The student can show the final answer of the problem solving b. Students can write the final answer in an accordance with the conclusion in question

Data Collection Techniques

During the first stage of the analysis, the focus was on identifying students with elementary capabilities in Geometry. At the pre-experimentation stage, the in-depth analysis focused on the procedures used by the students to solve the proposed problems before and after the implementation of the two-column demonstrations. In other words, the analysis involved identifying the indicators of Newman's errors when students used double column demonstrations and when they did not (see Table 1).

- Stage 1: A survey consisting of 33 items was applied to obtain data concerning the students' knowledge of geometry. Among the categories to be considered, the elementary knowledge of the students was recorded in topics related to angles in parallel lines, classification of triangles, remarkable points and straight lines in a triangle, elements of the circumference and angles at the circumference. All students who scored over 65 points were taken. (see in Appendix B)

The phases described below are part of the pretest/posttest design for a single group. Stage 2 presents a pretest G with a control group 0_1 prior to an experimental treatment. After the stimulus, a posttest X with control group 0_2 is presented in stage 3.

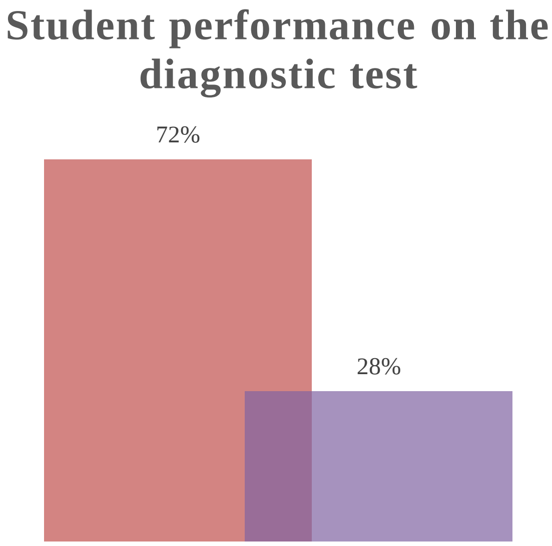
- Stage 2: On a different day, a first test was administered before the intervention period consisting of an Olympic test of two problems to be solved in approximately two hours in which students were instructed to answer the problems with the knowledge demonstrated in phase 1.

- Stage 3: A two-month intervention period was introduced where students were taught geometry problem solving strategies (see Introduction), then, a second test was administered; this test consisted of a geometry problem to be solved by the student using prose writing or two-column demonstration.

RESULTS AND DISCUSSION

Initial Survey

A total of 32 students nationwide participated in this study, in phase 1 the total student population has been considered for the analysis, and the results show the following



On the diagnostic test, 72% (23/32) of the participants scored above 60 points out of 100, with the remaining 28% (9/32) scoring below 60 points. To continue with the study, we have taken all of the students who obtained a score above 65 points, which represents 43.7% of the students (14/32).

- *Pre-experiment*

Pre-exam

Upon reviewing the answers submitted by the students (0_1) and comparing them with the indicators of Newman's error analysis, it appears that they made errors on both proposed problems, as shown in the table 2

Table 2. *Percentage of errors made by students in the pre-exam based on Newman Error.*

Category of Newman Error	Geometry Pre – Test		Total
	1*	2*	
Reading Error	14.2%	0%	14.2%
Comprehension Error	7.1%	7.1%	14.2%
Transformation Error	85.7%	0%	85.7%
Process Skill Error	100%	0%	100%
Encoding Skill Error	92.8%	0%	92.8%

As a matter of clarification, the percentage indicates the total number of students who had Newman errors. As an example, if a student was detected with a reading error in problem one (1*), then that student is no longer counted in problem two (2*). As a result, the percentages shown in problem 2 relate to students who presented a Newman error for the first time during the investigation.

Furthermore, it is important to note that there was no use of two column demonstrations in this problem as well as in the entire exam. Instead, they used a prose exposition. Below are the problems applied to the pre-test and an analysis of the solutions students presented.

Problem 1. Collinear points & equal segments

(1) Let ABC be an acute-angled triangle, Γ its circumcircle and M the midpoint of BC. Let N be a point in the arc BC of Γ not containing A such that $\angle NAC = \angle BAM$. Let R be the midpoint of AM, S the midpoint of AN and T the foot of the altitude through A. Prove that R, S and T are collinear.

(1) Let α and ω be two circles such that ω passes through the center of α. ω intersects α at A and B. Let P be any point on the circle ω. The straight lines PA and PB intersect α again at E and F respectively. Prove that $AB = EF$.

