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Comment

Comment on Stilmant et al. Flow at an Ogee Crest Axis for a Wide Range of Head Ratios: Theoretical Model. *Water* 2022, *14*, 2337

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Abstract: Critical flow in irrotational motion is important in theoretical hydrodynamics and dam hydraulics. Therefore, the commented paper is relevant in theory and practice. It deals with an approximation for critical flow based on a set of simplified irrotational flow equations in the gravity field. The underlaying model equations were found by the discussers to strongly rely on Jaeger's work. Therefore, some important aspects need a detailed clarification. Jaeger's velocity profile was determined here by a possibly novel procedure starting from the irrotational flow relations in the complex potential plane. It was shown that, though not perfect, a theory assuming critical crest conditions gives consistent estimates of the discharge coefficient, crest flow depth, bottom pressure head, and velocity profile. A new method for computing the flow profile over an ogee crest is presented by simultaneous determination of the discharge coefficient and the real critical point position using the Bélanger–Böss theorem, resulting a physically based determination of the critical point in spillway flow. It is demonstrated that Jaeger's curvature parameter K is not a universal value, such that neither the current comment nor the discussed paper are therefore "free" from empirical parameters.

Keywords: critical point; irrotational flow; ogee profile; spillway

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The topic of critical flow in irrotational motion is important in theoretical hydrodynamics, and results in tools relevant in practice, given the utility in dam hydraulics. Therefore, the discussed paper [1] is both pertinent in theory and practice. It deals with an approximation for critical flow based on a set of simplified irrotational flow equations in the gravity field. The underlaying model equations were found by the discussers to strongly rely on Jaeger's work [2,3]. His theory was specifically applied to an ogee crest by Castro-Orgaz and Hager [4], in Appendix F of this book. Given the similarities of the paper developments with previously available results not commented on, in the discussers' opinion, some important aspects need a detailed clarification. The comment is rather long, given new material presented not available in Castro-Orgaz and Hager [4]. It should not be understood as a dismissal of the article contribution; rather, it is intended to complement it and clarify important aspects of the topic.

The comment is structured as follows: (i) Jaeger's theory for irrotational flow over curved beds, (ii) application: flow over an ogee profile, and (iii) concluding remarks.

1. Jaeger's Theory for Irrotational Flow over Curved Beds

The authors based their work in Equation (16) on the commented paper defining the bed-normal variation of the velocity profile component parallel to the bed. All the theory and practical developments rely on this equation, and the authors refer to Peltier et al. [5] for the development of this relation. However, this velocity profile was proposed by Jaeger [2] and applied by Castro-Orgaz and Hager [4] (henceforth NHF17) in Appendix F of the book, and used to derive a complete irrotational flow theory, including the integrated discharge, specific energy, free surface profile definition, and location of the critical flow or singular point downstream of the ogee crest. Below, we detail step-by-step relevant aspects of the theory.

1.1. General Hydrodynamic Statements

Consider steady irrotational flow over an ogee spillway (Figure 1). A general re-derivation of Jaeger's [2] theory is presented here by a possibly novel procedure starting with the fundamental relations of the theory or harmonic functions, well known in classical hydrodynamics [6,7]. Let $W = \phi + i\,\psi$ be the complex potential, with ϕ and ψ the potential and stream functions, respectively, and i the imaginary unit. The logarithmic hodograph is then $\ln(\mathrm{d}W/\mathrm{d}z) = \ln V - \mathrm{i}\theta$, with V the modulus of the velocity vector and θ the inclination of the streamlines with the horizontal (Figure 1a). In the complex potential plane, $\ln V$ and θ are harmonic functions, and they must satisfy the Laplace equations [6], namely,

$$\frac{\partial^2 \ln V}{\partial \psi^2} + \frac{\partial^2 \ln V}{\partial \phi^2} = 0, \tag{1}$$

$$\frac{\partial^2 \theta}{\partial \psi^2} + \frac{\partial^2 \theta}{\partial \phi^2} = 0. \tag{2}$$

The complex potential plane and logarithmic hodograph plane are connected through the Cauchy–Riemann equations, e.g.,

$$\frac{\partial \ln V}{\partial \phi} = -\frac{\partial \theta}{\partial \psi}, \tag{3}$$

$$\frac{\partial \ln V}{\partial \psi} = \frac{\partial \theta}{\partial \phi}. \tag{4}$$

Now, consider Equation (4). Integration of it from the channel bed up to the free surface along an arbitrary curve with ϕ = const. produces

$$\frac{V}{V_s} = \exp\left(-\int_0^q \frac{\partial \theta}{\partial \phi}\mathrm{d}\psi\right) \tag{5}$$

This is a general mathematical statement for any equipotential of the flow net. Indeed, this a fundamental equation derived by Jaeger [2], as will be shown. The velocity V is given in natural coordinates (s, n) along and normal to the streamlines (Figure 1b) [8,9] by the statements:

$$V = \frac{\partial \psi}{\partial n} = \frac{\partial \phi}{\partial s}. \tag{6}$$

