

IMPROVING GRAPH-BASED DETECTION OF SINGULAR EVENTS FOR PHOTOCHEMICAL SMOG AGENTS

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1 ABSTRACT

2 Recently, a set of graph-based tools have been introduced for the
3 identification of singular events of O_3 , NO and temperature time series, as well
4 as description of their dynamics. These are based on the use of the Visibility
5 Graphs (VG). In this work, an improvement of the original approach is
6 proposed, being called Upside-Down Visibility Graph (UDVG). It adds the
7 possibility of investigating the singular lowest episodes, instead of the highest.
8 Results confirm the applicability of the new method for describing the
9 multifractal nature of the underlying O_3 , NO , and temperature. Asymmetries in
10 the NO degree distribution are observed, possibly due to the interaction with
11 different chemicals. Furthermore, a comparison of VG and UDVG has been
12 performed and the outcomes show that they describe opposite subsets of the
13 time series (low and high values) as expected. The combination of the results
14 from the two networks is proposed and evaluated, with the aim of obtaining all
15 the information at once. It turns out to be a more complete tool for singularity
16 detection in photochemical time series, which could be a valuable asset for
17 future research.

18 KEYWORDS

19 - Photochemical smog

20 - Visibility Graphs

21 - Singularity detection

22

23 1. INTRODUCTION

24 Among the problems related to atmospheric pollution, there is a matter of
25 special concern studied by environmental scientists in the recent years, the so-
26 called photochemical smog. Also known as “Los Angeles smog”, since it was
27 firstly noticed in that city in 1944, as a result of the observed damage on the
28 vegetation (NAPCA, 1970). It can be defined as the accumulation of gases and
29 aerosols as a result of reactions between nitrogen oxides (NO_x), certain volatile
30 organic compounds (VOCs) and oxygen under the influence of solar radiation.
31 A wide range of chemicals (ozone, aldehydes or hydrogen peroxides among
32 them) are created in the process (Guicherit and van Dop, 1977). Typically, this
33 phenomenon is more prominent when a city is more populated and warmer.
34 Among the gases involved, there are two which are extensively researched due
35 to the many harms associated to them and their quantitative importance: the
36 tropospheric ozone (O_3) and the nitrogen dioxide (NO), being the second a
37 precursor of the first one. It must be stressed that both of them (O_3 and NO)
38 have a serious impact on human health (Cheng et al., 2020; Kampa and
39 Castanas, 2008; Yue et al., 2018). Furthermore, a recent study has
40 demonstrated that O_3 produces harsh effects on the economy due to a
41 reduction of the crop yield (Miao et al., 2017).

42 During the last decades, investigation on complex networks and their
43 applications has been carried out in many works (Boccaletti et al., 2006; Gan et
44 al., 2014; Newman, 2003; Stam, 2010). A complex network can be understood
45 as a graph (a set of nodes and edges as will be further explained) which
46 exhibits nontrivial topological properties and is often used to model and
47 describe real systems. Furthermore, in the recent years there have been a
48 considerable amount of works seeking ways to represent nonlinear time series
49 as complex networks (Zou et al., 2019). This includes manuscripts based on
50 recurrence networks, transition networks and visibility graphs. The main
51 potential of these approaches is the vast number of tools that there exist to
52 analyze networks from a computational perspective. Authors highlight the
53 centrality parameters, since they are essential to this work. They are used to
54 quantify the importance of the nodes within a graph and will be introduced and
55 used later in the text.

56 Among the new methodologies previously described, there is one that has
57 been recently used to investigate environmental time series (Carmona-Cabezas
58 et al., 2019b; Donner and Donges, 2012; Pierini et al., 2012). This methodology
59 received the denomination of Visibility Graph (VG) algorithm (Lacasa et al.,
60 2008). As it has been demonstrated several times, the complex networks
61 obtained through this method inherit the main features of the original time series
62 and therefore can be used to describe them (Lacasa et al., 2009; Lacasa and
63 Toral, 2010).

64 Besides describing the nature and main features of the time series, another
65 possibility implies the detection of singularities within these signals. For that
66 purpose, many techniques have been used. One example is the Hölder

67 exponent, which is based on multifractal properties of the system (Loutridis,
68 2007; Shang et al., 2006). By looking at the information retrieved from the
69 transformed complex network, it is also possible to detect singular points, as it
70 has been explored in several works recently (Bielinskyi and Soloviev, 2018;
71 Carmona-Cabezas et al., 2019b, 2020). In particular, the unusually large values
72 of the cited centrality parameters associated to each node, can provide much of
73 the information that could be derived from the time series.

74 In the presented work, a new approach is introduced to improve this
75 detection of singular points in a time series from photochemical smog variables
76 (pollutant concentration and temperature), using the VG. The motivation behind
77 it was the fact that regular VG criterion associates the highest connectivity to
78 the points with largest concentration. Therefore, singular events that have low
79 value are overlooked by the original technique. The proposed improvement
80 analyzes the original and inverted series and combines their parameters for a
81 wider point of view.

82 The pursued aim with this work is to test the application range and possible
83 advantages or pitfalls of the proposed improvement. By doing that, authors
84 intend to explore how this advance could complement the identification of
85 singular episodes of pollutant time series (which could be potentially extended
86 to others apart from O_3 and NO). Being that the case, future researchers will
87 benefit from a more thorough technique for detecting unusual low and high gas
88 concentrations, with different criteria, as a result.

89 2. MATERIALS AND METHODS

90 2.1. Data

91 For this work, measurements of tropospheric ozone (O_3), nitrogen dioxide
92 (NO) and temperature have been used. All of them correspond to hourly time
93 series, being recorded in 2017. In the last part of this manuscript, months
94 corresponding to different seasons are selected. The reason for this lies in the
95 fact that, as explained before, this work seeks improving a previous one
96 (Carmona-Cabezas et al., 2020), and therefore the same months have been
97 used for clearer comparison. The station where they were collected is called
98 San Fernando (36°27' N, 6°12' W), which is located in the province of Cádiz
99 (southern Iberian Peninsula) and administered by the Consejería de
100 Medioambiente (Regional Environmental Department) of Andalusia and the
101 European Union.