Figure 7. Problem 1 applied options in the first pre-experiment test

- *Common writing style of students*

To begin with, only those students who showed transformation error (85.7%) wrote a procedure to solve the problem, among which the following are the most important ones

- *A solution that consists only of the figure's drawing*

The students who presented these solutions represented 35.7% (5/14) of the group. In general, these solutions presented the construction of the figure described in the problem without attempting to resolve the problem in any other way. Some examples are shown below

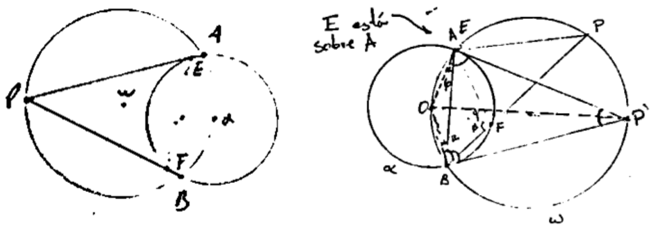


Figure 8. Proposed solution illustrating the problem. Source: based in students answers.

- *A calculation sheet containing only the mathematical approach*

The solutions presented by seven students (50%) contained the construction of the figure described in the problem as well as a series of mathematical calculations, but with no specific wording, order, or conclusion as shown by the encoding error (92.8%). In order to understand this, a procedure of this type is presented

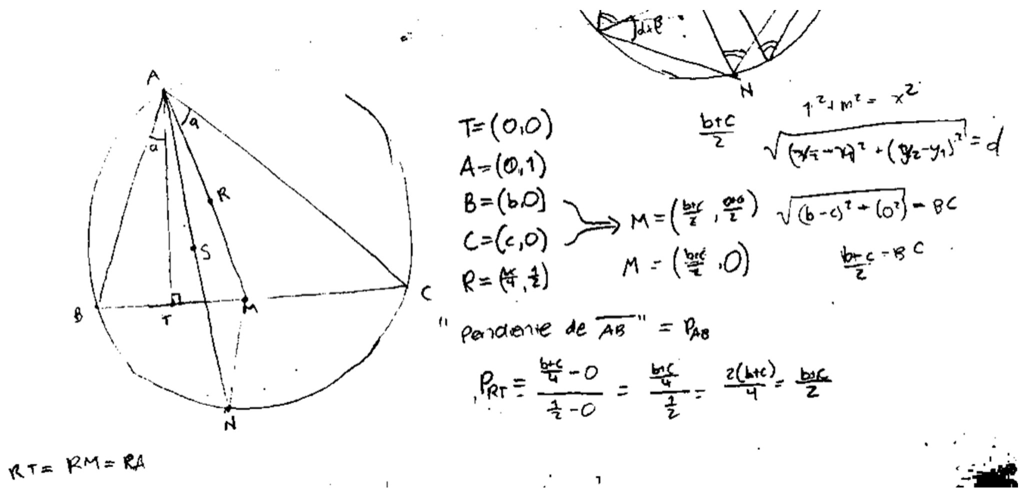


Figure 9. Solution presented by a student showing only mathematical calculations. Source: based on student responses.

Upon inspection, it is clear that this is a draft of the problem, as it illustrates both the figure of the problem and an unwritten solution. Last but not least, the other two students left the problem blank.

Problem 2. Twin Segments

(2) In the acute triangle ABC, $\angle A = 45^o$. Points O, H are the circumcenter and orthocenter of ABC, respectively. D is the foot of altitude of B. Point X is the midpoint of arc AH of the circumcircle of triangle ADH containing D. Show that $DX = DO$

Figure 11. problem 2 proposed in the pre-test

This problem was answered by a small number of students (6/14) corresponding to the 42.8% because they spent so much time doing the problem 1. However, the solutions presented will be shown, among which the following are highlighted

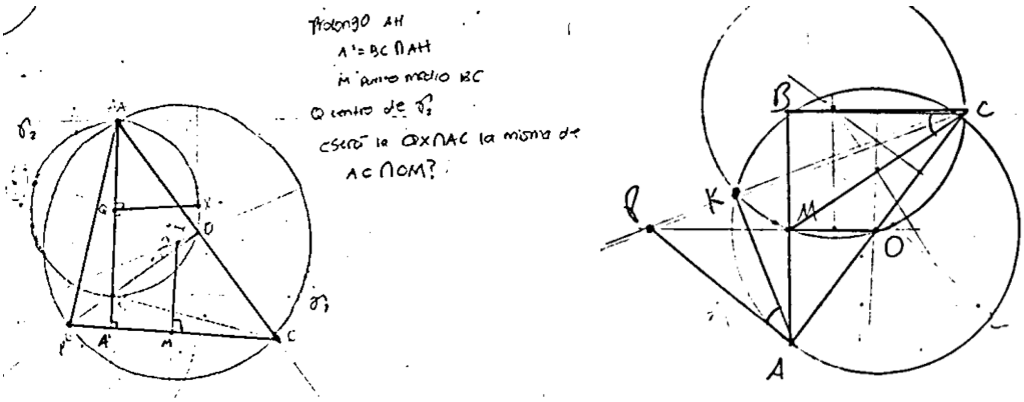


Figure 12. Student-submitted solutions to pre-test problem 2. Source: based on students' answers.

