QÜESTIIÓ, vol. 26, 3, p. 443-459, 2002

ANALYSIS ON THE INDIVIDUAL EFFICIENCY PREDICTION IN THE COMPOSED ERROR FRONTIER MODEL. A MONTE CARLO STUDY

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This study seeks to analyse some important questions related to the Stochastic Frontier Model, such as the method proposed by Jondrow et al *(1982) to separate the error term into its two components, and the measure of efficiency given by Timmer (1971). To this purpose, a Monte Carlo experiment has been carried out using the Half-Normal and Normal-Exponential specifications throughout the rank of the γ parameter. The estimation errors have been eliminated, so that the intrinsic variability of the conditional of u given ε can be evaluated. In addition, the behaviour of the mean and mode as point estimators of u is investigated. The results have yielded some interesting findings. We have observed that both the point estimates and the mean efficiency are more precise in cases of lower efficiency. This occurs when the variable that generates the inefficiency outweighs the one that picks up the errors out of the control.*
The change in order found between the estimated efficiency and its true value is misleadingly high especially for low γ, which underlines the risk of estimating at these values.

Keywords: Bias, MSE, point estimator, production frontier, Jondrow formulae

AMS Classification (MSC 2000): 62F12, 65C05

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– Received July 2001.
– Accepted October 2002.

1. INTRODUCTION

A number of studies have already been carried out on the subject of productivity. Nowdays, the two methodologies most used to estimate efficiency by means of the Frontier Production are Linear Programming applying DEA, and the Stochastic Frontier Production (Henceforth SFP, also called 'Composed error'). The second of these will be the subject of the present work.

The Stochastic Frontier Production function was proposed independently by Aigner, Lovell and Schmidt (1977) and by Meeusen and Van den Broeck (1977). The general model of an SFP is as follows:

$$y_i = X_i * \beta + \varepsilon_i \qquad (i = 1, 2, \ldots N) \quad \varepsilon_i = v_i - u_i$$

where y_i denotes the output for observation i, X_i the vector of inputs for observation i, β is a vector of parameters, N is the sample size, and the variable error ε_i collects the difference between the systematic part of the model noted v_i and the observed values u_i.

The Composed Error presumes that the error variable not only catches the effect of the inefficiency but also that another error exists which is not controllable by the firms. This last is also included in the composed error. It is therefore assumed that the ε variable is generated as the difference between a stochastic variable v (not controllable, symmetric, and defined between $-\infty$ and ∞) and the inefficiency variable u which will always be positive and assymmetric. The v variable is assumed to be Normally distributed with mean zero and variance σ_v^2, and the component u is either Exponential, Half-normal, Truncated Normal or Gamma.

Once a specification is presumed for the u component, the estimation of the production frontier model is accomplished by assigning the corresponding distribution to ε as the difference between v and u. The residues are attained as an immediate result from the estimation ($\varepsilon_i = y_i - X_i * \hat{\beta}$). These may be considered as estimates of the error terms. However, the problem of decomposing these estimates into separate estimates of the constituent parts has remained unsolved for some time. Of course, the average technical inefficiency $-$the mean of the distribution of u_i- is easily calculated, but the main issue is how to get the relative importance of the elements and thus, to be able to match the results.

The study of efficiency by means of the SFP model was significantly developed thanks to the contribution of J. Jondrow, C. A. K. Lovell, I. S. Materov, and P. Schmidt (Henceforth, JLMS) who in 1982 proposed a formulae which made it possible to separate the two components of the ε variable.

The above mentioned formula constituted a milestone, and has remained widely used ever since, regardless of its possible weaknesses. Concerning this, we must point out that the estimation of a particular value of u by means of the mode or the expectation of the whole conditional distribution introduces a concentration effect into the distribution of the estimated inefficiency.

Although Olson *et al.* (1980) conducted a noteworthy experiment dealing with the Stochastic Frontier Production, the objective of the research focussed on the mean efficiency estimation using Corrected Least Squares.

The results of recent Monte Carlo experiments in which the JLMS performance was analysed can be found in Coelli (1995) and Kumbhakar and Löthgren (1998). In these papers the bias and the variance of the mean efficiency estimate were studied, considering the gamma parameter as a source of variation. The conclusion was that both the bias and the precision present their worst results when gamma takes central values: i.e. near 0.5.

Nevertheless, nothing in relation with the accuracy of the individual estimation of the efficiency has been taken into consideration in previous research. We consider that special attention should be paid to the change in the order in efficiency that occurs in the estimation process. It would be worthwhile to carry out further studies into the evaluation of the error introduced by the JLMS method.