102 According to the Köppen-Geiger classification, the zone where the data is
103 collected is labelled as "Csa", as it is most of the Mediterranean basin. "Csa"
104 regions are characterized by warm temperatures with summers that are
105 regularly hot and dry. Furthermore, two of the most important industrial centers
106 in the region (Huelva and Bay of Algeciras) are located relatively close to the
107 study area. As a result of the mentioned conditions, this selected place is
108 propense to accumulation of tropospheric ozone (O_3) and nitrogen dioxide (NO)
109 (Domínguez-López et al., 2014).

110 2.2. Visibility Graph

111 As it was introduced before, in the last decade, a new method to analyze
112 one dimensional series was introduced (Lacasa et al., 2008). This technique
113 transforms these series into a different mathematical entity: a graph or network.
114 Therefore, it was given the name Visibility Graph, because of its resemblance to

115 the original one used in architecture for space analysis (Lozano-Pérez and
116 Wesley, 1979; Turner et al., 2001). One of the main features of the VG is that it
117 has been demonstrated that it inherits properties of the original time series that
118 it is obtained from (Lacasa et al., 2009, 2008; Lacasa and Toral, 2010). For
119 instance, a periodic series would result on a regular graph after applying it.

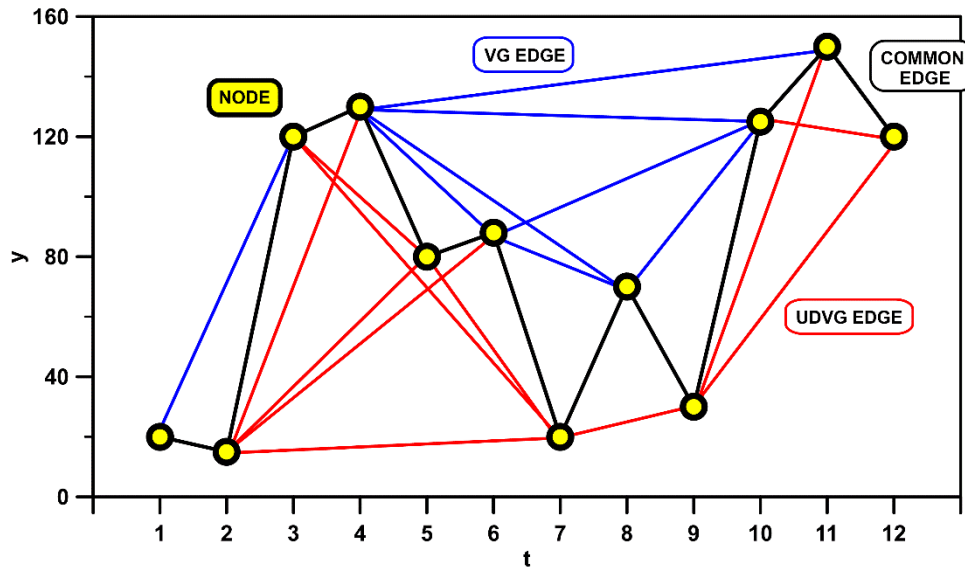
120 In general, a graph can be understood as a set of *nodes* and *edges* that link
121 them. In the context of VG, the nodes correspond to the points in the time
122 series. Thus, it is necessary to establish the criterion for linking them and so
123 establishing the *edges*. The basic idea is that two nodes are connected to each
124 other if a line between them can be drawn and it does not pass below any other
125 point in the signal. That is, two points (t_a, y_a) and (t_b, y_b) are connected in the
126 graph (have visibility) if any point (t_c, y_c) between them ($t_a < t_c < t_b$) fulfills:

$$y_c < y_a + (y_b - y_a) \frac{t_c - t_a}{t_b - t_a} \quad (1)$$

127 From the VG method described, it is easy to see that the nodes with highest
128 connectivity (also known as *hubs*) will be usually the ones with the unusual
129 greatest values in the time series. This approach comes in handy in order to
130 describe these points with higher magnitude; nevertheless, if one is interested
131 on what happens with the opposite case (i.e. minimal unlikely values), the
132 indicated technique is not suitable for describing them. That is indeed one
133 disadvantage of employing VG for detecting singular points in a time series.

134 In a recent work, a variation of VG was presented (Soni, 2019) in order to
135 explore new approaches to gain information about a time series. There, the
136 concept of a signed complex network is introduced. The basic idea behind that
137 method is that some of the edges will have a positive sign, while some other will

138 be negative. The regular VG computed as explained before corresponds to the
139 positive edges of this signed graph. On the other hand, the negative
140 connections are obtained also from the regular VG but performed this time over
141 the “upside-down” time series. That is, instead of using the original series $f(t)$,
142 the converted series $-f(t)$ is used. This new graph was employed as a whole,
143 in order to obtain series of clusters from the network and to analyze multivariate
144 correlations, as an extension of previous works (Lacasa et al., 2015; Sannino et
145 al., 2017). Nevertheless, the purpose of the work introduced here is to
146 investigate the possibility of applying this idea for improving the detection of
147 singular points in a time series, such as O_3 and NO concentration, or
148 temperature. For that reason, the positive and negative parts need to be
149 obtained separately, as some of the parameters that will be further explained
150 cannot be retrieved from a signed network (e.g. the betweenness centrality).
151 For clarity reasons, the “positive” network will be given the name of regular VG
152 in the text; while for the “negative” one, the term Upside-Down VG (UDVG) will
153 be used. In Figure 1, an example of the two types of network is shown.



154

155 Figure 1: Example of computation of the regular VG (blue lines) and the UDVG
 156 (red lines) to a sample time series and resulting graphs. Black lines indicate the
 157 common edges.

158

159 It must be highlighted as well that all the edges of the two graphs are different,
 160 except for those connecting each node to its nearest neighbors in the time
 161 series. In Figure 1, the first statement is seen by looking at the blue and red
 162 edges, while the second one reflects in the black ones that both regular VG and
 163 UDVG have in common. In other words, the elements of the adjacency matrices
 164 fulfill: $a_{ij}^{VG} + a_{ij}^{UDVG} \leq 1; \forall j \neq i \pm 1$. This means that the elements surrounding
 165 the main diagonal are equal and the others cannot be $a_{ij} = 1$ simultaneously in
 166 the two matrices.