Based on the solutions presented for problem 2, it is evident that 100% of these solutions were drafts of the problem provided by the students. This indicates that the students did not complete the problem within the time limit

Final Test

As a result of the pre-exam, an intervention process was conducted using geometry problem-solving strategies described in the introduction and demonstrating how to solve problems using two columns. Based on the national test of the Virtual Mathematics Competition 2022 described in (Cerros, E., et at, 2022) the ability of students to use these demonstrations was assessed, the results of which will be presented below.

Problem 5 Let Γ be a circumference with diameter AB, and C at the circumference. The line through point B perpendicular to BC intersects the angle bisector of $\angle ACB$ at the point D. E is the foot of the altitude from B to AD and M is the midpoint of the segment CD. Show that B, D, E, and M are concyclic.

Figure 13. Problem used as a final exam in this study

Table 3. *Percentage of errors made by students in the final exam based on Newman Error.*

Category of Newman Error	Geometry Final Test *CVM 2022 - Problem 5*	Total
Reading Error	7.1%	7.1%
Comprehension Error	14.2%	14.2%
Transformation Error	28.5%	28.5%
Process Skill Error	35.7%	35.7%
Encoding Skill Error	35.7%	35.7%

In this second test, students show fewer Newman errors in Process Skill and Transformation, which indicates that students had ideas on how to proceed with their solution after it had been transformed into mathematical elements. To perform a more detailed analysis, a table will be displayed listing the participants *(0_2)* and the types of solutions they presented.

Table 4. *Write-ups of problem 5 of the Virtual Mathematics Competition*

Participants *(0_1)*	Type of solution written by the participant
J_1	Prose style writing
J_2	Two – Column demonstration writing
J_3	Two – Column demonstration writing
J_4	Prose style writing
J_5	Prose style writing
J_6	Two – Column demonstration writing
J_7	Non-written wording
J_8	Two – Column demonstrations & Prose style writing
J_9	Blank wording
J_{10}	Prose style writing
J_{11}	Prose style writing
J_{12}	Prose style writing
J_{13}	Blank wording
J_{14}	Blank wording

The blank solutions represent students who did not answer the problem and, therefore, were not included in the final analysis shown in Table 3. In the following sections, we will present solutions that were solved using two-column demonstrations

- *J_2's attempted solution*

The solution presented by J_2 shows the data written in two – columns demonstrations and an almost forced attempt to draw a direct conclusion from the data, as shown below.

