Developing an Instrument for Analyzing Mathematics and Mathematics Education Ideas in the Spanish Press of the 18th Century

Maria José Madrid 1,*, Alexander Maz-Machado 2, Astrid Cuida 3 and Cristina Pedrosa-Jesús 2

Abstract: Old mathematics books and textbooks have focused different researchers onto the history of mathematics and mathematics education. However, books are not the only information source for this field; for example, researchers can also study periodical-type publications from the past (such as diaries, weeklies, newspapers, etc.). Considering this, this study developed an instrument to analyze publications about mathematics and mathematics education included in newspapers, weeklies, journals, etc., which were not exclusively devoted to science, from the perspective of the history of mathematics and mathematics education. In order to do so, a descriptive research focused on the analysis of historical texts was carried out using the content analysis technique. The different labels and categories of this instrument are here exemplified by the categorization of some entries included in several periodical publications published in the 18th century in Spain.

Keywords: history of mathematics and mathematics education; press; 18th century; instrument

MSC: 97-03

1. Introduction

Textbooks have been and still are a relevant tool in the teaching of different school subjects, including mathematics. Their role in the classroom has been reflected in different research studies that have been carried out, for example by Villella and Contreras [1]. In addition, many researchers in mathematics education have designed methodologies, models or instruments to analyze different aspects of mathematics textbooks; for example, Vásquez and Alsina [2] designed a model to analyze how probability is treated in primary education Chilean mathematics textbooks, Monterrubio and Ortega [3] constructed a model for assessing mathematics textbooks, structured around a series of organizers which evaluate different aspects, etc.

Old textbooks and old mathematics books have also played a relevant role as primary sources to learn more about the history of mathematics and mathematics education. Many examples of studies in the field involving books can be mentioned, for example, Frejd [4] made a content analysis and a comparison among five Swedish textbooks in algebra from the period around the year 1800, Barrantes-Hernández and Picado-Alfaro [5] analyzed the adoption of the Metric System in Costa Rica, based on mathematical and didactic specificities recognized in textbooks for training primary education teachers between 1885 and 1914, Abed and Al-Absi [6] analyzed, through content analysis, the types of mathematical disciplines in Jordanian Elementary Textbooks during 1970–2013, etc.
Within this line of research on the history of mathematics and mathematics education, different authors have presented methodological proposals, which develop different models, methodologies, or instruments to compare and analyze mathematics old books and textbooks. Among them, Schubring [7] presented a methodology to analyze historical textbooks focusing on Lacroix’s books. Authors such as Ruiz De Gauna, Dávila, Etxeberria, and Sarasua [8] presented a methodology to analyze different mathematics text books from the secondary level, published in Spain from 1970 to 2005, and they include categories such as a didactic approach to the mathematics contents, graphic–symbolic language, mathematical problems and exercises, as well as methodological innovations and learning–teaching processes. González and Sierra [9] presented a tool for textbook analysis, taking as an example the evolution of the concept of critical points in Spanish textbooks throughout the 20th century.

However, there are other historical sources which can also provide information about the history of mathematics and mathematics education. An example of this is the study of the catalogues of school supplies presented by Carrillo [10] or Drake [11], who analyzed teachers’ narrative descriptions of themselves as learners and teachers of mathematics in order to understand the implementation of mathematics education reforms.

Researchers have also considered the role that periodic publications have as an information source for this field. For example, Zelbo [12] analyzed two recreational mathematics columns from the end of the nineteenth century which appeared in American periodic publications, Georgia’s Sunny South and Delaware’s Delaware Gazette and State Journal. Furthermore, Meavilla and Oller [13] studied the mathematical contents included in the periodic publication Miscelánea Turolense published between 1891 and 1901 in Spain; Useche [14] studied the role of mathematics instruction in Zoel García de Galdeano’s journal El Progreso Matemático.

Due to this, we have considered the possibilities of periodic publications as a source for learning more about the history of mathematics and mathematics education, focusing in this case on the press published in Spain in the 18th century.

In the 18th century, especially since the second half of the century, the press developed in Spain, with a growing number of readers and the emergence of different formulas such as the cultural press and criticism press, together with the already existing informative press (fundamentally political and military) [15].