We have conducted a Monte Carlo experiment in order to investigate the effect of the main sources of variation on the estimation error and the change in the efficiency order. These sources are: The value of the gamma parameter, the inefficiency distribution, the sample size and the employed formula. In respect of this last, either the mode or the expectation may be used.

The present paper is organised as follows. In Section 2 the JLMS method is briefly introduced, in section 3 the design of the Monte Carlo study is described. Section 4 reports the main results of the experiment and finally, Section 5 shows the conclusions of the study.

The JLMS formulation

The JLMS method widely develops the use of SFP models in empirical applications. The researchers arrived at the formulae for the separation of the u_i component from the residues ε_i which last contains information on u_i. They proceeded to consider the conditional distribution of u_i, given ε_i, arguing that this distribution contains whatever information ε_i yields about u_i. Thus, either the mean or the mode of this distribution can be used as a point estimate of u_i. They explicitly dealt with the commonly assumed cases of half-normal and exponential for u_i, expressed in the two following theorems:

The half-normal case

Theorem 1. *The conditional distribution of u given ε is that of a N (μ_*, σ_*^2) variable truncated at zero.*

$$v_i \approx N(0, \sigma_v^2) \qquad u_i \approx |N(0, \sigma_u^2)|$$

$$\sigma^2 = \sigma_u^2 + \sigma_v^2 \qquad \mu_* = -\frac{\sigma_u^2 \varepsilon}{\sigma^2} \qquad \sigma_*^2 = \frac{\sigma_u^2 \sigma_v^2}{\sigma^2}$$

where «≈» means «distributed as» and μ_* and σ_*^2 are the mean and the variance of the conditional distribution of *u* given ε, respectively.

Hereafter, we will drop the subscript (*i*) for the sake of simplicity. Jondrow et al defined λ as $\lambda = \frac{\sigma_u}{\sigma_v}$ and produced the following two point estimators:

$$E(u|\varepsilon) = \sigma_* \left[\frac{f\left(\frac{\varepsilon\lambda}{\sigma}\right)}{1 - F\left(\frac{\varepsilon\lambda}{\sigma}\right)} - \left(\frac{\varepsilon\lambda}{\sigma}\right) \right]$$

$$\begin{aligned} M(u|\varepsilon) &= -\varepsilon\left(\frac{\sigma_u^2}{\sigma^2}\right) && \text{if} \quad \varepsilon \le 0 \\ &= 0 && \text{if} \quad \varepsilon > 0 \end{aligned}$$

where the sign «|» means «conditioned to» and $f(.)$ and $F(.)$ are the probability density and distribution functions, respectively, of a standard normal random variable.

The exponential case

Theorem 2. *The conditional distribution of u given ε is that of a N $(-\sigma_v A, \sigma_v^2)$ variable truncated at zero.*

The parameter A was defined as $A = \frac{\varepsilon}{\sigma_v} + \frac{\sigma_v}{\sigma_u}$, deducing the point estimators for *u*:

$$E(u|\varepsilon) = \sigma_v \left[\frac{f(A)}{1 - F(A)} - A \right]$$

$$\begin{aligned} M(u|\varepsilon) &= -\varepsilon - \frac{\sigma_v^2}{\sigma_u} && \text{if} \quad \varepsilon \le -\frac{\sigma_v^2}{\sigma_u} \\ &= 0 && \text{if} \quad \varepsilon > -\frac{\sigma_v^2}{\sigma_u} \end{aligned}$$

where $f(.)$ and $F(.)$ have been defined above.

The estimation of the u component is carried out by replacing the appropriate parameters in the above formulation. Obviously, as JLMS noted, the intrinsic variability of the conditional distribution of u given ε is independent of sample size, given that ε contains rudimentary information on u.

3. DESIGN OF THE MONTE CARLO STUDY

The main parameters of the experiment are: σ^2(the variance of ε), γ (variance ratio) and the sample size (N). Due to the invariance results noted by OSW (1980) only one value of the variance ($\sigma^2 = 1$) is considered.

The variance ratio γ is taken to reflect the percentage contribution of the variance of u to the total variance of the error term in the data generating process.

The random terms v_i, $i = 1, \ldots, N$, are drawn from a Normal distribution $N(0, \sigma_v^2)$ and the technical inefficiency terms u_i, from either a Normal truncated at zero from below or an Exponential distribution. ε_i is obtained by means of the expression $\varepsilon_i = v_i - u_i$ $(i = 1, 2, \ldots, N)$.

No regression is involved and we assume that the usually unobservable ε and its two components are known beforehand.