167 2.3. Centrality parameters

168 One of the most widely used approaches to characterize graphs and
 169 complex networks is based on the analysis of the most important nodes within.
 170 It is done by employing the so-called centrality parameters, which are evaluated
 171 at each node, giving an idea about how “central” each one is, in relation to the

172 rest of them. This concept was firstly used for studying social networks and
173 transferred to other fields of research afterwards (Agryzkov et al., 2019; Joyce
174 et al., 2010; Liu et al., 2015). The actual meaning of a central node may vary
175 depending on the actual parameter used to evaluate the network. Here, authors
176 focus on three of them: the degree, betweenness and closeness centrality,
177 which have been used to describe physical systems in previous works
178 (Carmona-Cabezas et al., 2020; Donner and Donges, 2012; Mali et al., 2018).

179 The first centrality parameter that will be explained is the degree. In a graph,
180 the number of edges which are connected to a given node i is defined as the
181 degree of that node (k_i), i.e., $k_i = \sum_j a_{ij}$, being a_{ij} the elements of the
182 adjacency matrix. Once the degree for each node is obtained, the degree
183 distribution $P(k)$ can be computed. This quantity has been proven to be able to
184 characterize the nature of the studied signal (Lacasa et al., 2008; Mali et al.,
185 2018). In fact, degree distributions that can be adjusted to a power law $P(k) \propto$
186 $k^{-\gamma}$ correspond to scale free networks which comes from fractal series, as it
187 was discussed by (Lacasa et al., 2009; Lacasa and Toral, 2010). The reason for
188 this is the effect of hub repulsion (Song et al., 2006). A hub is a node from a
189 graph with unlikely greater number of links, and so, higher degree. Therefore,
190 the right tail of degree distributions is dominated by these nodes and, after
191 being represented in a log-log plot, they can be fitted by a simple linear
192 regression.

193 The other two employed parameters cannot be understood without defining
194 the shortest path (SP) quantity first. SP is a measurement of the number of
195 different edges that connect two distant nodes. Given a pair of nodes (i, j) ,
196 different possible paths between them are available. Some of them (not

197 necessarily unique) will have the minimum possible number of edges and, thus,
 198 they will be the minimal paths known as SP. It must be regarded that it has an
 199 important presence in the definition of the betweenness and closeness
 200 centrality. The betweenness of a node i can be computed by the following
 201 expression:

$$b_i = \sum_{\substack{j=1 \\ j \neq i}}^N \sum_{\substack{k=1 \\ k \neq i, j}}^N \frac{n_{jk}(i)}{n_{jk}} \quad (2)$$

202 Where n_{jk} is the number of SP's from node j to k , whereas $n_{jk}(i)$ is the
 203 number of those SP's that contain the node i . A high betweenness can be
 204 interpreted as a node which is passed through by SP's connecting the rest of
 205 nodes.

206 Lastly, the closeness centrality is obtained as shown in the following
 207 expression:

$$c_i = \frac{1}{\sum_{j=1}^N d_{i,j}} \quad (3)$$

208 There, the closeness of each node c_i is computed from the so-called
 209 distance matrix D , where each element $d_{i,j}$ corresponds to the SP from node i
 210 to j . Therefore, this quantity accounts for how close a given node is to the rest
 211 of the network, in terms of edges needed for other nodes to be reached.

212

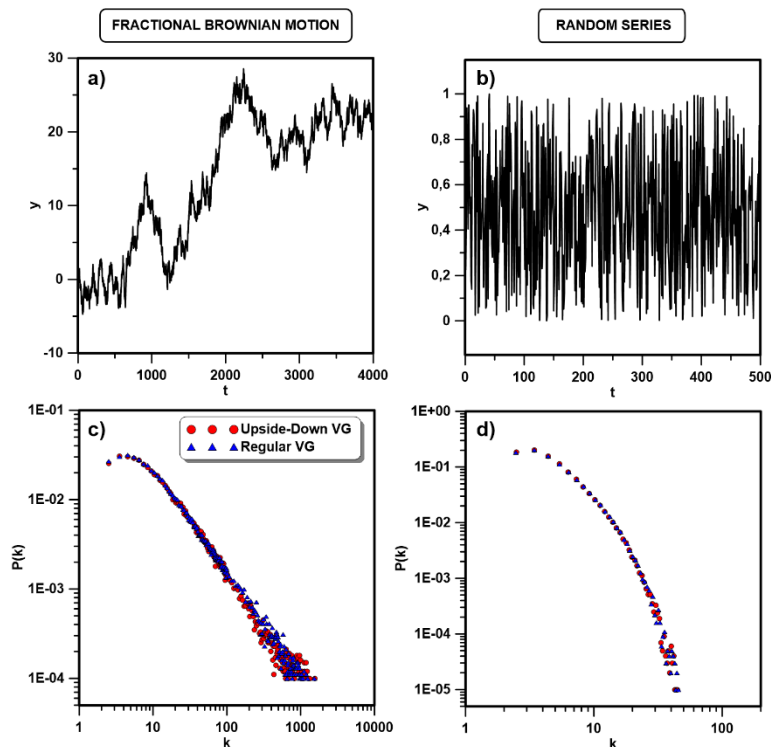
213 3. RESULTS AND DISCUSSION

214 3.1. Degree distributions

215 After the proposed methodology has been explained, authors have analyzed
216 firstly how the UDVG degree distribution differs from the regular VG with the
217 same time series. It served as a preliminary study, before tackling the
218 identification of relevant points in the signal, which is the main objective of this
219 work.

220 Figure 2 represents two theoretical time series have been employed to test
221 the method. The first one of them is obtained from a fractional Brownian motion
222 with Hurst exponent $H = 0.5$ and 10^4 points. The second one corresponds to a
223 random series with 10^5 points. The reason for choosing them is that they are
224 standard well-known series that are frequently used within this type of studies
225 with VGs. This figure shows the series (a and b) and their respective degree
226 distribution computed for both approaches (c and d). It can be inferred that the
227 distributions that arise from using the UDVG are almost identical to the VG
228 ones. Therefore, at least for these types of series, the VG and UDVG degree
229 distributions describe the same properties of the underlying time series.

230 In the case of the fractional Brownian motion, they also present curves
231 which can be adjusted to the same power law. Thus, this might indicate that
232 UDVG would be also suitable for describing scale-free networks, such as those
233 extracted from these type of series, which are fractal (Lacasa et al., 2009). For
234 the random time series, the result is a distribution with a tail that follows an
235 exponential trend, as expected (Lacasa et al., 2008).