The press occupied an important place in the political, economic, and cultural life of Spain in the 18th century and it contributed to the dissemination of the Enlightenment thought in the country, although periods of abundant publications are interspersed with periods of few publications for political, religious, as well as economic, reasons [16].

However, with the exception of the official informative newspapers, which had a considerable circulation for the time, the public was a minority. In particular, it belonged to the nobility, the clergy and mostly to the middle classes, even of a relatively modest status, and in this sense, the press played a levelling role in the Spanish society, although it left out an immense illiterate majority [15].

Another relevant fact is that in the Spanish and Latin American press, since the middle of the 18th century, periodical publications whose initial purpose was not scientific, included reviews of scientific books, articles on technique or applied science, etc. [17].

Among these scientific ideas, it is possible to find ideas about mathematics and mathematics education in different periodicals published in this century in Spain, as stated in Madrid and López-Esteban [18]. Considering this, it is necessary to develop a methodological framework that allows analyzing the 18th century Spanish press from the perspective of the history of mathematics and mathematics education; this will allow the categorization of the different entries and will facilitate their analysis. In order to do so, the aim of this paper was to present an instrument for the analysis of mathematical and mathematical education contents in the Spanish press of the 18th century. This instrument 

will allow us to characterize these contents, assessing which were more widespread at the
time, which were of greater interest, etc.

2. Materials and Methods

This research has an exploratory, descriptive, ex post fact, and qualitative nature. The
method followed was historical research. It was focused on the analysis of old texts from
the perspective of the history of mathematics and mathematics education. The analysis
technique used was content analysis, a technique widely used in research on the history
of mathematics and mathematics education [19,20].

In order to carry out the study, we did a pilot exploratory experience, where we
created an instrument with several labels from the analysis of two Spanish weeklies [21].

After this experience, we have re-elaborated the previous instrument and created, in
the first place, a system of categories. Then we have distributed the previous labels into
these categories, and also new ones which were not considered before. Therefore, we have
developed a second and more complete version of our instrument which includes the
following 4 categories and main labels (Figure 1).

![Figure 1. Scheme of the labels and main categories included in our instrument.](image)

This instrument was validated through a triangulation of experts in Research
Methods in Education and History of Mathematics and Mathematics Education.

The units of analysis defined were each of the entries in the periodical publications
that included mathematics or mathematics education content. These entries were read,
analyzed, and subsequently categorized. In order to do this categorization, we first
collected general information about the publication using Table 1. Then, we classified each
entry in the publication analyzed into one of the four categories listed in Figure 1.
Depending on this initial classification, we used Tables 2–5 to place each entry into one of
the labels and, within each label, we analyzed whether it included the required
information and, if so, how that information was detailed.
Table 1. Form for the analysis of the general data in the press.

<table>
<thead>
<tr>
<th><strong>General Data</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Name of the publication:</td>
</tr>
<tr>
<td>Number:</td>
</tr>
<tr>
<td>Number of pages dedicated to mathematics or mathematics education/number of pages per number:</td>
</tr>
<tr>
<td>Author/s (if indicated):</td>
</tr>
</tbody>
</table>

Table 2. Form for the analysis of mathematics and mathematics education ideas in the press.