Four sample sizes ($N = 25, 50, 100, 200$) and nine variance ratios γ ($\gamma = 0{,}1, 0{,}2, \ldots, 0{,}9$) are considered in the study. 2000 Monte Carlo replications are involved in each combination. This gives 80000 generated data sets.

We calculated three estimates of u_i, namely, two using the JLMS expressions for the expected value ($\hat{u}_{i(E)}$), and the mode ($\hat{u}_{i(M)}$) and the third by the use of the Timmer method ($\hat{u}_{i(T)}$). As a result, three estimates of efficiency were obtained, calculated as

$$e^{-\hat{u}_{i(E)}}, \quad e^{-\hat{u}_{i(M)}}$$

and

$$e^{\hat{u}_{i(T)}} \quad \text{respectively.}$$

$\hat{u}_{i(T)}$ are attained as:

$$\hat{u}_{i(T)} = \varepsilon_i - (\text{máx}\,\varepsilon) \quad (i = 1, 2, \ldots, N)$$

where $(\text{máx}\,\varepsilon)$ denotes the maximum ε value of the considered sample, which contains $\hat{u}_{i(T)}$.

Thus, $\hat{u}_{i(T)}$ represents the estimated efficiency of the i-th element calculated as a deterministic frontier approach.

In each Monte Carlo replicate, the following measures are observed:

- *Disparity (D)*

  We propose this statistic in order to provide more detail on the reliability of the estimates. D is defined as follows:

$$D(\theta) = \frac{1}{N}\sum_{i=1}^{N}\left|\hat{\theta}_i - \theta_i\right| \quad (i = 1, 2, \ldots, N)$$

  where θ_i denotes the i-th true value of θ, $\hat{\theta}_i$ the estimation of θ_i and N the sample size. Thus, D can be applied to $D(\hat{u}_E)$ and $D\left(e^{-\hat{u}_E}\right)$

$$D\left(\hat{u}_{(E)}\right) = \frac{1}{N}\sum_{i=1}^{N}\left|\hat{u}_{i(E)} - u_{i(E)}\right|$$

$$D\left(e^{-\hat{u}_{(E)}}\right) = \frac{1}{N}\sum_{i=1}^{N}\left|e^{-\hat{u}_{i(E)}} - e^{-u_{i(E)}}\right|$$

- *Mean Square Error of the mean estimated efficiency. (MSEE)*

  This measure is obtained differently from that in Kumbhakar and Löthgren (1998). It is calculated using the following expression:

$$MSEE = \frac{1}{2000}\sum_{j=1}^{2000}\left(\sum_{i=1}^{N}\frac{e^{-\hat{u}_i}}{N} - E\left(e^{-u_\gamma}\right)\right)^2 + \left(\frac{\sum_{j=1}^{2000}\sum_{i=1}^{N}e^{-\hat{u}_i}}{2000} - E\left(e^{-u_\gamma}\right)\right)^2$$

$$(i = 1, 2, \ldots, N)$$
$$(N = 25, 50, 100, 200)$$

  where $E\left(e^{-u_\gamma}\right)$ is the expected value of the efficiency distribution, and will depend on the considered γ.

- *Variance of the estimated efficiency (V)*

  This statistic is calculated as:

$$V = \frac{1}{2000}\sum_{j=1}^{2000}\left(\sum_{i=1}^{N}\frac{e^{-\hat{u}_i}}{N} - E\left(e^{-u_\gamma}\right)\right)^2$$

- *Bias of the mean estimated efficiency (B)*

  This measure is once again calculated, with relation to $E\left(e^{-u_\gamma}\right)$.

$$B = \left(\frac{\sum_{j=1}^{2000} \sum_{i=1}^{N} e^{-\hat{u}_i}}{2000} - E\left(e^{-u_\gamma}\right) \right)$$

  This is expressed as $MSEE = V + B^2$.

- *Order changes of efficiency (O)*[1].

  O is defined as: $O = \frac{1}{N}\sum_{i=1}^{N} \left|O_i - \hat{O}_i\right|$

  Where O_i indicates the order corresponding to the *i*-th element within its original efficiency ranking and $\hat{O}_i$ indicates the same aspect on the base of the estimated values of u[2]

- *Inter-sample category changes (C)*

  Once we have O_i and $\hat{O}_i(i = 1, 2, \ldots, N)$ both series are ranked and divided into 3 categories −low, medium and high efficiency− in order to compute the elements whose categories have changed after the estimation, that is (C).

Additionally, several correlations between different elements are reported for each reiteration.