236

237 Figure 2: Top: fractional Brownian motion signal with Hurst exponent $H = 0.5$
 238 and 10^4 points (a) and 500 points from a random series (b). Bottom: The degree
 239 distribution computed from the complex networks obtained for both series, by
 240 employing the VG and UDVG.

241

242

243 Once it was observed that UDVG and VG obtain the same results for the
 244 theoretical time series, authors have tested the photochemical time series,
 245 which are the focus of this study. These correspond to three different signals:
 246 two from O_3 and NO_2 concentration, and the other one temperature, all of them
 247 from the same year (2017), as previously stated. The reason behind choosing
 248 one complete year in this particular part of the study is that for a reliable
 249 comparison of degree distributions, a considerable amount of points in the time
 250 series is required (Carmona-Cabezas et al., 2019a). Since the resolution of the
 251 measurements is one hour, it has been observed after several tests that taking
 252 only monthly samples for comparison could give misleading results. It should be

253 underscored that the same is not true for the later analysis of singular episodes,
254 since that is a local study and does not depend in the extension of the pollutant
255 time series.

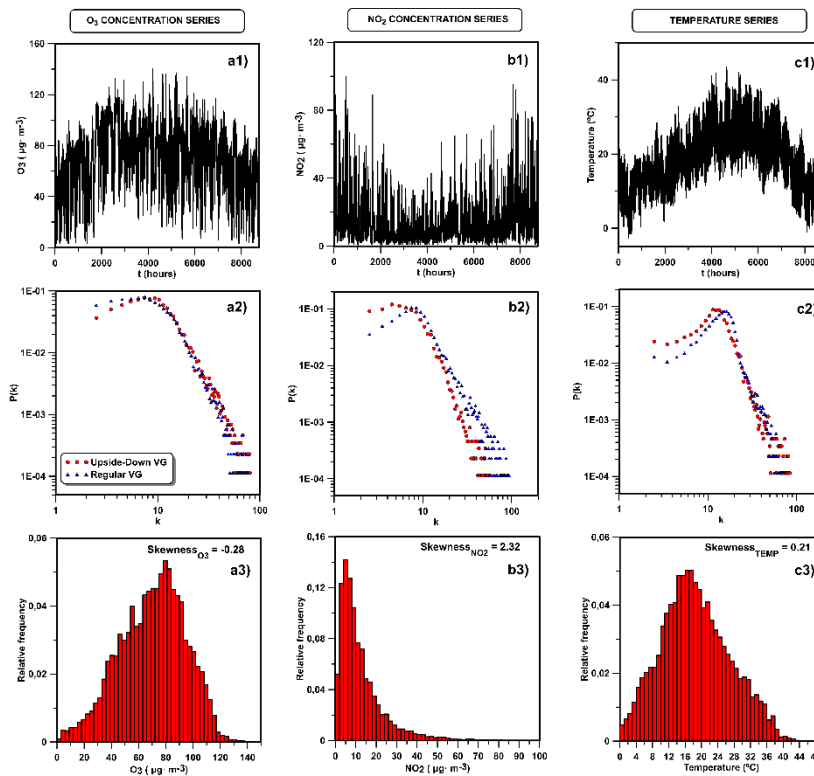
256 The three signals are depicted in the upper part of Figure 3 (a1, b1 and c1).
257 Conversely to what was observed in the previous case, now Figure 3 (a2, b2
258 and c2) display slight differences between the degree distributions of the classic
259 VG and the UDVG. Nevertheless, a clear power law behavior is observed in
260 every case. This is in accordance with previous works, where ozone (O_3) and
261 nitrogen dioxide (NO) time series exhibited scale-free behavior, as a
262 consequence of the multifractal nature of its dynamics (He, 2017; Pavón-
263 Domínguez et al., 2015). The observed contrasts are more pronounced in the
264 case of the nitrogen dioxide (NO) concentration time series, clearly showing a
265 marked difference in the slope of the distribution tail (the γ -exponent). Authors
266 attribute this effect to the difference between the three underlying time series.
267 The ground-level ozone signal exhibits a pattern that equally presents singular
268 minima and maxima, and the same can be said about the temperature.
269 Therefore, the frequency distributions of their concentrations will have roughly
270 symmetric shapes. On the other hand, the same cannot be argued for the
271 nitrogen dioxide (NO) concentration. Minima and maxima values are not
272 distributed evenly along the time series, which is clear in Figure 3c. The maxima
273 are rather infrequent and singular in comparison to the minima, which are much
274 more common, as most of the values are very close to zero. Therefore, one
275 could expect the probability distribution of the concentration of nitrogen dioxide
276 (NO) to be non-symmetric. To investigate that, the most suitable method is to
277 inspect the skewness (S) for each time series. This quantity describes the

278 asymmetry of the probability distribution of a given real measure around its
279 mean. When skewness is equal to zero, it means that the distribution is
280 symmetric respect to its mean, being the opposite case ($S \neq 0$) for non-
281 symmetric distributions. In Figure 3, probability distributions of concentrations
282 and temperature are depicted with their respective skewness value. In the case
283 of O_3 and temperature (Figure 3 a3 and c3), the distributions are almost
284 symmetric, as mentioned before, with skewness close to zero ($S_{O_3} = -0.28$ and
285 $S_{temp} = 0.21$). Despite this, a mild deviation between the regular and inverted
286 distributions can be observed, leading to the low negative skewness that is
287 observed. On the contrary, a positive skewness value ($S_{NO_2} = 2.32$) of nitrogen
288 dioxide (NO) concentration is clearly seen (Figure 3b3), i.e. low values with
289 respect to the mean are highly frequent. Therefore, the concentration of
290 nitrogen dioxide (NO) in San Fernando reaches low peaks many times during
291 the month, while the high accumulations of this noxious gas are much rarer.