We seek to link the complex potential plane with the system of natural coordinates. The definition of curvature is thus needed, given by

$$\kappa = \frac{\partial \theta}{\partial s}. \tag{7}$$

Using Equations (6) and (7) in Equation (4) produces, after some elementary manipulation,

$$\frac{\partial V}{\partial n} = \kappa V, \tag{8}$$

which is the differential equation defining the variation of V with the curvilinear coordinate n measured along the equipotential curves [8–10]. It is available in Rouse [8] and

Prandtl [11], and it is a result also derived by Jaeger [2,3]. Its integral produces a fundamental result by Jaeger [2] as follows:

$$\frac{V}{V_s} = \exp\left(-\int_0^N \kappa \mathrm{d}n\right), \tag{9}$$

where N is the length of the equipotential curve. Jaeger [2] presented it changing the variable n to elevation z by using the streamline inclination angle θ. Equation (9) was used in Peltier et al. [5] without commenting on Jaeger's earlier work. Equation (9) and the resulting integrated discharge using it were clearly stated by Kozeny [12] along with Bernoulli's equation for analysis of irrotational flows.

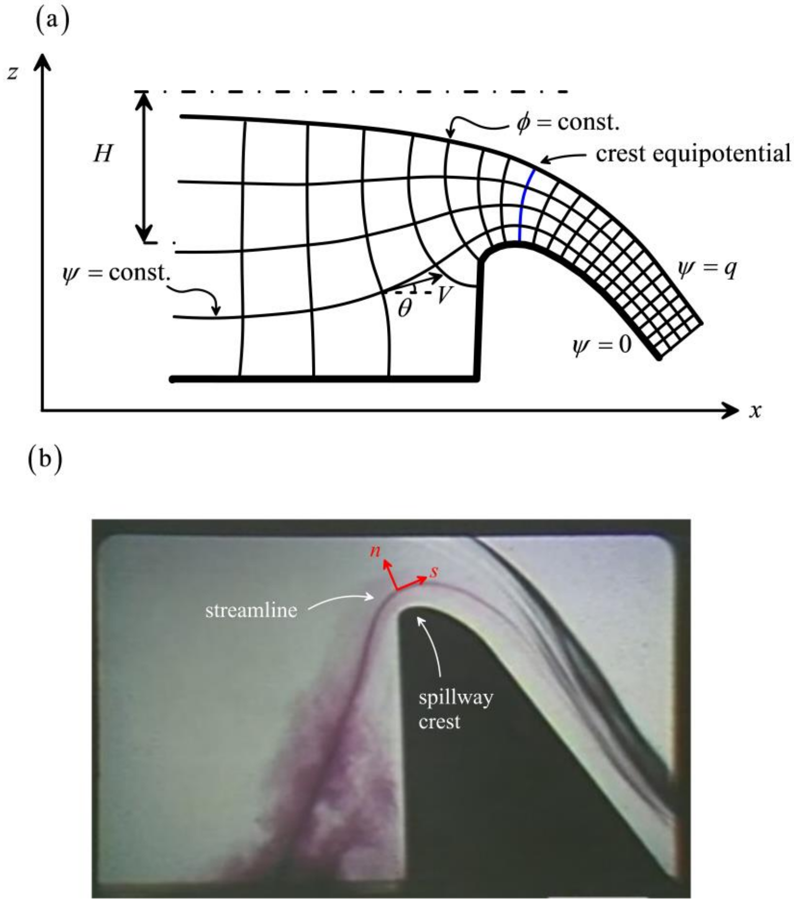

Figure 1. Irrotational flow over a spillway. (**a**) Sketch of the flow net depicting the streamlines ψ = const. and equipotential lines ϕ = const. The crest equipotential is marked in blue. The head over the ogee weir is H and the unit discharge q, determining the discharge coefficient $C_d = q/(gH^3)^{1/2}$. (**b**) ogee crest model test by Hunter Rouse, showing a streamline curving along the crest. Natural coordinates (s, n) marked using the colored streamline for reference (image courtesy of IIHR Hydroscience and Engineering, Iowa, from Hunter Rouse's educational film "Fluid motion in a gravitational field", 1961).

This elementary demonstration starting with the basic aspects of complex variables theory seems to be lacking in books and papers, and, possibly, this has limited in a way making a more general and fundamental recognition of Jaeger's irrotational developments, as in Peltier et al. [5] or the discussed paper [1]. This derivation demonstrates that Jaeger's integral Equation (9) is general and applies to any equipotential curve of the flow net sketched in Figure 1. Further merit of Jaeger [2] lies in the way he approximated analytically this integral, to be commented on below. The development for the crest equipotential curve (see blue line in Figure 1a) setting critical flow is just another aspect of his development, but we stress here that his work applies to the entire flow net, as demonstrated above. Thus, Jaeger's theory is not limited to critical flow at a weir crest, as seems to be suggested in the discussed paper; this is just another aspect of some of Jaeger's developments.