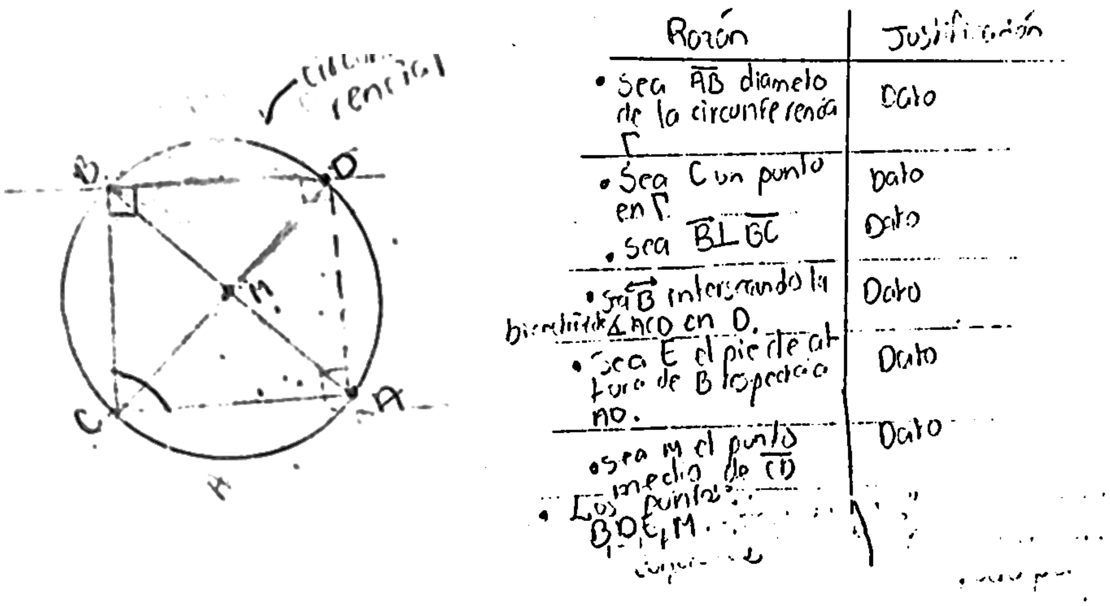


Figure 14. Two – column demonstration that only reflects the data of the problem. Source: based on student responses.

Authors such as (Weiss M., et al, 2009) state that something must be demonstrated in order to proceed to the next step in a two-column demonstration. As for the latter, it can be stated that J_2 lacked clear ideas for demonstrating based on the information provided.

- *J_3's nearly correct solution*

In the solution presented by J_3, he shows the figure and the wording written in two column demonstrations

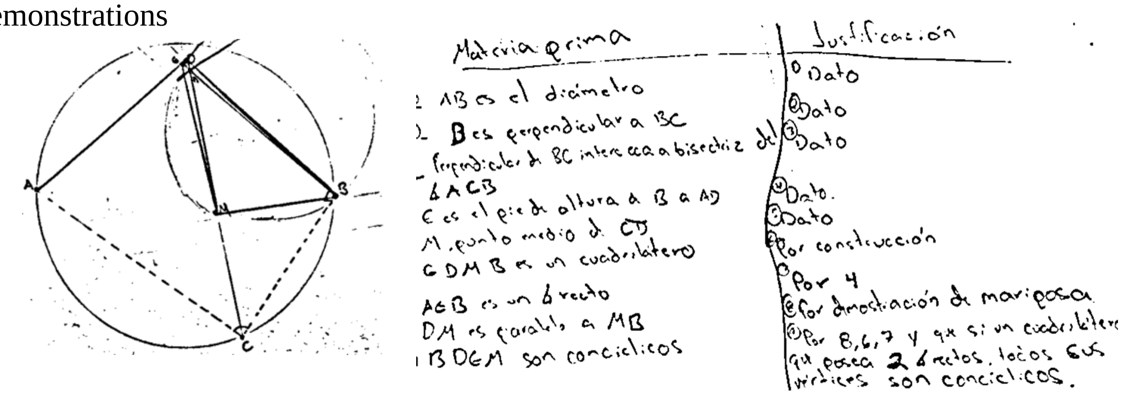


Figure 15. Solution presented by J_3. Source: based on students' answers.

In order that J_3's solution can be understood by the readers, a translated version will be provided in Table 5.

Table 5. *Translation of the Two – Column demonstrations solution presented by J_3*

N°	Statement	Reason
1	The segment *AB* is the diameter	Given
2	The line thought point *B* is perpendicular to *BC*	Given
3	The line perpendicular to *BC* intersects the	Given

	angle bisector of ACB	
4	*E* is the foot of altitude from *B* to *AD*	Given
5	*M* is the midpoint of the segment *CD*	Given
6	*GDMB is a quadrilateral*	
7	AEB =90	Definition of foot of the altitude and step 4
8	*DM* is parallel to *MB*	Definition of the butterfly theorem
9	The points *B*, *D*, *E* and *G* are concyclic	Based on steps 8, 7, 6 and the fact that if a quadrilateral has two right angles all its vertices are concyclic

In the same manner as J_2, J_3 started by drawing a diagram and writing down the data shown in the two-column demonstration. After that, he tried to demonstrate that a set of points lies within the circle shown in the introduction. In general, he starts from step 7 by finding at least two right angles with the same diameter and states that *A*, *B*, *C,* and *D* are cyclic if *ABC* and *ADC* are 90 degrees.

Based on that idea, he states in step 7 that the *AEB* angle is straight (90˚) as defined by the foot of the altitude, which is correct. Nevertheless, the problems begin at step 8, when J_3 mentions that *DM* is parallel to *MB* and justifies this by using the butterfly theorem. As you may know, it is immediately false because two parallel segments should not share the same point, which in this case is M. Furthermore, the butterfly theorem can only be applied when it has been established that a quadrilateral is cyclic.

Finally, he immediately states that *B*, *D*, *E* and *G* are concyclic, which would not have been correct even if step 8 had been error-free because with his idea he was only demonstrating that "If angles *AEB* and *DMB* are 90˚, then *A*, *B*, and *D* are cyclic" which is trivial and does not even need to be demonstrated.