292 For the case of temperature, this symmetry can be interpreted as the
293 relatively regular behavior of day and night values, meaning that the
294 appearances of singular episodes of low and high temperature will be linked
295 during the year, depending on the meteorological conditions of each season.
296 On the other hand, one could expect this difference between nitrogen dioxide
297 (NO) and tropospheric ozone (O_3), regarding the symmetry of the degree
298 distribution. Both gases are correlated through the simplified photochemical
299 reaction $NO + O \leftrightarrow O_3 + NO$. Production and destruction of ozone will occur
300 during day and night times respectively. The photolysis that leads to the ozone
301 accumulation and the reach of the photostationary state regularly happens
302 during mid-day, when there is radiation available. The sense of the reaction is

303 reverted during nighttime in the absence of light. Although the quantitative
304 concentration levels may vary depending on factors such as wind speed,
305 temperature or mixing height, the distributions of maxima and minima could be
306 expected to be symmetric as it is seen for the O_3 . However, the nitrogen dioxide
307 (NO) intervenes in other reactions that could lead to the appearance of singular
308 minima in its concentration. One example is the aldehyde production through
309 interaction with VOCs, which results on a lower rate of NO - NO reaction. For a
310 deeper understanding of this, further analysis with NO and VOCs time series
311 would be necessary.

312 Authors would like to point out that, for the previous theoretical series this
313 relation is also observed, being their computed skewness values very close to
314 zero ($S_{brownian} = 0.13$ and $S_{random} = 3.10 \cdot 10^{-4}$), as expected since their
315 distribution were almost perfectly coincident for VG and UDVG.



316

317 Figure 3: Top: Ozone (O_3), nitrogen dioxide (NO_2) concentration and
 318 temperature annual temporal series (a1, b1 and c1). Middle: Degree
 319 distributions computed with regular VG and UDVG (a2, b2 and c2). Bottom:
 320 frequency distributions of the pollutant concentrations and temperature time
 321 series (a3, b3 and c3).

322

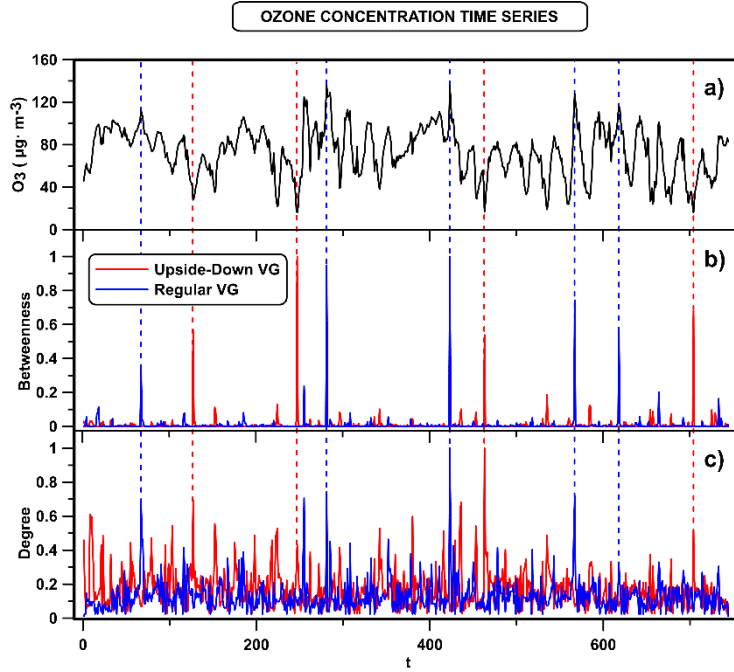
323 3.2. Identification of hubs

324 After the preliminary study of probability distributions has been carried out, a
 325 pointwise study of NO and O_3 concentrations and temperature is undertaken
 326 here. Figure 4 depicts a comparison between the hubs computed by applying
 327 the VG on the unvaried time series and those of the inverted one. Now only one
 328 real time series is shown, because the actual interest here is to observe the
 329 differences between UDVG and regular VG when detecting the singular
 330 extremes. In this case, only one month (July) from the ozone concentration time
 331 series was chosen for the sake of clarity (see Figure 4a). This month was
 332 chosen because, in this location, July is the period of the year were the most

333 severe episodes of ozone pollution occur. In the next two figures (Figure 4b and
334 c), the normalized betweenness and degree values are shown for both
335 networks (blue is for the original VG, while red for the UDVG). Only these two
336 centrality parameters were chosen in this case because they have clearer
337 signals. The three centrality parameters presented in the methodology section
338 of this work will be used in later discussions.

339 It can be regarded in Figure 4 the fact that both networks (the regular and
340 inverted one) are able to identify extrema in the time series in a complementary
341 manner, as anticipated. While the regular VG hubs correspond maximal
342 episodes of tropospheric ozone concentration (which has been already used),
343 the UDVG obtained ones do the same with minima of the concentration. These
344 latter correspond to the nighttime, when the photochemical reaction is
345 unbalanced towards *NO* formation in the absence of radiation. The actual
346 physical interpretation of the different centrality parameters can be observed in
347 the previous related work (Carmona-Cabezas et al., 2020) for the regular VG
348 hubs. Additionally, it will be explained for the UDVG case in the last figures.

349 Moreover, it must be noticed how the hubs from betweenness coincide with
350 those of the degree, while the opposite case is not always true. Therefore, the
351 first one may be a more selective approach to identify singular nodes in a
352 signal, as it has been discussed in a previous work (Carmona-Cabezas et al.,
353 2019b). This filtering feature might be useful for the use of this technique on
354 environmental series where the density of extrema is considerably high.



355

356 Figure 4: a) Ozone concentration time series from a selected month (July). b)
 357 and c) Normalized betweenness and degree centrality values, respectively,
 358 for each point in the time series, from the two graphs studied (VG in blue and
 359 UDVG in red). The dashed lines are used to highlight the hubs positions and
 360 compare them in the three plots.

361 Once the difference between the VG and UDVG hubs has been discussed,
 362 authors propose an approach for combining the information given by both
 363 networks. The aim is to yield a more complete technique for future
 364 investigations to analyze pollutant time series. The combined parameters tested
 365 here simply consist on adding each VG centrality parameter to the opposite of
 366 the one computed using UDVG. To make it clearer, for the betweenness,
 367 degree and closeness:

$$\begin{cases} b_i^{comb} = b_i^{VG} - b_i^{UDVG} \\ k_i^{comb} = k_i^{VG} - k_i^{UDVG} \\ c_i^{comb} = c_i^{VG} - c_i^{UDVG} \end{cases} \quad (4)$$

368 This transformation is useful for the identification of singularities or extreme
 369 values, considering both the minima and maxima values. It is based on the fact
 370 that when the VG maps a hub, the UDVG will not, since they are

371 complementary, as exposed in the Methodology section. Thus, the hubs
372 information is not lost by this procedure, because their values will not be
373 canceled out for the case of extremes. Consequently, it improves the
374 differentiation between regular and singular values. This results in the derived
375 combined degree signal being smoother and clearer than in the separated case.
376 For the combined betweenness, there will be almost no difference in the
377 smoothness, since the values that do not correspond to skyline hubs are
378 practically zero in any case.