4. RESULTS

In this section we present the main results of the Monte Carlo experiment, which reveal a similar performance by the assigned specifications for ε (Normal-Halfnormal (N-HN) and Normal-Exponential (N-E)).

We present all the relevant tables at the end of the paper.

[1] This measure will not be applied to $\hat{u}_{i(M)}$, because this estimator produces several zero values, i.e. full efficiencies, and the order would be distorted.

[2] Note that O values will be identical when using either $e^{\hat{u}_{i(E)}}$ or $e^{\hat{u}_{i(T)}}$.

Tables 1 and 2 illustrate the average of D results obtained for $\hat{u}_{i(E)}$, $\hat{u}_{i(M)}$ and $e^{\hat{u}_{i(E)}}$, $e^{\hat{u}_{i(M)}}$ respectively. They show the data yielded at $\gamma = 0{,}5$, which is taken as an intermediate point in order to compare their performances through different sample sizes. Looking at tables 1 and 2, we see that D is always greater for the mode than for the expected value, which bears out the hypothesis that a high estimated error is introduced by the use of the JLMS expression for the mode as a point estimator. This fact leads us to drop $\hat{u}_{i(M)}$ from further consideration.

The figures indicate that the sample size does not affect D.

Turning our attention to the disparity of $\hat{u}_{i(E)}$ $\left(D\left((\hat{u}_{i(E)}\right)\right)$, we see in table 3 that a maximum was reached, located at the central γ value, and the minimum at the extremes, underlining the fact that this measure of disparity depends on σ_*^2, which, as is shown in table 4, has a maximum at intermediate values of γ.

In contrast to this, we see that the behaviour of $\left(D\left(e^{-\hat{u}_{i(E)}}\right)\right)$ is slightly different from the last one, having a maximum at $\gamma \approx 0{,}3$.

Also, it is interesting to note that D is unaffected by the sample size (see tables 3 and 5).

We found a different type of behaviour with both taken specifications of ε. Thus, the N-HN shows higher disparity than the N-E case. Their $\left(D\left(e^{-\hat{u}_{i(E)}}\right)\right)$ values show a crossing point, having the N-E lower D values than the N-HN from $\gamma = 0{,}3$ to $\gamma = 0{,}9$.

The overall performance of V and MSEE is similar to that presented by Kumbhakar and Löthgren (1998), in spite of the fact that our methodological procedure differs from theirs. Both measures have a comparable pattern (tables 6 and 7), having a maximum at approximately $\gamma = 0{,}4$, and a decreasing trend until it reaches the value of 0.9. The causes of this pattern are the same as those mentioned for D (High values of σ_*^2 at central γ's) behaviour.

Both statistics improve as sample size increases. Thus, for $N = 200$ the above mentioned decreasing trend is more marked, especially in the variance case.

The above noted suggests that more reliable results will be attained for high N and, moreover, at extreme values of γ.

With regard to the bias (B) (table 8), it is noteworthy that B is not affected by the sample size. B turns out to be always negative, following the same pattern as V, given that the highest absolute values of B are ranged between $\gamma = 0{,}3$ and 0,5.

The study of the O statistic is crucial. The ranking of individuals or firms could be more relevant than efficiency in itself. Consider, for instance, the cases of applications seeking causes of inefficiencies by the 2-stage method; under this procedure the elements are generally divided into 3 categories, depending on their ranking. This indicates the importance of O.

Table 9 shows that the predominant factor is γ. Thus, O gradually decreases from 28 (27) at $\gamma = 0{,}1$ down to 10 (8) at $\gamma = 0{,}9$ in the N-HN (N-E) case. This bears out the assumption that the greater the γ, the greater variance of u and, therefore, it will be more difficult for the generated $u_{i's}$ to have their orders altered after the estimation.

Inspection of the data on C (Tables 10 and 11) shows that the N-HN distribution presents a lower C than N-E for any γ value.

A high C can be seen especially for small and medium γ's. Consequently, about 30 % of the individuals belonging to the high efficiency category fall, after the estimation, into the group of medium efficiency, and 20 % into the lowest category. Obviously, this fact reveals a severe weakness of the JLMS method and may lead to erroneous conclusions.

The sample size does not affect the total number of category changes C, although the composition of the changes is modified. Tables 12 and 13 show the total changes between the extreme categories and the total changes among the nearest categories respectively.

It can also be observed that a significant decrease of C occurs at γ's very close to 0.9.

Moving onto the correlation study, one thing is immediately noticeable: the relationships between u and v with ε respectively (table 14), reflect that ε is indeed made up of both components. Therefore, the higher the γ, the higher (lower) the correlation of $u(v)$ with ε.