1.2. Equipotential Lines

A fundamental idea of Jaeger [2,3] also pursued in Appendix F of NHF17 is to approximate the equipotential lines by the flow depths defined normal to the channel bed, instead of vertically, thereby producing a good approximate irrotational flow model for a curved bed structure, where the bed-normal velocity component is neglected. We will use the authors' notation for ease of comparison. The situation is depicted in Figure 2.

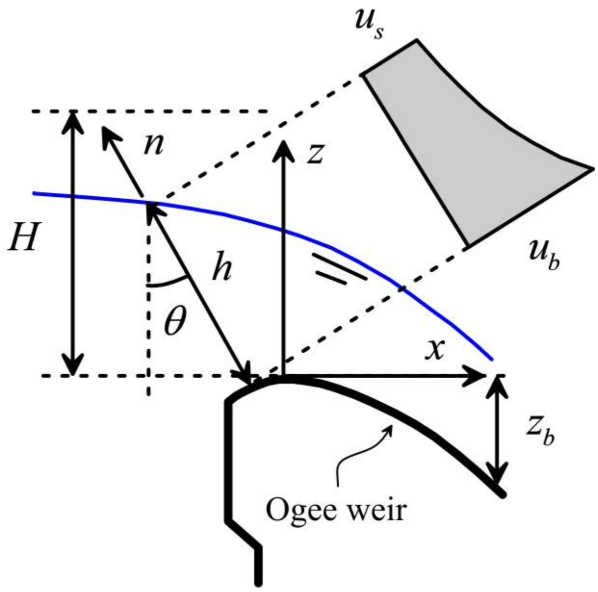

Figure 2. Potential flow over curved bottom from Jaeger [2] (adapted from NHF17); note that the bed-normal velocity component is neglected.

1.3. Taylor Expansion of Streamline Radius of Curvature

Once the shape of the equipotential lines is approximated to the bed-normals, the variation of the streamline radius of curvature $r = 1/\kappa$ in Equation (9) needs to be approximated. In NHF17, the Taylor expansion of r around the bed was considered as

$$r(\eta) = r_b + \left(\frac{\partial r}{\partial \eta}\right)_b n + \left(\frac{\partial^2 r}{\partial \eta^2}\right)_b \frac{\eta^2}{2} + \ldots\ldots \tag{10}$$

and its truncation as

$$r(\eta) \approx r_b + \left(\frac{\partial r}{\partial \eta}\right)_b \eta = r_b + K\eta. \tag{11}$$

This implies

$$K = \left(\frac{\partial r}{\partial \eta}\right)_b. \tag{12}$$

Equation (12) is equivalent to Equation (18) of the paper. This approximation was introduced in Peltier et al. [5] and the discussed paper without stressing its origin: it is the original approximation by Jaeger [2], and its interpretation as a truncated Taylor expansion was first performed by Montes [13].

1.4. Velocity Profile

Inserting (11) into Equation (9) produces [2,4]

$$\frac{u}{u_s} = \left[\frac{r_b + K\,h}{r_b + K\eta}\right]^{1/K}, \quad K \neq 1. \tag{13}$$

For convenience, we use r_b in the absolute value (NHF17), and K is Jaeger's theory parameter. Elementary manipulation permits writing of Equation (13) as a function of the bed velocity u_b, resulting in Equation (16) of the commented paper, demonstrating it is identical to Jaeger's theoretical velocity profile for flows over curved beds.

1.5. Free Surface Definition

The coordinates of the free surface points (x_s, z_s) in Figure 2 are as follows (NHF17):

$$z_s = z + \cos\theta \cdot h,\; x_s = x - \sin\theta \cdot h, \tag{14}$$

which are Equations (11) in the paper. Note that by using this definition use of curvilinear coordinates is not really needed.

1.6. Discharge and Specific Energy

Integration of Equation (13) in the direction normal to the bed was accomplished by Jaeger [2] and NHF17 as follows:

$$q = \int_0^h u\mathrm{d}\eta \;= \frac{u_s r_b}{K-1}\left[\left(1 + \frac{Kh}{r_b}\right) - \left(1 + \frac{Kh}{r_b}\right)^{\frac{1}{K}}\right], \quad \text{for } K \neq 1, \tag{15}$$

where the total energy head H is related to the surface velocity by Bernoulli's equation as follows (NHF17, Appendix F):

$$H = z_b + h\cos\theta + \frac{u_s^2}{2g} = \text{const.} \tag{16}$$

From Equations (15) and (16),

$$q = \frac{\sqrt{2g(H - z_b - h\cos\theta)}r_b}{K-1}\left[\left(1 + \frac{Kh}{r_b}\right) - \left(1 + \frac{Kh}{r_b}\right)^{\frac{1}{K}}\right]. \tag{17}$$

Equation (17) is equivalent to Equation (29) of the paper. The transformation is easily achieved using the ratio $\lambda = u_b/u_s$, and by applying Bernoulli's equation to introduce the bed pressure p_b.

The fundamental equations of the paper are Equations (16) and (29), which are equivalent to Equations (13) and (17), which we stress here are fundamental elements of Jaeger's theory [2,4,14].