- *An analysis of the graph construction proposed in J_6's solution.*

The most common error in solving Geometry problems in Mathematics Olympiads is to rely solely on the construction of the figure, ignoring the theoretical definitions, which should not be overemphasized. The majority of J_6*'s* errors are due to the construction of the figure. This poses some errors, and on the basis of that similar errors are made throughout the demonstration.

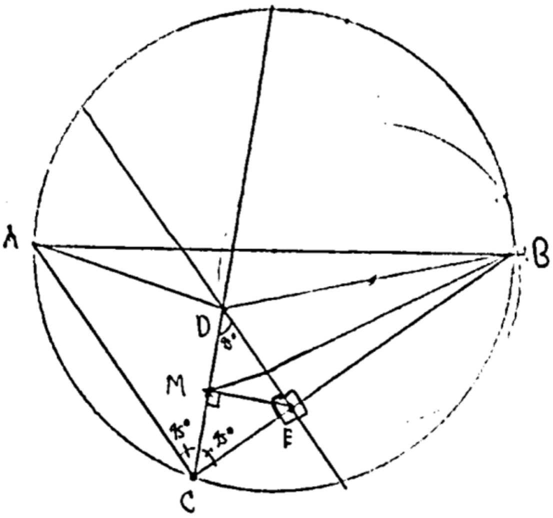


Figure 16. Representative graphic made by J_6. Source: based in the student's answers

Taking a look at the original problem and what a drawing should contain, we notice the following about the J_6 graphic:

- The line through *B* that is perpendicular to *BC* does not exist.
- In the graphic showed the point *E* is not the foot of the altitude from B to *AD*
- There is a line perpendicular to *BC*, but from what can be observed, the student believed that the line through *B* and the line perpendicular to *BC* are the same.

Based on all these observations, it is evident that J_6's solution involving two – column demonstrations is incorrect. The triggering factor in this case was not the use of two – column demonstration, but rather the drawing's inadequate construction.

Prose-Style-Two-Column demonstrations

One of the most elegant solutions reported in (Cerros, E., et al, 2022) was from J_8, a student considered in the sample, her solution includes a prose demonstration but within a two – column demonstration. The style and authenticity of the solution is unique.

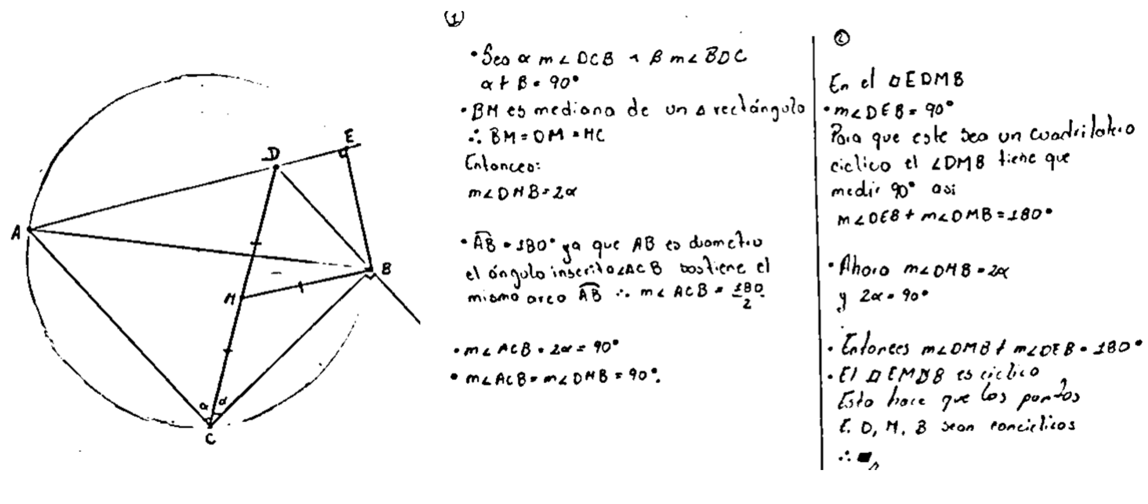


Figure 17. Prose-style-two-column demonstration solution. Source: based of the students' responses

Table 6. *Translation of the Prose – Style – Two – Column demonstrations solution presented by J_8*