Since the components u and v have been generated independently, we observe that the correlation u and v is close to zero.

The correlation of u with $\hat{u}$ (table 15) shows that the estimation is improved as γ rises, and that we must be cautious when applying the JLMS expressions at low values of γ. The JLMS method produces a negative correlation of $\hat{u}$ with $\hat{v}$, which decreases as γ increases. Also, a close relationship appears between ε and $\hat{u}_{(E)}$, yielding an almost exact correlation, but negative.

The existing correlation between the true efficiency and $e^{-\hat{u}_{i(E)}}$ is one of the main results. As can be seen in table 16 the higher the γ value, the greater the correlation. However, this correlation is very weak at low γ's, which underlines the risk of estimating at these values.

The correlation between the efficiency estimated by Timmer and the true efficiency, when v is different from zero, tends to be higher as γ rises. Obviously, the error is caused by the fact that the Timmer method assumes $v = 0$. Therefore, the lower the v, the higher the correlation. This correlation is always lower than that existing between e^{-u_i} and $e^{-\hat{u}_{i(E)}}$, demonstrating that from this point of view, the JLMS method, using the expected value, outperforms the Timmer one for any γ considered.

The estimates of the efficiency given by JLMS and Timmer are correlated. However, the lower the γ, the lower the correlation obtained ($r = 0{,}7$-$0{,}95$).

5. CONCLUSIONS

The present study investigates the behaviour of the Jondrow *et al.* method to estimate individual technical efficiency by means of a Monte Carlo experiment.

The following conclusions can be drawn from the results:

- The expectation estimator of u conditional on epsilon outperforms the one calculated using the mode.
- The theoretical variance of the estimator proposed by JLMS has been empirically contrasted and has been shown to be superior for intermediate values of gamma, but inferior for the extreme values. The bell-shaped performance influences considerably the disparity measures as well as the variance and MSE.
- The bias is negative in all cases, implying an underestimation of the mean efficiency.
- The sample size affects the MSE of the efficiency, improving the precision the higher the size. However, the bias and the disparity measures are not affected.
- Slight differences in the disparity and the mean variation of orders among the Normal-Half-normal and Normal-Exponential are found.
- The deterministic frontier function approach yields worse results for any value of gamma different from one, being more imprecise when gamma is lower.
- The mean variation of orders is unaffected by the variance of the point estimator of u. Thus, the former decreases monotonically when the variance ratio rises.
- It can be concluded that both the point estimates and the mean efficiency are more precise in cases of lower efficiency. That is, when the variable that generates the inefficiency outweighs the one that picks up the errors out of the control.
- The change in order found between the estimated efficiency and its true value is misleadingly high, especially for low γ. More precisely, approximately 40 % of the firms change from one group to another. Therefore, the picture that emerges is rather disappointing, since the potential conclusions drawn can be distorted.

6. REFERENCES

Aigner, D., Lovell, C. A. K. and Schmidt, P. (1977). «Formulation and Estimation of Stochastic Frontier Production Function Models», *Journal of Econometrics*, 6, 21-37.

Battese, G. E. and Coelli, T. J. (1993). «A Stochastic Frontier Production Function Incorporating a Model for Technical Inefficiency Effects», *Working Papers in Econometrics and Applied Statistics*, 69, University of New England, Armidale.

Coelli, T. J. (1995). «Estimators and Hypothesis Tests for a Stochastic Frontier Function: A Monte Carlo Analysis», *Journal of Productivity Analysis*, 6, 247-268.

Hjalmarsson, L., Kumbhakar, S. C. and Heshmati, A. (1996). «DEA, DFA and SFA: A Comparison», *Journal of Productivity Analysis*, 7, 303-327.

Horrace, W. C. and Schmidt, P. (1996). «Confidence Statements for Efficiency Estimates from Stochastic Frontier Models», *Journal of Productivity Analysis*, 7, 257-282.

Jondrow, J., Lovell, C. A. K., Materov, I. S. and Schmidt, P. (1982). «On the Estimation of Technical Inefficiency in the Stochastic Frontier Production Function Model», *Journal of Econometrics*, 19, 233-238.

Kumbhakar, S. and Löthgren, M. (1998). «A Monte Carlo Analysis of Technical Inefficiency Predictors», *Working Paper Series in Economics and Finance*, 229, March 1998. Stockholm School of Economics.

Meeusen, W. and van den Broeck, J. (1977). «Efficiency Estimation from Cobb-Douglas Production Function with Composed Errors», *International Economic Review*, 18, 435-444.