2. Application: Flow over an Ogee Profile

2.1. Assumption of Critical Flow at the Ogee Crest

In the discussed paper, it is acknowledged that Jaeger's approximation for critical flow produces a good C_d estimation, but it is indicated that the prediction of the crest bottom pressure or velocity profile may not be consistent. It is argued that deviations in velocity predictions affect the model results. We comment on this view as follows based on Montes [13,14]. Setting critical flow $\partial q/\partial h = 0$ at the weir crest produces the minimum specific energy [13,14].

$$\frac{H}{r_b} = \frac{h}{r_b} + \frac{1}{2K}\frac{\left(1+\frac{Kh}{r_b}\right) - \left(1+\frac{Kh}{r_b}\right)^{1/K}}{1-\frac{1}{K}\left(1+\frac{Kh}{r_b}\right)^{1/K-1}}, \tag{18}$$

the discharge coefficient

$$C_d = \left[2\left(1-\frac{h}{H}\right)\right]^{1/2}\left(\frac{H}{r_b}\right)^{-1}\frac{1}{K-1}\left[\left(1+\frac{Kh}{r_b}\right) - \left(1+\frac{Kh}{r_b}\right)^{\frac{1}{K}}\right], \tag{19}$$

and the crest bottom pressure

$$\frac{p_b}{\rho g H} = 1 - \left(1-\frac{h}{H}\right)\left(1+\frac{K\,h}{r_b}\right)^{2/K}. \tag{20}$$

Montes [13] detailed these derivations, also available in NHF17. Now, these relations are compared with Hager's [15] measurements in Figure 3 using $K = 2.2$, as proposed by Montes [13] and Castro-Orgaz and Hager [4]; the comparison includes C_d, h/H_D, $p_b/(\rho g H)$ and the velocity profile $u/(2gH_D)^{1/2}$. The results for the first three quantities are a rejoinder of results by Castro-Orgaz and Hager [4], presented here for convenience to depict the overall behavior of the approximation. Velocity profiles are newly compared here. Note that the discharge coefficient, crest water depth, and bottom pressure here are reasonably well modeled. Further, the predicted velocity profile is also in reasonable agreement with observations. This demonstrates that this approximation to Jaeger's theory is consistent. Of all quantities, that showing largest deviations from experiments is the water depth, but the model is not inconsistent, as seemed to be suggested in the discussed paper. For this level of approximation, the predicted ratio of crest flow depth to energy head from our results is $h/H = 0.687$ on average. As Hager [15] found, experiments indicate $h/H = 0.75$ on average, the discrepancy of theory being since flow conditions at the crest are not truly critical. Flow profile computations detailed in the next section were used to obtain the ratio h/H at the crest from a more rigorous theoretical treatment, and it was found to be close to 0.75, as experiments indicate.

With regard to practice, Hager [15] proposed the semi-empirical equation

$$C_d = \left(\frac{2}{3}\right)^{3/2}\left(1+\frac{4\frac{H}{H_D}}{9+5\frac{H}{H_D}}\right), \quad \frac{H}{H_D} < 3, \tag{21}$$

which is recommended in dam hydraulic practice [16]. Both are compared in Figure 3 with Jaeger's critical flow computations. Note that both are reasonably close to each other for practical use. Therefore, Jaeger's [2] critical flow approximation to his more general irrotational flow theory is a consistent theoretical model which produces reasonably good predictions.