Part 1	Part 2
• Let α measure of the angle *DCB* and β the measure of the angle *BDC* • $\alpha + \beta = 90°$	In the □EDMB • $m\measuredangle DEB = 90°$ For this to be a cyclic quadrilateral the $\measuredangle DMB$ has to order 90° as follows $m\measuredangle DEB + m\measuredangle DMB = 180°$
• The line *BM* is median of a right triangle ∴ *BM* = *DM* = *MC* As a consequence, $m\measuredangle DMB = 2\alpha$	• Now, $m\measuredangle DMB = 2\alpha$ As we already know, $2\alpha = 90°$
• The arc AB is equal to 180° because *AB* and the inscribed angle containing ∡*ACB* contains the same arc *AB* $\therefore m\measuredangle ACB = 90°$	• Thus, $m\measuredangle DMB + m\measuredangle DEB = 180°$
• $m\measuredangle ACB = 2\alpha = 90°$ • $m\measuredangle ACB = m\measuredangle DMB = 90°$	• ∴ □EDMB is cyclic. This in turn makes points E, D, M and B cyclic.

As seen in the solution shown, J_8 arranges the solution in part 1 and part 2 where part 1 represents everything that is extracted directly from the problem data and part 2 in how this extracted information assists in proving that the quadrilateral formed by the points *E*, *D*, *M* and *B* is cyclic.

The black dots represent a step in the demonstration, which precedes each previous point, this demonstration is considered two – column because the style is the same. However, it is also considered a prose style demonstration because the justifications for each step are written indirectly. J_8's solution is a very uncommon style of writing problem solutions in Math Olympiads. In fact, solutions like this may even represent a new type of writing in Mathematics Olympiads

CONCLUSIONS

In the survey, 43.7% of the students demonstrated knowledge of angles in parallel lines, classification of triangles, remarkable points and straight lines in a triangle, elements of the circumference and angles at the circumference.

In the pre-experiment, prior to the experimental treatment in pre-test *G*, it was observed that the control group 0_1 had a minimal understanding of Euclidean geometry, but no practical methods or techniques to solve problems, in other words, they had no notion of how to proceed with the demonstration.

Once the post-test *X* was applied, it was observed that the control group 0_2 ordered the information contained in the problem better and were able to draw conclusions from this; however, it is important to note that some students were unable to solve problems using techniques.

This type of demonstration serves as a tool for helping students sort the data presented in the problem. However, at the same time, it makes no sense to sort the data correctly if the students do not understand the mathematical strategy for solving a particular problem. Only students who are familiar with a straightforward strategy for solving a particular problem will be able to use two-column demonstrations effectively.

ACKNOWLEDGEMENTS

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APPENDIX A. SUPPLEMENTARY DATA

Cerros, E., Aguilera, M., Luna, F., Roy-Choudhury, M., Martinez, A., Oziel, E., Callison, L., Vasquez, L., & Romero, D. (2022), “A collection of solutions created by participants in the Virtual Mathematics Competition 2022”, *Mendeley Data*, DOI: https://doi.org/10.17632/8dbrr7xpxg.1

REFERENCES

Department of Mathematics, University of Toronto. (2000). Writing Up Solutions. http://www.math.toronto.edu/barbeau/writingup.pdf

Djukić, D., Janković, V., Matić, I., & Petrović, N. (2011). The IMO Compendium: A Collection of Problems Suggested for the International Mathematical Olympiads: 1959-2009 Second Edition. Springer New York.

Bellanca, J., & Stirling, T. (2011). Classrooms without borders: Using internet projects to teach communication and collaboration. *Teachers College Press.*

Chen, E. (2021). Euclidean geometry in mathematical olympiads (Vol. 27). American Mathematical Soc.

Engel, A. (2008). Problem-solving strategies. Springer Science & Business Media.

Ramos Palacios, L. A. (2006). Una estrategia metodológica para desarrollar olimpiadas matemáticas en el nivel medio del sistema educativo hondureño. [Tesis de Maestría, Universidad Pedagógica Nacional Francisco Morazán].

Hang, K. H., & Wang, H. (2017). Solving problems in geometry: Insights and strategies for mathematical Olympiad and competitions (Vol. 10). World Scientific Publishing Company.

Herbst, P. G. (2002). Establishing a custom of proving in American school geometry: Evolution of the two-column proof in the early twentieth century. *Educational Studies in Mathematics*, 49(3), 283-312. https://doi.org/10.1023/A:1020264906740

Idris, R. (2017). Mengatasi kesulitan belajar dengan pendekatan psikologi kognitif. *Lentera pendidikan: jurnal ilmu tarbiyah dan keguruan*, 12(2), 152-172. https://doi.org/10.24252/lp.2009v12n2a3

Moncada, J. M. (1904). *Educación, trabajo y ciencia: (método de enseñanza integral)*. Retrieved from https://www.cervantesvirtual.com/nd/ark:/59851/bmcpk120

Newman, A. (1983). The Newman language of mathematics kit: Strategies for diagnosis and remediation. Sydney: Harcourt Brace Jovanovich Group.