<table>
<thead>
<tr>
<th><strong>Mathematics–Mathematics Education Ideas</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Statement: verbal, includes graphical representations, mathematical language,...</td>
</tr>
<tr>
<td>Mathematical content: arithmetic, algebraic, geometric, calculus, etc.</td>
</tr>
<tr>
<td>Context: mathematical, commercial, technical, measurement, scientific, business, etc.</td>
</tr>
<tr>
<td>Interest aroused and continuity: solutions for the problem have been published or debate about it has been generated later.</td>
</tr>
<tr>
<td>Up-to-date content: whether it is classic content or includes the latest discoveries in mathematics.</td>
</tr>
<tr>
<td>Connections: they have been published in other journals, the author has published mathematics books, the author has published in other journals, etc.</td>
</tr>
<tr>
<td>Correct/Incorrect.</td>
</tr>
<tr>
<td>Solving method used, solving scheme, mathematical language used, graphical representations (if included) and solution.</td>
</tr>
<tr>
<td>Solution proofs or variants of the problem.</td>
</tr>
<tr>
<td>Corrections: corrections have been included for the solutions.</td>
</tr>
<tr>
<td>Mathematical content: arithmetic, algebraic, geometric, calculus, mathematics education, etc.</td>
</tr>
<tr>
<td>Context: mathematical, commercial, technical, measurement, scientific, education, business, etc.</td>
</tr>
<tr>
<td>Interest aroused and continuity: solutions for the problem have been published or debate about it has been generated later.</td>
</tr>
<tr>
<td>Up-to-date content: whether it is classic content or includes the latest discoveries in mathematics.</td>
</tr>
<tr>
<td>Connections: they have been published in other journals, the author has published mathematics books, the author have published in other journals, etc.</td>
</tr>
<tr>
<td>Reflect opinions, attitudes or beliefs about mathematics and mathematics education.</td>
</tr>
<tr>
<td>Other mentions to mathematics and mathematics education, offers for more contents, etc.</td>
</tr>
</tbody>
</table>
Table 3. Form for the analysis of the information about institutions where mathematics was taught and learnt in the press.

<table>
<thead>
<tr>
<th>Institutions Where Mathematics Was Taught and Learnt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advertisements</td>
</tr>
<tr>
<td>Exams</td>
</tr>
<tr>
<td>Syllabus</td>
</tr>
<tr>
<td>Other</td>
</tr>
</tbody>
</table>

Table 4. Form for the analysis of the information about publications which disseminate mathematics knowledge in the press.

<table>
<thead>
<tr>
<th>Publications Which Disseminate Mathematics Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advertisements</td>
</tr>
<tr>
<td>Reviews of mathematics books</td>
</tr>
<tr>
<td>Other</td>
</tr>
</tbody>
</table>

Table 5. Form for the analysis of the information about mathematicians, mathematics teachers or people related to mathematics in the press.

<table>
<thead>
<tr>
<th>People: Mathematicians, Mathematics Teachers or People Related to Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job search</td>
</tr>
<tr>
<td>Biographical texts</td>
</tr>
<tr>
<td>Other</td>
</tr>
</tbody>
</table>

Table 6 shows an example of how the categorization was made using the instrument.

Table 6. Example of the categorization for an entry from Correo de Madrid.

<table>
<thead>
<tr>
<th>Number</th>
<th>Category and Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correo de Madrid 30/12/1789 [22] (pp. 2593–2597) included a more than 4-pages 2-column biography of Descartes, with comments such as: what most immortalizes him is the application of Algebra to Geometry; through which he has shown the mutual help that sciences can give to each other.</td>
<td>Category: People: Mathematicians, mathematics teachers or people related to mathematics; Label: Biographical texts</td>
</tr>
</tbody>
</table>

The search and location of the weeklies and newspapers analyzed was carried out through several digital libraries: the Hispanic Digital Library from the Spanish National
3. Results

Here we exemplify the different categories of our instrument showing different entries from 18th century Spanish periodical publications.

Mathematics–mathematics education ideas

Problem posing and resolution

Correo de Madrid 23/07/1788 [23] (p. 1087) included an entry from Francisco Calvo, where he said that he believed the following algebraic question was impossible to solve, but he was unable to demonstrate its impossibility. The question was: find three square numbers whose differences and the differences of their roots are square numbers.

The author wanted to know if the question was possible or impossible, and if it was impossible, he was looking for the rigorous algebraic demonstration of its impossibility.

Also, in Correo de los ciegos de Madrid 5/1/1787, an author, who called himself El Andaluz Alto de fantasía proposed the following arithmetic problem: four saleswomen have an unknown number of limes in a proportion. Knowing that together the first and fourth one have 81 limes, and the second and third one have 69 limes: how many limes have each one of them?" [24] (p. 104).