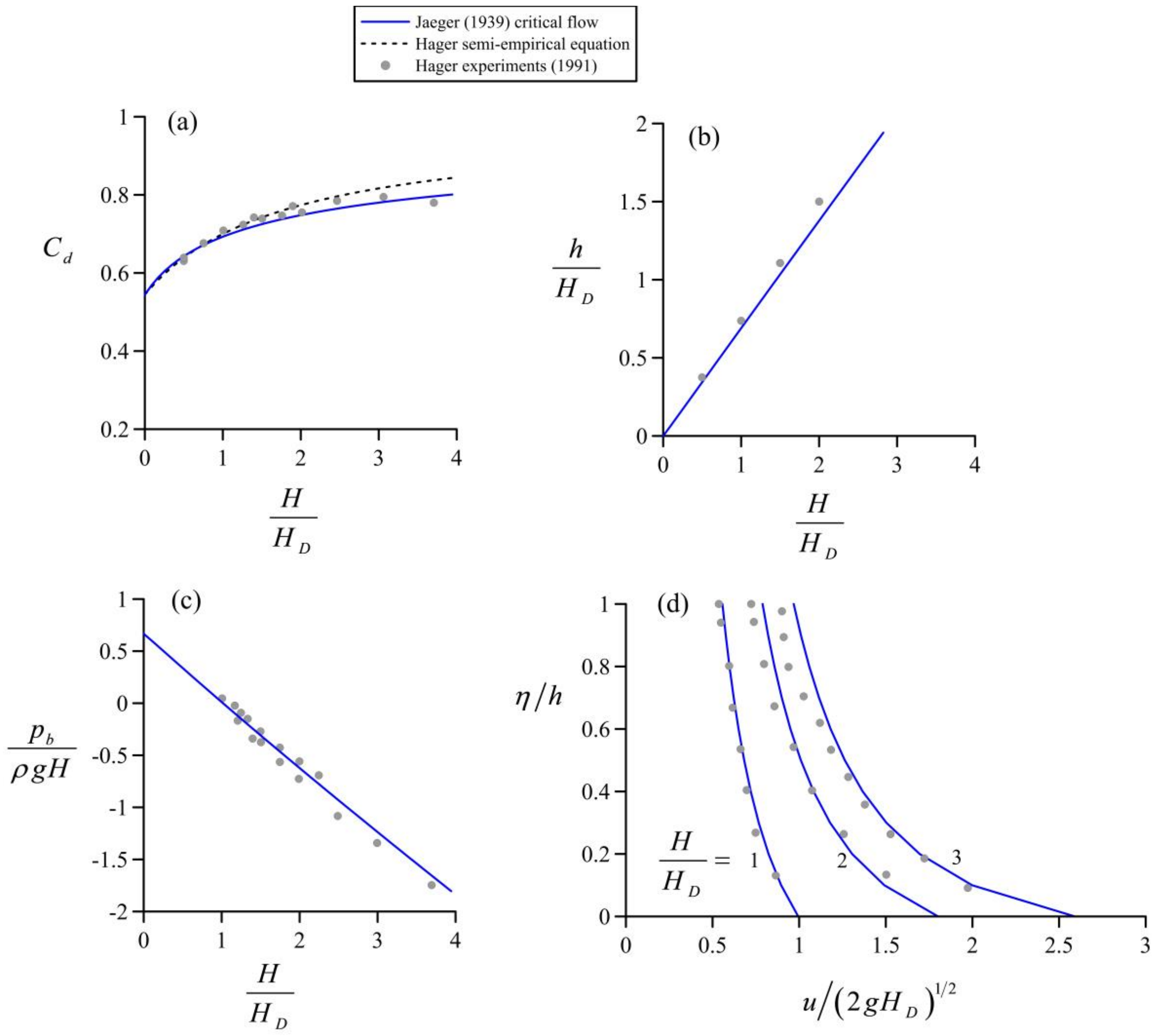

Figure 3. Jaeger's [2] theory applied to an ogee crest assuming critical flow at the apex. Comparison of modeled C_d, h/H_D, $p_b/(\rho g H)$ and $u/(2gH_D)^{1/2}$ with experiments by Hager [15] (**a**) Discharge coefficient, (**b**) crest flow depth, (**c**) crest bottom pressure head, (**d**) crest velocity profiles.

2.2. Free Surface Profile Determination Using the Bélanger–Böss Theorem

The discussed paper states that the real critical flow section (aside from how critical flow is defined) is shifted downstream of the crest. This was previously found by Ishihara et al. [17], and specifically discussed in NHF17 for an ogee weir. New results by Castro-Orgaz and Hager [18] highlighted it further. Nothing of these findings was mentioned in the discussed paper, however, so we detail here some fundamental issues of relevance.

In our analysis, we consider the continuous bed profile proposed by Hager [15,19] based on earlier work by Knapp [20]:

$$\overline{Z} = -\overline{X} \cdot \ln \overline{X}, \tag{22}$$

where

$$\begin{aligned} \overline{Z} &= 2.705(Z + 0.136) \text{ with } Z = z_b/H_D, \\ \overline{X} &= 1.3055(X + 0.2818) \text{ with } X = x/H_D. \end{aligned} \tag{23}$$

The main advantage of this proposal is that the bed curvature is continuous in the crest apex, removing the discontinuity of the standard WES profile. Selection of the crest apex curvature value becomes thus arbitrary for the WES profile [19]. Models like Jaeger's theory or the Boussinesq equations rely on assuming that the streamlines, including the bed profile, are continuous and differentiable (NHF17). In Jaeger's theory at least, continuous bed derivatives up to $\mathrm{d}^2 z_b/\mathrm{d}x^2$ are needed. Given that the bed profile is a streamline in irrotational flow models, attempts to make flow profile simulations introducing curvature discontinuities may introduce instabilities. Given that the Hager [19] and WES profiles had negligible differences in the crest domain, we applied Jaeger's [2] theory to Hager's [19] crest profile.