Pérez, G. (1996). Historia de la educación superior de Honduras: antecedentes históricos 1733 – 1847

Pérez, G. (1997). Historia de la educación Superior de Honduras: Ciento cincuenta años de vida universitaria

Portillo, A. (2003). La Educación Superior en Honduras 1949-2000 Vol. III. Bosquejo Histórico de las Unidades Académicas (pp. 1-239).

Robert Osserman, Are proofs in high school geometry obsolete? J. Math. Behavior, to appear.

Valencia, M. R. M. (2014). Honduras: Origins, Development, and Challenges in the Teaching of Mathematics. Mathematics and Its Teaching In The Southern Americas: With An Introduction By Ubiratan D'ambrosio, 10, 265. https://doi.org/10.1142/9789814590570_0011

Weiss, M., Herbst, P., & Chen, C. (2009). Teachers' perspectives on "authentic mathematics" and the two-column proof form. *Educational Studies in Mathematics*, 70(3), 275-293.

Wu, H. (1996). The role of Euclidean geometry in high school. *Journal of mathematical behavior*, 15(3), 221-238.

Xiong, B., & Lee, P. Y. (Eds.). (2007). Mathematical Olympiad in China: problems and solutions. World Scientific

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APPENDIX B. APPLIED SURVEY

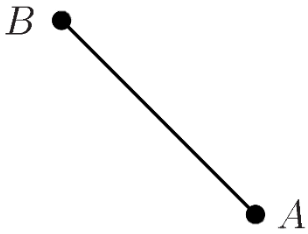
Competencia Virtual de Matemáticas
Geometry Diagnostic Olympiad Assessment
Name and Surname: ______________________
E-mail: ______________________

1. *Fundamental Concepts*

Determine the geometric notion and label it beneath the illustration

1

2



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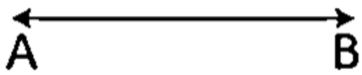

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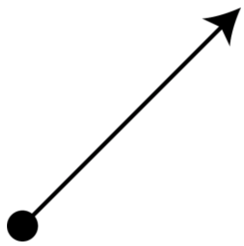
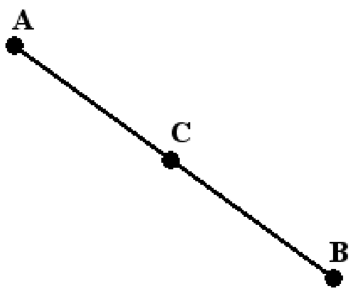


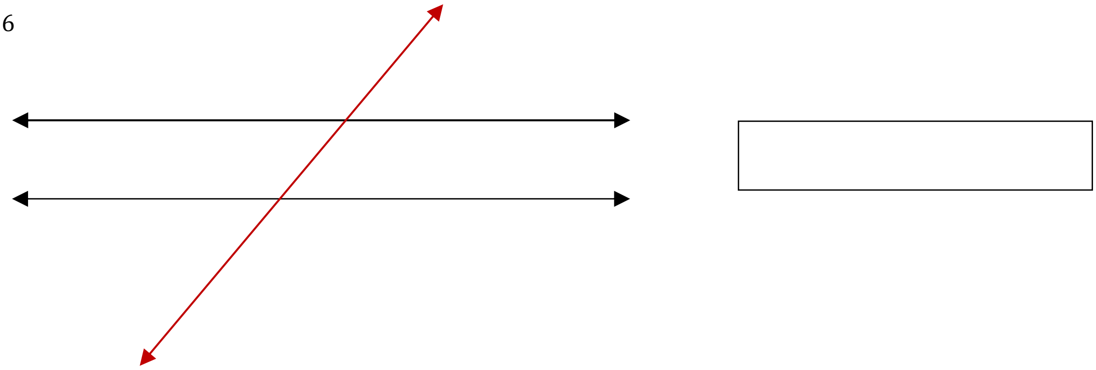
If $AC = BC$ then C is:

5



What is the name of the red line?

6



2. ***Geometric Properties***

Provide the appropriate sentence completion for the geometric element mentioned in the sentence.

7. If 3 or more points are on the same line, they are: ________________________________
8. If we have 3 lines passing through the same point, they are: _______________________
9. If we extend two lines and they never intersect, they are: __________________________
10. If the intersection of two lines forms a 90-degree angle, the lines are: ______________

3. ***Triangle Properties and Important Points of Triangles***

Circle the correct answer.