In current notation, we could pose this problem as:

\[
\begin{align*}
\frac{x}{y} &= \frac{z}{t} \\
x + t &= 81 \\
y + z &= 69
\end{align*}
\]

In number 33 from Correo de los ciegos de Madrid 30/1/1787 [25] (pp. 131–132), this problem received two answers.

An author, who indicated just its initials I.V.N.D., explained that he had discovered the solution of this problem in a dream. He dreamt that he was in a large square used only for selling limes, where there were an infinite number of saleswomen’s stalls, all divided in groups of four women. The saleswomen were determined to solve the problem, and in order to do so, they took the limes from one woman, put them on another one, and took them off again and again. The author tried to help them using this method, but he was unable to do so. When the author had tried for more than three hours, a man appeared and told him: if you cut limes in half, you give one half to each saleswoman, then you give 4 limes to the second one, 8 limes to the third one and 16 limes to the fourth one (by doing so, you would have a double proportion), and in the end, you divide the remaining limes equally into them, you will have the solution. The author concluded that this solution is that the first one had 26 and a half, the second one 30 and a half, the third one 38 and a half and the fourth one 54 and a half.

Another author (P.V.L.) said that the first one had 18 limes, the second one 27 limes, the third one 42 and the fourth one 63 limes.

Not only was this problem solved here, but it is also possible to find connections among different periodical publications. In particular, Diario curioso, erudito, económico y comercial 19/2/1787 [26] (pp. 210–211) included an answer by the author T.M., who also commented the previous solutions.

In the first place, he criticized the solution given by I.V.N.D, saying that the solution 26½, 30¼, 38½, 54½ does not make sense, and that the other solution that the author
insinuates, $30 \frac{1}{2}, 34 \frac{1}{2}, 38 \frac{1}{2}, 46 \frac{1}{2}$, was also a nonsense, because the proportion is not verified and he considered this was normal, because writers who calculate when dreaming are exposed to such disappointments.

Then he said that the solution by P.V.L. is correct, but this problem is an indeterminate one, which composes the most elegant and, at the same time, the most inventive part of the arithmetical and geometrical analysis, and lays the foundations to the doctrine of curves, in which consists the perfection of sublime geometry. Due to that, the author wanted to solve it more completely, and he presented eight solutions with whole numbers using a table (Table 7), he added that strictly speaking they could be sixteen solutions, just by placing the limes in retrograde order, with the last one being the first one, the penultimate one becoming the second one, etc.

![Table 7](image)

More examples of these ongoing exchanges in problem posing and problem solving can be found for example in Correo de Cadiz [27,28].

Connections among periodical publications happened also for example in Semanario de Salamanca 9/7/1795 [29] (pp. 28–29), which included the solution for a problem about the calculation of the curve, whose tangents are all the reflected radii, given a curve and a luminous point in its plane. This problem was posed by D. Martin Broussein in the journal Diario de Madrid on the 7 June.

**Informative texts**

Semanario Literario de Cartagena 12/1/1787 [30] included a four-page publication titled Antigüedad de los caracteres Arithmeticos (Antiquity of the Arithmetical Characters), which talked about the numerical characters in Egyptian hieroglyphs and the connection of the hieroglyphs with the current numerical characters.

In response to this publication, J. Alconchel sent to Semanario Literario de Cartagena 23/2/1787 a two-page publication on the formation of numbers, and he even offered to send more texts about the origin and progress of arithmetic, geometry, etc. [31] (pp. 62–64).

Correo de Madrid 29/12/1790 [32] (p. 91) included an entry on children’s learning of arithmetic, stating for example that the Pythagorean table should be known by heart.

**Opinion texts**

Opinions about mathematics and its learning can be found, for example, in Semanario Salamanca 28/4/1798 [33] (pp. 57–61); here, a woman, who indicated just her initials A. F., spoke about her opinions about mathematics and its learning. Among her comments, she considered mathematics a universal, useful, and necessary science and young men should know it.

Also, in Semanario de Salamanca 28/2/1795 [34] (pp. 127–133), the author Pedro Alonso de la Auecilla included a seven-page entry, where he expressed his hateful opinions about mathematics; in fact, he declared himself an enemy of it.