The specific energy for irrotational flow reads

$$H = z_b + h\cos\theta + \frac{V_s^2}{2g} = E + z_b = \text{const}, \tag{24}$$

where V_s is the absolute velocity including the bed-normal component. A generalized definition of critical flow is now sought from Equation (24). Rewrite Equation (24) as follows:

$$H = z_b + h\cos\theta + \alpha^2 \frac{q^2}{2gh^2} = \text{const}; \quad \alpha = \frac{V_s}{q/h}. \tag{25}$$

The coefficient α shall not be confused with a Coriolis coefficient. Now, we define critical flow from the minimum specific energy condition [21,22], resulting from Equation (25) in

$$\frac{\partial E}{\partial h} = 0 \Rightarrow \cos\theta - \alpha^2 \frac{q^2}{gh^3}\left(1 - \frac{h}{\alpha}\frac{\partial \alpha}{\partial h}\right) = 0. \tag{26}$$

The resulting minimum specific energy is then

$$E_{\min} = h\cos\theta\left[1 + \frac{1}{2}\left(1 - \frac{h}{\alpha}\frac{\partial \alpha}{\partial h}\right)^{-1}\right]. \tag{27}$$

Equation (26) is fully general, and no approximations are invoked thus far. Note that we set the extremum condition using the free surface specific energy instead of the average energy head as proposed by Jaeger [3], given that both are identical in irrotational motion. Our statement is simpler, given that a depth-averaged pressure coefficient is lacking. Now, simplifications arise once we set the specific value of α from Jaeger's theory, namely,

$$\alpha = \frac{h/r_b(K-1)}{\{1 + (Kh/r_b)\} - \{1 + (Kh/r_b)\}^{1/K}}, \tag{28}$$

resulting in the specific energy equation stated in NHF17, Appendix F, for flows over curved beds:

$$E = h\cos\theta + \frac{q^2}{2gh^2}\left[\frac{h/r_b(K-1)}{\{1 + (Kh/r_b)\} - \{1 + (Kh/r_b)\}^{1/K}}\right]^2 = \text{const} \tag{29}$$

Inserting Equation (28) into Equation (26) permits, at each coordinate x of the ogee weir, determination of a value of the critical depth $h_c(x)$, e.g., Equation (26) becomes a function $F(x, h_c) = 0$ which is easily solved numerically. The curve $h_c(x)$ is the critical depth profile, and, as presented here, it is a generalization of the concept used in gradually varied flows [23] to curvilinear non-hydrostatic irrotational motion. Inserting the critical depth obtained at each x into Equation (27) permits computation of the minimum specific energy $E_{\min}$ and the linked minimum total head $E_{\min}(x) + z_b(x)$.

In NHF17, for a fixed H over an ogee weir, C_d was determined by a trial-and-error procedure, upon computing fully sub- and supercritical profiles along the ogee profile, until both profiles approached in a transition point.

Here, we have devised a new and systematic procedure as follows based on the Bélanger–Böss theorem [3,24,25]. For convenience, we remark that, given Equation (25), this theorem states that the conditions [$\partial q/\partial h = 0$, H = const.] and [$\partial E/\partial h = 0$, q = const.] are mathematically equivalent to define the critical flow section [3].

The algorithm developed is as follows:

1. Divide the ogee profile in various discrete nodes i where the equations will be solved.
2. Select H and assume a C_d; start typically with $(2/3)^{3/2}$.
3. Determine the critical depth profile $h_c(i)$ from Equations (26) using Equation (28).
4. Compute $E_{\min}(i)$ at each x of the mesh for the corresponding value of the critical depth by using Equation (27).
5. Compute $H_{\min}(i) = E_{\min}(i) + z_b(i)$ at each x position.
6. Determine the maximum of $H_{\min}(i)$, $H_{\max}$.
7. If $H_{\max} < H$ go back to 2 and increase C_d. If $H_{\max} > H$, then reduce C_d.
8. Once the critical point is located ($H_{\max} = H$, within a prescribed tolerance), numerically solve Equation (29) from it in the upstream direction to determine a subcritical flow profile. Repeat the process in the downstream direction from the critical point and determine the supercritical root of Equation (29) at each position.

Once the progress converges, C_d is the maximum possible for H, and, at a particular section of the spillway, the actual head equals that of critical flow, e.g., the specific energy is a minimum there. This new method resulted in simpler computations than a classical mathematical analysis of singular points [4,17,22]. By using the Bélanger–Böss theorem, the procedure relies on purely physical aspects, namely, the extremum conditions. Note that the exact coordinates of the singular point are not determined, but a discrete node of the mesh approximates it. Using enough nodes in the mesh, the coordinates of the singular point are approximated with excellent accuracy for practical purposes.

The computed flow profiles of spillway flow using $K = 2$ are presented in Figure 4, showing that Jaeger's [2] equations with the Bélanger–Böss theorem to define critical flow are a reasonably good model for flow over ogee weirs, in addition to consisting of an important generalization of open-channel hydraulic computations. For the simulations, the h/H ratio at the crest was close to 0.75, in agreement with experimental data by Hager [15].

Finally, the bottom pressure head, obtained by applying Bernoulli's equation,

$$\frac{p_b}{\rho g} = H - z_b - (H - z_b - h\cos\theta)\left(1 + \frac{Kh}{r_b}\right)^{2/K}, \tag{30}$$

was analyzed. It was found that the pressure was somewhat above experimental data, pointing out that $K = 2$ may not be accurate. This issue is critically examined in Figure 5, where free surface and relative bottom pressures [15] for a computation with $K = 1.6$ are included for H/H_D =1 and compared with experiments and the former simulation results from $K = 2$. First, observe that the free surface moves up, but the displacement is small, indicating that the flow depth computations are insensitive to the value of K if kept in a reasonable domain. Castro-Orgaz and Hager [4] suggested $1.5 < K < 2$. Now, observe the bottom pressure head curves; the displacement is significant. For $K = 1.6$, the agreement with measured pressures [15] is fair. For the sake of completeness, the crest velocity profile was determined from the simulation using $K = 1.6$ and it is compared with experiments in the same figure, also showing fair agreement with observations. Note that the deviation of the predicted C_d from Equation (21) is again acceptable.