11. Equilateral triangles have angled that measure:

a) 60 degrees
b) 40 degrees
c) 90 degrees
d) 120 degrees

12. Which distinctive theorem can be applied to find the hypotenuse of a right triangle?

a) Thales' Theorem
b) Pythagorean Theorem
c) Fermat's Little Theorem

Complete the statement by writing the missing word or number.

13. The sum of the interior angles of a regular polygon is 2,880. How many sides does the polygon have? ___________________________

14. The sum of the interior angles of a triangle is ___________________________________

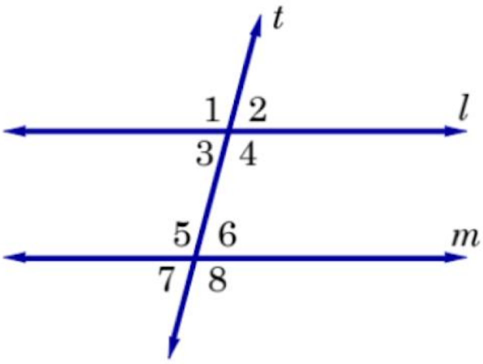
15. The __________________________ is the line or line segment where the lines intersect that divide an angle into two equal parts.

16. The __________________ is the line that extends from a vertex of a triangle to the point where it forms a 90-degree angle with the opposite side or its extension.

17. The ____________________ is the line that passes through the midpoint of a segment and forms 90-degree angles with it.
18. The _____________________ is the segment that connects a vertex of a triangle with the midpoint of the opposite side.
19. The ___________________ is the point where the angle bisectors of a triangle intersect.
20. The ___________________ is the point of intersection of the altitudes of a triangle.
21. The ___________________ is the point where the medians of a triangle intersect.
22. The ___________________ is the center of the circle circumscribed around a triangle.

4. *Angles in Parallel Lines*

Select all the options that apply.



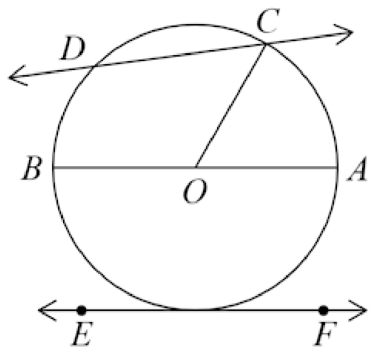
	∡ 1 = ∡ 5	∡ 2 = ∡ 6	∡ 3 = ∡ 7	∡ 4 = ∡ 8	∡ 3 = ∡ 6	∡ 4 = ∡ 5	∡1 = ∡8	∡2 = ∡7
Angulos Correspondientes	☐	☐	☐	☐	☐	☐	☐	☐
Angulos Alternos externos	☐	☐	☐	☐	☐	☐	☐	☐
Angulos Alternos Internos	☐	☐	☐	☐	☐	☐	☐	☐

5. Elements of the Circumference

Identify the terms of the image.

23

24



26 $\overline{CO}$

(a) Secant (b) Chord (c) Radius (d) Diameter (e) Tangent

27 $\overleftrightarrow{EF}$

(a) Secant (b) Chord (c) Radius (d) Diameter (e) Tangent

28 $\overline{CD}$

(a) Secant (b) Chord (c) Radius (d) Diameter (e) Tangent

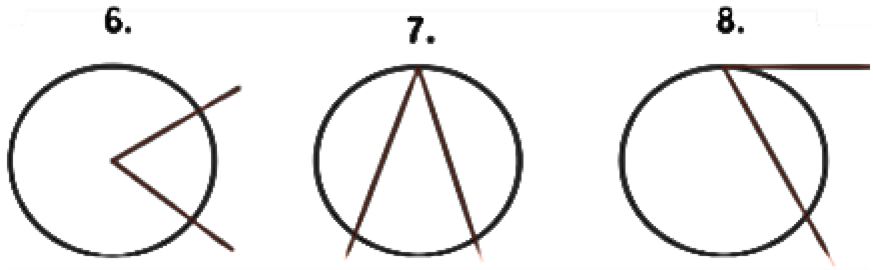
29 $\overleftrightarrow{EF}$

(a) Secant (b) Chord (c) Radius (d) Diameter (e) Tangent

30 $\overline{AB}$

(a) Secant (b) Chord (c) Radius (d) Diameter (e) Tangent

6. Angles at the Circumference



31. What is the angle represented beneath 6?

(a) Central angle (b) Inscribed angle (c) semicircular angle

32. What is the angle represented beneath 7?

(a) Central angle (b) Inscribed angle (c) semicircular angle

33. What is the angle represented beneath 8?

(a) Central angle (b) Inscribed angle (c) semicircular angle