**Institutions where mathematics was taught and learnt**

Advertisements
Diario de Madrid 4/8/1794 [35] (pp. 873–874) and Semanario erudito y curioso de Salamanca 19/8/1794 [36] (pp. 120–122) included information on a job advertisement for a professor of mathematics at the institution Estudios Reales de Madrid. The future professor should teach the different branches of mathematics per 2 h every school day during the academic year, alternating and dividing his teaching with other professor, and his salary would be 13200 reales de vellón (Spanish old coin) per year. In order to achieve this job, the candidates should know Latin and they had to pass an opposition exam.

Exams
Diario curioso, erudito, económico y comercial 17/07/1787 [37] (p.71) included information about mathematics exercises in Reales Estudios de San Isidro (Madrid). In particular, it indicated that Vicente Ruiz would discuss about finding the divisors, both of first and second degrees, in the equations of higher degrees.

Syllabus
Ambrosio Alvarez Enciso published in Diario curioso, erudito, económico y comercial 26/5/1787 [38] (pp. 594–595) and Diario curioso, erudito, económico y comercial 27/5/1787 [39] (pp. 598–600) his plan for a public trade school, which should have an arithmetic, geometry, and geography teacher, and he added that arithmetic and geometry should be learned in the second year.

Publications which disseminate mathematics knowledge
Reviews
Some publications included reviews about mathematics books, for example, in Diario curioso, erudito, económico y comercial 24/7/1786 [40] (pp. 101–102), a review of a book about trigonometry written by Cagnoli: Tratado de Trigonometria rectilínea y esférica, que contiene métodos y fórmulas nuevas, con aplicación á la mayor parte de los problemas de Astronomía [Treatise on rectilinear and spherical Trigonometry, which contains new methods and formulae, with application to most Astronomy problems] is included.

This review included information on the purpose of the author with this book, its characteristics, its contents, the mathematical knowledge necessary to be able to read it, people who could find it useful, etc., and also positive comments and appreciations about it, for example: this book will be very well received by those who study mathematics, by astronomers, geographers, and even by geometers.

Book advertisements
Also in Diario curioso, erudito, económico y comercial 28/03/1787 [41] (p. 360), we can find book advertisements, for example, the book Arithmética teórica y práctica en compendio de las cuentas mas usuales y corrientes para el comercio de los Reynos de Castilla, Aragon, Valencia, Navarra y Mallorca, como tambien el modo para saber girar los cambios sobre las mas principales plazas de Europa, y algunas otras noticias de geometria para los agrimensores [Theoretical and practical arithmetic in a compendium of the most usual and current accounts for the trade of the kingdoms of Castile, Aragon, Valencia, Navarre and Majorca, as well as the way to know how to change on the most important squares of Europe, and some other geometric news for surveyors], written by Fr. Fermin de los Arcos, could be found at the Printing and Bookshop of Don Alfonso López, in de la Cruz Street.

Correo de los ciegos de Madrid 31/10/1786 [42] (p. 26) included an advertisement about Semanario literario y curioso from Cartagena. It indicated that since Friday 1st September, a newspaper was published in this city [Cartagena] entitled: Semanario literario y curioso. Its aim was to show the progress of Natural History, Physics, Mathematics, Medicine, Chemistry, Surgery, Agriculture, Fine Letters, and Arts, compiling some of their principles and foundations.

Mathematicians, mathematics teachers or people related to mathematics
Biographies
Correo de Madrid included, in several numbers, biographies about different philosophers, physicians, mathematicians, etc., for example about Leibnitz in Correo de Madrid, 9/1/1790 [43] (pp.2617–2619). This 3-page, two-column biography tells about
Leibnitz’s appearance, his life, his family, his studies, his publications, his controversies, etc. For example, it included the following comment: The dispute of the Englishmen who attributed to Newton the discovery of the differential calculus, attributed to him by the Society of London, took his life [Leibnitz’s]. As for this invention, it is certain that Leibnitz published his Elements of Calculus in 1684, and Newton published his Method of Fluxions in 1687. And this Englishman confessed in a letter that Leibnitz had previously found this method.