Olson, J. A., Schmidt, P. and Waldman, D. W. (1980). «A Monte Carlo Study of Estimators of Stochastic Frontier Production Functions», *Journal of Econometrics*, 13, 67-82.

Timmer, C. P. (1971). «Using a Probabilistic Frontier Production Functions to Measure Technical Efficiency», *Journal of Political Economy*, 79(4), 776-794.

Table 1. Disparity of u estimates based on JLMS point estimators.

gamma = 0.5	Size	N-HN.(E)	N-E.(E)	N-HN(M)	N-E.(M)
	25	0.38179651	0.34505828	0.42633239	0.43844921
	50	0.38180968	0.34532386	0.42599261	0.43909834
	100	0.38155264	0.34387603	0.42533376	0.43793732
	200	0.38143671	0.34412331	0.42505889	0.43927048

Table 2. Disparity of efficiency estimates based on JLMS point estimators.

gamma = 0.5	Size	N-HN.(E)	N-E.(E)	N-HN(M)	N-E.(M)
	25	0.156645445	0.1668595	0.194236911	0.248554577
	50	0.156475167	0.166527231	0.193707896	0.248804567
	100	0.156298243	0.165771439	0.193260251	0.248286296
	200	0.156290733	0.165932704	0.193123864	0.249090726

Table 3. Disparity of u estimates based on JLMS point estimators.

γ	N-HN.N = 25	N-HN.N = 50	N-HN.N = 100	N-HN.N = 200	NE.N = 25	NE.N = 50	NE.N = 100	NE.N = 200
0.1	0.2393	0.2376	0.2374	0.2383	0.2196	0.2195	0.2192	0.2191
0.2	0.3147	0.3136	0.3144	0.3150	0.2886	0.2869	0.2868	0.2873
0.3	0.3556	0.3565	0.3572	0.3566	0.3252	0.3259	0.3231	0.3241
0.4	0.3773	0.3766	0.3790	0.3764	0.3404	0.3415	0.3408	0.3411
0.5	0.3818	0.3818	0.3816	0.3814	0.3451	0.3453	0.3439	0.3441
0.6	0.3731	0.3722	0.3707	0.3714	0.3370	0.3363	0.3356	0.3361
0.7	0.3443	0.3460	0.3460	0.3456	0.3146	0.3149	0.3148	0.3146
0.8	0.3001	0.3005	0.3011	0.3018	0.2771	0.2762	0.2774	0.2769
0.9	0.2274	0.2277	0.2287	0.2274	0.2134	0.2138	0.2122	0.2129

Table 4. Variance of u given epsilon distribution.

γ	σ_*^2
0.1	0.2108
0.2	0.3261
0.3	0.3788
0.4	0.3883
0.5	0.3667
0.6	0.3220
0.7	0.2596
0.8	0.1833
0.9	0.0961

Table 5. Disparity of efficiency estimates based on JLMS point estimators.

γ	N-HN.N = 25	N-HN.N = 50	N-HN.N = 100	N-HN.N = 200	NE.N = 25	NE.N = 50	NE.N = 100	NE.N = 200
0.1	0.1529	0.1523	0.1523	0.1527	0.1495	0.1497	0.1496	0.1493
0.2	0.1704	0.1704	0.1704	0.1707	0.1713	0.1708	0.1706	0.1708
0.3	0.1709	0.1720	0.1719	0.1716	0.1752	0.1762	0.1750	0.1754
0.4	0.1658	0.1655	0.1664	0.1656	0.1719	0.1730	0.1730	0.1731
0.5	0.1566	0.1565	0.1563	0.1563	0.1669	0.1665	0.1658	0.1659
0.6	0.1444	0.1440	0.1440	0.1436	0.1567	0.1565	0.1564	0.1560
0.7	0.1282	0.1279	0.1285	0.1282	0.1427	0.1422	0.1428	0.1426
0.8	0.1085	0.1089	0.1087	0.1092	0.1246	0.1242	0.1241	0.1241
0.9	0.0813	0.0816	0.0820	0.0816	0.0964	0.0963	0.0957	0.0959

Table 6. Variance of efficiency estimates based on JLMS point estimators.