Considering now both sets of simulations for $K = 2$ and 1.6 in Figure 5, improved accuracy is not claimed here. Rather, a warning statement is conveyed: The coefficient K is unknown in advance, the value $K = 2$ is not generally valid, and some features, like the bed pressures, are more sensitive to it than the free surface position. Note that K is theoretically linked to the way the streamlines change their curvatures along a given equipotential, near the bed, which is also a streamline. It is thus clear that the geometry of the flow net in the vicinity of the bed will impact K, which will not only be different from 2, but variable along the bottom and with the head, e.g., $K = K(x/H_D, H/H_D)$. Possibly, this is the strongest theoretical barrier of the theory.

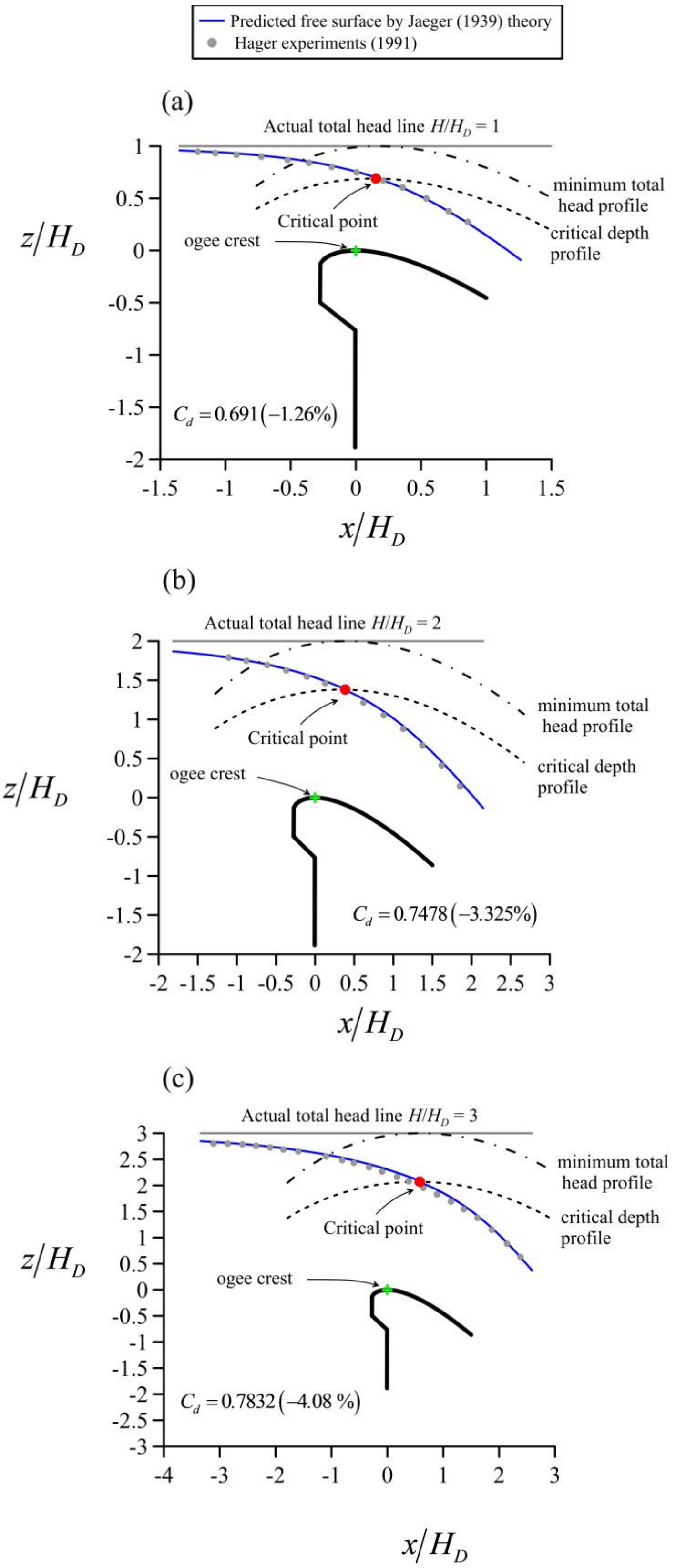

Figure 4. Computed flow profiles of spillway flow using K = 2 based on the theory by Jaeger [2] and the Bélanger–Böss theorem [3]. The critical point is indicated, along with the critical depth profile and the minimum total energy head profile. Deviations of the predicted C_d from the semi-empirical Equation (21) are also plotted. (**a**) H/H_D = 1, (**b**) H/H_D = 2, (**c**) H/H_D = 3.