379 For clarity reasons, the same structure as in a previous work (Carmona-
380 Cabezas et al., 2020) has been followed for the plots. Hence, the combined
381 betweenness is computed first and from it, the five most pronounced peaks are
382 chosen automatically. Equivalent results can be yielded by selecting a greater
383 number of peaks, as it has been tested. A criterion for it was indicated in the
384 mentioned previous work (Carmona-Cabezas et al., 2020). Afterwards, the
385 remaining centrality measures were analyzed in the positions where the first
386 peaks are located. It must be highlighted that all the plotted parameters are
387 normalized to the maximum absolute value of each one, for the sake of
388 comparison.

389 In Figure 5, this explained procedure is performed using the series of
390 tropospheric ozone previously introduced. As it is easily seen, the accordance
391 between the different studied parameters is adequate, as it was expected. The
392 combination of the VG and UDVG still preserve the capability to identify
393 extrema by the different centrality parameters. The smoothest series
394 corresponds to that of the betweenness as previously explained, followed by the
395 degree and finally by the closeness. It is in accordance to what was observed

396 using the less complete method in the previous paper (Carmona-Cabezas et al.,
397 2020). It must be stressed that the order of the magnitude of the different peaks
398 is not conserved in the different combined centrality parameters. For instance,
399 in Figure 5c, the peak 2 is the most negative one, while in Figure 5d and Figure
400 5e are the peaks 3 and 5 respectively. This is due to the different physical
401 meanings of each parameter related to the concentration time series. Therefore,
402 this should be taken in consideration if different parameters are used to
403 compare ozone (or other pollutant) extreme concentration episodes in future
404 studies.

405 The first one of them is the combined betweenness (Figure 5c). In order to
406 understand the usefulness of this parameter to the photochemical pollution, it
407 must be pointed out that in a previous study (Carmona-Cabezas et al., 2019b)
408 skyline hubs were related to values of the series which can give more
409 information about its upper envelope. In short, one of the detected O_3
410 singularities may be considered as an unlikely high episode of ozone
411 concentration in relation to other maxima in the same series. This means that if
412 ozone daily maximal concentrations were raising for several consecutive days,
413 a peak in the betweenness indicates that after this encountered, the trend is
414 likely to change to a downwards one. Conversely, translating this to the inverted
415 series (and the resulting UDVG), the same could be inferred about minimal
416 night O_3 concentrations. Environmentally speaking, a change in the tendency
417 could be a pointer to an alteration of the previous ambient conditions that would
418 lead to an abnormal shift in the height of the mixing layer, for instance.
419 Therefore, the combined betweenness can serve as a more complete warning
420 tool, pointing changes in the conditions that affect the temporal evolution of

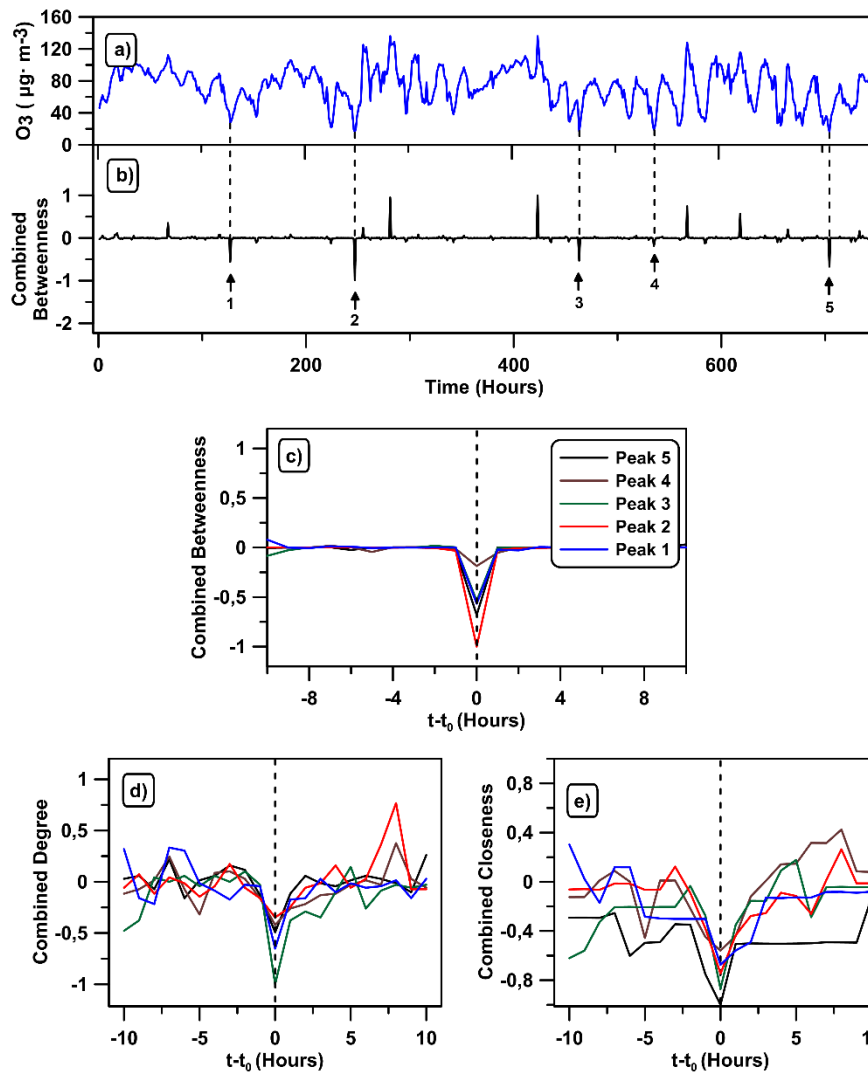
421 pollutants concentration, while the previous approach would only yield insight
422 on the upper one, limiting the analysis.