(N-NH specification)

γ	N = 25	N = 50	N = 100	N = 200
0.1	0.0009	0.0008	0.0008	0.0008
0.2	0.0019	0.0017	0.0016	0.0016
0.3	0.0027	0.0023	0.0021	0.0020
0.4	0.0030	0.0025	0.0022	0.0021
0.5	0.0030	0.0024	0.0021	0.0019
0.6	0.0029	0.0021	0.0018	0.0016
0.7	0.0028	0.0019	0.0015	0.0012
0.8	0.0026	0.0016	0.0011	0.0008
0.9	0.0026	0.0014	0.0008	0.0005

Table 7. MSEE of efficiency estimates based on JLMS point estimators.
(N-NH specification)

γ	N = 25	N = 50	N = 100	N = 200
0.1	0.0016	0.0015	0.0015	0.0015
0.2	0.0035	0.0031	0.0031	0.0031
0.3	0.0046	0.0042	0.0041	0.0039
0.4	0.0051	0.0044	0.0042	0.0041
0.5	0.0048	0.0041	0.0038	0.0037
0.6	0.0042	0.0035	0.0032	0.0031
0.7	0.0038	0.0029	0.0025	0.0022
0.8	0.0031	0.0021	0.0016	0.0013
0.9	0.0028	0.0016	0.0009	0.0006

Table 8. Bias of efficiency estimates based on JLMS point estimators.
(N-NH specification)

γ	N = 25	N = 50	N = 100	N = 200
0.1	−0.0269	−0.0266	−0.0269	−0.0271
0.2	−0.0389	−0.0381	−0.0390	−0.0389
0.3	−0.0442	−0.0437	−0.0440	−0.0436
0.4	−0.0457	−0.0442	−0.0447	−0.0447
0.5	−0.0419	−0.0418	−0.0420	−0.0423
0.6	−0.0369	−0.0372	−0.0377	−0.0380
0.7	−0.0323	−0.0320	−0.0321	−0.0316
0.8	−0.0229	−0.0233	−0.0237	−0.0229
0.9	−0.0134	−0.0133	−0.0136	−0.0130

Table 9. Mean order variation of efficiency estimates based on JLMS point estimators.

γ	N.HN	N.E
0.1	28.0008	27.3777
0.2	25.5301	24.6233
0.3	23.4530	22.3785
0.4	21.7847	20.2589
0.5	19.8739	18.1641
0.6	18.0790	16.1377
0.7	16.0540	13.9268
0.8	13.5566	11.3914
0.9	10.2587	8.2966

Table 10. Changes of category after the estimation of efficiency. N-HN error specification.

γ	Total Changes	Changes from 1 to 2	Changes from 1 to 3	Changes from 2 to 1	Changes from 2 to 3	Changes from 3 to 1	Changes from 3 to 2
0.1	57.564	9.89	7.336	9.342	11.83	7.884	11.282
0.2	53.274	9.64	5.516	8.986	11.808	6.17	11.154
0.3	49.562	9.234	4.374	8.8	11.39	4.808	10.956
0.4	45.928	8.682	3.424	8.302	11.048	3.804	10.668
0.5	42.574	8.034	2.496	7.912	10.818	2.618	10.696
0.6	38.246	7.42	1.764	7.242	10.028	1.942	9.85
0.7	33.856	6.7	0.928	6.628	9.336	1	9.264
0.8	28.412	5.672	0.402	5.692	8.122	0.382	8.142
0.9	20.256	4.172	0.092	4.14	5.88	0.124	5.848

Table 11. Changes of category after the estimation of efficiency. N-E error specification.

γ	Total Changes	Changes from 1 to 2	Changes from 1 to 3	Changes from 2 to 1	Changes from 2 to 3	Changes from 3 to 1	Changes from 3 to 2
0.1	58.572	9.844	7.734	9.344	11.958	8.234	11.458
0.2	55.378	9.398	6.288	8.848	12.278	6.838	11.728
0.3	51.706	8.948	4.876	8.234	12.386	5.59	11.672
0.4	48.88	8.496	4.032	7.816	12.252	4.712	11.572
0.5	45.738	8.214	3.076	7.516	11.928	3.774	11.23
0.6	42.632	7.666	2.388	7.146	11.522	2.908	11.002
0.7	38.348	6.826	1.544	6.442	10.996	1.928	10.612
0.8	33.696	5.996	0.924	5.836	10.008	1.084	9.848
0.9	25.474	4.464	0.242	4.386	8.07	0.32	7.992

Table 12. Total number of changes between categories 1-3, 3-1.