Advertisements

*Diario curioso, erudito, económico y comercial* 12/09/1787 [44] (p. 299) included the following advertisement: a teacher of girls is wanted for a place not far from this Court, her salary will be 5 reals per day and house. It is required that she knows well the skills proper to her gender; and also, she should read, write and count regularly.

Not only teachers who count were required, counting was a skill demanded for other jobs, for example: *Diario curioso, erudito, económico y comercial* 3/8/1787 [45] (p. 140) included the following advertisement: It is wanted a servant between 18 and 20 years old, who can write well and count.

On the other side, some people who looked for a job, included mathematical knowledge as a skill in their advertisement, for example: *Diario de Madrid* 10/1/1788 [46] (p.40) included: Servants. A 21-year-old young man, well versed in arithmetic, is looking for a place inside or outside Madrid or for America.

*Diario curioso, erudito, económico y comercial* 1/08/1787 [47] (p. 131) included an advertisement from someone who said is educated in mathematics, algebra, machinery, and hydraulics, knows how to draw, illuminate, measure land, level it, and make plans for any building and he was looking for a job to instruct children in any of these sciences, or for any other purpose.

4. Discussion and Conclusions

Periodical publications can be a relevant source to show the presence of mathematics and mathematics education beyond formal or regulated education. In particular, these kinds of publications show the social and cultural aspects of mathematics, and how different people used a popular and social element such as newspapers, weeklies and other periodicals, a priori not specialized in science, to disseminate mathematical ideas to the Spanish society of the 18th century.

Therefore, we consider it necessary to develop an instrument that should help researchers to categorize and assess the different mathematical entries found in different publications. In order to do so, the instrument developed here allows us to categorize and analyze the different publications with mathematical or mathematics education contents included in periodical publications from the 18th century, showing their possibilities of use within the field of the history of mathematics and mathematics education.

Through it, we can learn more about the authors of mathematics books, institutions dedicated to the teaching of mathematics, mathematics teachers, books published at the time, opinions about mathematics and its teaching and learning, etc. In addition, the analysis of the problems and solutions included helps to dive into the different methods used to solve problems, the different solutions and comments given by different authors, the associated graphical representations, etc. It also serves as basis for more complex studies, which may use these categories and labels to investigate in a particular aspect.

Furthermore, our instrument is useful not only for Spanish 18th periodical publications; in Latin America, some publications included mathematics contents, for example, in *Papel Periódico de Santafé de Bogotá* 09/03/1792 [48] (pp. 39–40), an author (L.A.) talked about agriculture, and about a tree, whose crown measured 38 Castilian varas (length measure), then he calculated, among other things, the radius (19), perimeter (using the Archimedean proportion 7 to 22 he obtained 119 varas and 3/7), the area (1130 and a half square varas) and the number of people that can take shelter under this tree, considering half a square vara for each person: 2261 people.
Finally, many authors have considered the use of the history of mathematics and mathematics education in the mathematics classroom [49], and in particular, the value of studying original sources as an activity that might be integrated into the teaching of mathematics [50]. We consider that periodical publications can also provide useful examples to use in the classroom and to generate meaningful debates between students (for example, by recreating the debates that arose between authors at the time), and in this sense, our instrument will provide teachers a useful tool to categorize the different entries and to evaluate its usefulness for each situation.

**Author Contributions:** Conceptualization, M.J.M., A.M.-M., A.C. and C.P.-J.; methodology, M.J.M. and A.M.-M.; formal analysis, M.J.M. and A.C.; investigation M.J.M., A.M.-M., A.C. and C.P.-J.; writing—original draft preparation, M.J.M., A.M.-M., A.C. and C.P.-J.; writing—review and editing, M.J.M. and C.P.-J.; funding acquisition, A.M.-M. All authors have read and agreed to the published version of the manuscript.

**Funding:** This study has been partially carried out within the research project of the Plan I+D+i of Spanish Ministry of Economy and Competitiveness (FEDER funds) EDU2016-78764-P.

**Data Availability Statement:** Not applicable.

**Conflicts of Interest:** The authors declare no conflict of interest.

**References and Notes**