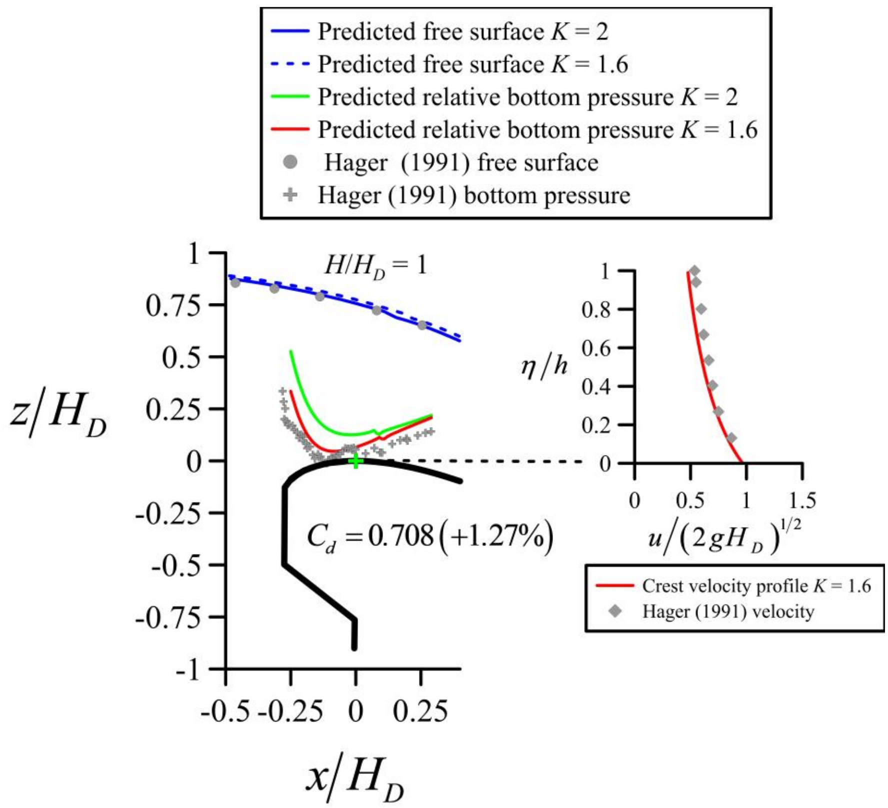

Figure 5. Impact of the curvature parameter K in Jaeger's [1] theory on computed free surface, relative bottom pressure, and crest velocity profile for design conditions.

This analysis demonstrates that K is not universal and the only reason for the choice $K = 2.2$ at the crest by Montes [13], or 2 by Jaeger [2] and the authors, is purely empirical. Therefore, this is an empirical coefficient in the theory, contrary to the assertions in the discussed article. Arguing that $r' = -2$ was selected because it allows analytical computations, and not by fitting to observations, is not a complete statement, because in the case of having obtained poor agreement of their model results with observations, this choice would have been possibly discarded. Therefore, as proposed above, a reasonable choice is to consider that K (or $r' = -K$) is a free parameter and find the best way for choosing it in each application. Castro-Orgaz [26], for example, determined empirical correlations for the parameters in Fawer's theory [27] analyzing two-dimensional potential flow solutions, and this could also be performed for K. Another option is to resort to 2D flow measurements, as performed by Ramamurthy et al. [28]. Using the experimental results of Peltier et al. [5] it should be possible to fit a Taylor expansion near the bed and estimate K along the ogee weir.

3. Concluding Remarks

The following elements from our comments should be considered to complement the discussed paper:

Jaeger's velocity profile was determined by a possibly novel procedure starting from the irrotational flow relations in the complex potential plane. It was remarked that it is a general statement valid for any equipotential line of the flow net. This highlights that Jaeger's velocity profile is an integrated form of a Cauchy–Riemann condition linking the logarithmic hodograph with the complex potential plane.

The main equations used in the discussed paper for the velocity profile, integrated discharge, and free surface specific energy are originally due to Jaeger's irrotational theory.

Assuming critical flow at the ogee crest, particular relations results from Jaeger's general irrotational flow theory. It was shown that, though not perfect, that the theory gives consistent estimates of the discharge coefficient, crest flow depth, bottom pressure head, and velocity profile.

A new method for computing the flow profile over an ogee crest is presented by simultaneous determination of the discharge coefficient and the actual critical point position using the Bélanger–Böss theorem. It shows that Jaeger's theory results in a consistent and physically based determination of the critical point in spillway flow.

A critical analysis of the impact of the curvature parameter K on the flow features, namely, the free surface, piezometric bottom pressure head, and crest velocity distribution, was presented. It is demonstrated that K is not a universal value, and, indeed, the lack of precise knowledge of this parameter is the main limitation of the theory. Neither the current comment nor the discussed paper are therefore "free" from empirical parameters.

Based on the above results, the variation of K (or $-r'$) along the ogee weir and with the head is at the minimum theoretically relevant. Can the authors highlight this variation using the experimental results in Peltier et al. [5]?

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