γ	N = 25	N = 50	N = 100	N = 200
0.1	15.22	13.936	14.4415	14.111
0.2	11.686	10.748	11.214	10.927
0.3	9.182	8.32	8.7545	8.39075
0.4	7.228	6.145	6.507	6.2395
0.5	5.114	4.378	4.6	4.41525
0.6	3.706	2.81	3.027	2.8325
0.7	1.928	1.57	1.6335	1.4355
0.8	0.784	0.546	0.549	0.47975
0.9	0.216	0.07	0.052	0.03875

Table 13. Total number of changes between categories 1-2, 2-1, 2-3, 3-2.
(N-NH specification)

γ	N = 25	N = 50	N = 100	N = 200
0.1	42.344	43.734	43.426	43.574
0.2	41.588	42.716	42.389	42.4065
0.3	40.38	41.34	40.902	41.1335
0.4	38.7	40.09	39.548	39.613
0.5	37.46	37.91	37.578	37.6075
0.6	34.54	35.628	35.189	35.233
0.7	31.928	31.936	31.998	31.986
0.8	27.628	27.214	27.084	27.142
0.9	20.04	19.918	19.957	19.496

Table 14. Correlation u, v, epsilon. Sample Size = 100.

γ	$u - \varepsilon$.(N-HN)	$u - \varepsilon$.(N-E)	$u - v$.(N-HN)	$u - v$.(N-E)	$u - u_T$.(N-NH)	$u - u_T$.(N-E)	$v - \varepsilon$.(N-HN)	$v - \varepsilon$.(N-E)
0.1	−0.3124	−0.3075	0.0000	0.0050	0.3124	0.3075	0.9489	0.9484
0.2	−0.4452	−0.4401	−0.0016	0.0012	0.4452	0.4401	0.8942	0.8945
0.3	−0.5423	−0.5449	0.0040	−0.0068	0.5423	0.5449	0.8352	0.8380
0.4	−0.6280	−0.6230	0.0037	0.0009	0.6280	0.6230	0.7723	0.7764
0.5	−0.7038	−0.7006	−0.0011	−0.0026	0.7038	0.7006	0.7071	0.7094
0.6	−0.7704	−0.7656	0.0028	−0.0011	0.7704	0.7656	0.6313	0.6379
0.7	−0.8339	−0.8289	0.0008	0.0021	0.8339	0.8289	0.5468	0.5514
0.8	−0.8927	−0.8893	−0.0029	0.0008	0.8927	0.8893	0.4493	0.4505
0.9	−0.9472	−0.9467	0.0031	−0.0003	0.9472	0.9467	0.3145	0.3176

Table 15. Correlation between true components and estimates based on JLMS.
Sample Size = 100.

γ	$u - \hat{u}_{(E)}$ (N-HN)	$u - \hat{u}_{(E)}$ (N-E)	$\hat{u}_{(E)}) - \hat{v}_{(E)}$ (N-HN)	$\hat{u}_{(E)}) - \hat{v}_{(E)}$ (N-E)	$u - \varepsilon$. (N-HN)	$u - \varepsilon$ (N-E)
0.1	0.3182	0.3287	−0.9773	−0.9162	−0.9816	−0.9330
0.2	0.4581	0.4835	−0.9556	−0.8535	−0.9717	−0.9087
0.3	0.5606	0.6019	−0.9342	−0.7998	−0.9678	−0.9048
0.4	0.6484	0.6826	−0.9117	−0.7602	−0.9680	−0.9138
0.5	0.7244	0.7562	−0.8874	−0.7217	−0.9712	−0.9273
0.6	0.7892	0.8118	−0.8603	−0.6846	−0.9762	−0.9426
0.7	0.8487	0.8639	−0.8262	−0.6428	−0.9824	−0.9594
0.8	0.9026	0.9114	−0.7796	−0.5932	−0.9890	−0.9755
0.9	0.9517	0.9562	−0.7028	−0.5187	−0.9954	−0.9901

Table 16. Correlation between true efficiency and estimates based on JLMS. Sample Size = 100.

γ	$e^{-u} - e^{-\hat{u}_{(E)}}$(N-HN)	$e^{-u} - e^{-\hat{u}_{(E)}}$(N-E)	$e^{-u} - e^{-\hat{u}_{(T)}}$(N-HN)	$e^{-u} - e^{-\hat{u}_{(T)}}$(N-E)
0.1	0.3100	0.3116	0.2282	0.2078
0.2	0.4347	0.4391	0.3214	0.2869
0.3	0.5219	0.5290	0.3906	0.3528
0.4	0.5967	0.5918	0.4604	0.4077
0.5	0.6628	0.6554	0.5272	0.4740
0.6	0.7212	0.7085	0.5981	0.5391
0.7	0.7798	0.7639	0.6768	0.6195
0.8	0.8418	0.8253	0.7653	0.7148
0.9	0.9076	0.8963	0.8689	0.8350