423 Regarding the next complex network indicator, the combined degree (Figure
424 5d), many works have been devoted to its study (Pierini et al., 2012; Zhou et al.,
425 2017). It is known that a degree hub is associated to a specially high ozone
426 concentration episode (Carmona-Cabezas et al., 2019a). At the position where
427 the hubs are encountered, the gas has reached a peculiarly high concentration.
428 This condition is less restrictive, as every day it is fulfilled. As a result, the
429 number of this type of peaks is greater, compared to the betweenness. In this
430 case, a peak would not be necessarily associated to a change in the prior
431 tendency of the O_3 concentrations. When the regular VG results are combined
432 with the UDVG, the unlikely low values of concentration can be identified as
433 well. Again, the identification of rare concentrations of O_3 is improved by this
434 combination, getting at the same time the information from low and high values
435 from one single parameter. Here, the sense of “singularity” in the ozone is
436 referred only to its magnitude, and not to the trend of the previous and posterior
437 days, as in the betweenness.

438 Figure 5e) illustrates the closeness centrality results. In previous research,
439 this quantity was mainly used for theoretical purposes. Nonetheless, it was
440 demonstrated recently that it could identify singularities as the previous ones,
441 but with a different criterion (Carmona-Cabezas et al., 2020). The peaks of this
442 magnitude were related to high concentrations of ozone episodes surrounded
443 by concave up tendency. This quantity was found to be noisier than
444 betweenness and degree, and so it is as well here. As in the previous
445 parameters, now the combined quantity (more specifically the negative part),

446 gives additionally information about the points where a minimal rare
447 concentration value is found, surrounded by a concave down accumulation of
448 values (the reversed shape with respect to the regular VG). In the context of
449 photochemical pollution, it would mean that this could be used to identify daily
450 high concentrations (during the photostationary state) that somehow drop, due
451 for instance to unexpected atmospheric conditions.

452 The selected minima correspond to the 6th, 11th, 20th, 23rd and 30th of July,
453 all of them occurring between 6:00 AM and 7:00 AM (UTC+1) as could be
454 expected. The photochemical reaction is reverted during the nighttime and most
455 of the tropospheric ozone (O_3) (produced during the previous day) is
456 recombined with NO to yield NO in the absence of light. After this time, there is
457 radiation available and its concentration has an upward trend. In the previous
458 work, the singular high episodes between 2:00 PM and 6:00 PM (UTC+1),
459 which corresponds to the opposite case.



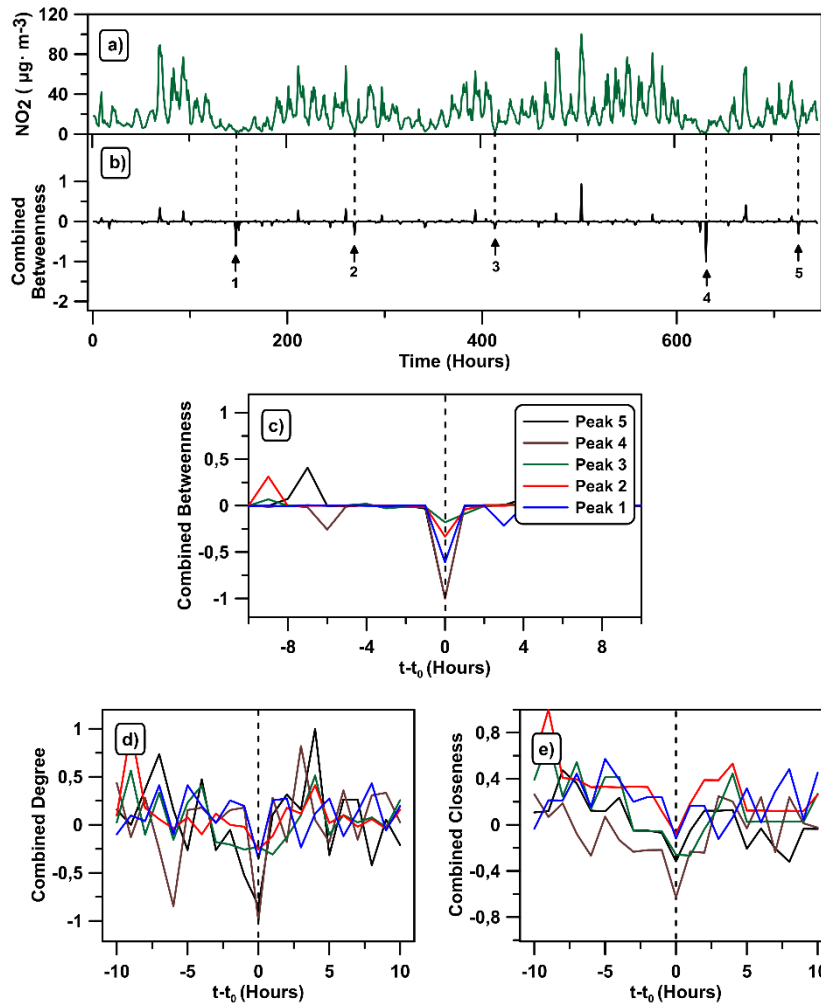
460

461 Figure 5: Ozone concentration time series (a) with the combined betweenness
 462 computed from the UDVG and VG (b). Plots from c) to e) show the complex
 463 network indicators: betweenness, degree and closeness in the selected five
 464 negative peaks.

465 The next graph (Figure 6) shows the results obtained for the nitrogen dioxide
 466 (NO_2) concentration time series, which as seen before, has a different minima
 467 and maxima behavior. For this study, the studied month is January as in the
 468 previous work, since in this region that is period of the year when less
 469 photochemical activity takes place. Therefore, reactions with other chemicals
 470 (such as VOCs to yield aldehydes) could play a more important role, leading to
 471 more singular extrema.

472 Once again, there is a good fit between the different parameters, although in
473 this case, the combined degree is noisier and not as clear as before. This might
474 be caused by the accumulation of low values of nitrogen dioxide (NO)
475 concentration that make the distribution to be more asymmetric, as discussed
476 (see Figure 3). The greater number of reactions that involve NO might increase
477 the number of singularities, being the degree noisier as a result.

478 In this case, the selected concentrations of NO are in the 7th, 12th, 18th, 27th
479 and 31st of January, between 2:00 AM and 5:00 AM (UTC+1). Regarding the
480 high singularities investigated in the previous work, there was no consistent
481 time frame where it could be encountered. Also, it is well known that there is a
482 marked difference between concentrations during weekends and weekdays
483 (Qin, 2004), which could be another possible cause for this uncertainty.



484

485 Figure 6: Nitrogen dioxide (NO_2) concentration time series (a) with the
 486 combined betweenness computed from the UDVG and VG (b). Plots from c) to
 487 e) show the complex network indicators: betweenness, degree and closeness in
 488 the selected five negative peaks.

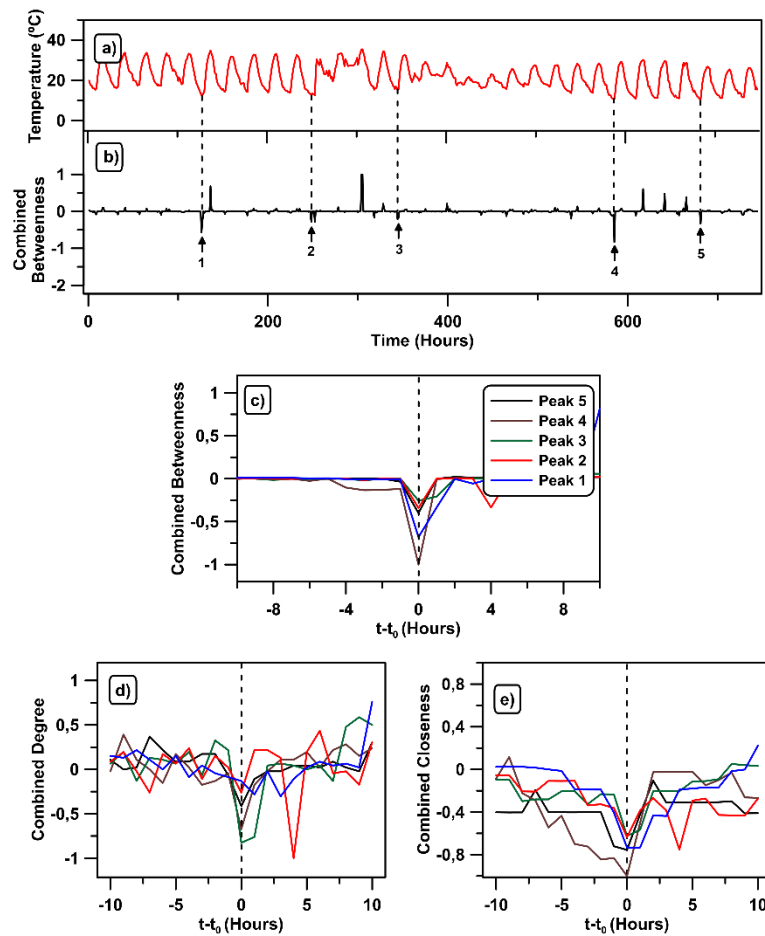
489 Finally, in Figure 7 the temperature time series is studied locally as in the
 490 previous two cases. Now the selected month is October, in order to observe
 491 singular episodes of this quantity. Due to the oceanic influence, the temperature
 492 is stable throughout almost all the year. Nevertheless, it is more unstable in
 493 autumn in this area, as discussed in previous works (Dueñas et al., 2004).

494 It is clearly seen that for temperature there is also concordance between the
 495 combined betweenness and the rest of combined centrality parameters. Now
 496 the combined degree signal has less noise than in the case of NO , except for

497 Peak 2. This one is more difficult to identify due to the fact that there are two
498 betweenness peaks very close to each other (see Figure 7 b).

499 The temperature singular minima that have been selected, following the
500 previous criterion, correspond to the 6th, 11th, 15th, 25th and 29th of October,
501 between 5:00 AM and 8:00 AM (UTC+1). It could be easily expected, since it is
502 the time when the minimum temperature is reached every day. Even during the
503 days in which the temperature becomes more unpredictable (around the middle
504 part of the month), these minima can be observed with a relatively constant
505 frequency.

506



507

508 Figure 7: Temperature time series (a) with the combined betweenness
509 computed from the UDVG and VG (b). Plots from c) to e) show the complex
510 network indicators: betweenness, degree and closeness in the selected five
511 negative peaks.

512 4. CONCLUSIONS

513 An improvement of a singularity detection technique is tested for its
514 application on photochemical time series in this manuscript. It adds the
515 possibility of describing singular minima and maximal singular values at the
516 same time, making it a more complete tool. Authors believe that it may have a
517 great potential for monitoring and analyzing pollutant and atmospheric time
518 series in the future.

519 The degree distributions obtained have been compared, proving that UDVG
520 inherits the nature of the original NO_2 , O_3 and temperature time series.
521 Moreover, different theoretical series have been tested, proving the suitability of
522 both VG and UDVG. It has been found that those distribution are coincident for
523 tropospheric ozone (O_3) and temperature, while they are not for the nitrogen
524 dioxide (NO_2). Their disparity has been related to the greater number of
525 reactions that involve NO_2 , such us its interaction with VOCs to yield aldehydes.
526 This must be investigated more in detail in a future study, applying different
527 complex networks tools developed to series of NO_x , VOCs and O_3 at the same
528 time.

529 Furthermore, the usefulness of UDVG for singular minima detection has
530 been successfully proven on the NO_2 , O_3 and temperature series. The
531 combination of VG and UDVG parameters (degree, betweenness and
532 closeness) is proposed as a more exhaustive method, compared to only
533 employing VG. Due to their complementary nature, these combinations store
534 the original information of the most central nodes, showing all the relevant
535 information at a glance. To authors' mind, this can widen the range of the

536 research applications of complex networks for photochemical pollution in a
537 future.

538

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545

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Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

1 **HIGHLIGHTS**

- 2 - Detection of singularities using graphs is improved by taking the inverted
3 series.
- 4 - Maxima and minima of pollutant series are identified by VG and UDVG
5 respectively.
- 6 - Asymmetries in the distribution of *NO* might be caused by reaction with VOCs.
- 7 - *NO* singularity identification is more difficult due to its more complex
8 dynamics.
- 9 - A more complete analysis tool is obtained by combining both approaches.

