Pattern Recognition 138 (2023) 109396

Contents lists available at ScienceDirect

Pattern Recognition

journal homepage: www.elsevier.com/locate/patcog

Assessing polygonal approximations: A new measurement and a comparative study

Nicolás Luis Fernández-García*, Luis Del-Moral Martínez, Ángel Carmona-Poyato, Francisco José Madrid-Cuevas, Rafael Medina-Carnicer

Department of Computer Science and Numerical Analysis, Córdoba University, Spain; and Maimonides Institute for Research in Biomedicine (IMIBIC), Avenida Menéndez Pidal s/n, Córdoba 14004, Spain

ARTICLE INFO

Article history:
Received 8 July 2021
Revised 24 November 2022
Accepted 5 February 2023
Available online 11 February 2023

Keywords:
2D Closed curve
Contour
Polygonal approximation
Performance evaluation

ABSTRACT

Two proposals related to the evaluation of polygonal approximations are presented in this document. First, a new measurement, called *normalized compression ratio and adjustment error* (NCA), to provide a fair evaluation of the performance of the polygonal approximations of 2D closed curves is proposed. Second, a new methodology for evaluation of measurements for assessing polygonal approximations is also proposed. This methodology is based on the *optimal quality curve* concept, which can characterize the performance of the measurements. A simple visual analysis of the *optimal quality curve* allows possible drawbacks or weaknesses of the measurement to be detected. The new evaluation methodology is used to compare the performance of the proposed NCA and the most popular measurements, such as Rosin's Merit, FOM or versions of FOM. Experiments show that NCA obtains the best results and, therefore, may be used to fairly evaluate the performance of polygonal approximations.

1. Introduction

The characterization of the image processing algorithms is essential to analyze new techniques and to improve the performance of real-world applications of computer vision research [1]. Digital image processing consists of some fundamental steps: image acquisition, preprocessing, segmentation, representation and description, and recognition and interpretation [2]. The present work focuses solely on the evaluation of a specific type of image representation: polygonal approximation of 2D closed curves or *contours*, which is a very important topic of shape representation [3]. Shape representation based on polygonal approximation is extensively used for constructing a characteristic description of a contour in the form of a series of straight lines or piece-wise linear approximation. This representation is very popular due to its simplicity, locality, generality and compactness [4], and it has been used in many applications, such as compression [5], digital cartography [6], shape classification [7] or remote sensing [8].

Given a *contour* $C = \{(x_i, y_i) | i \in \{1, \dots, N\}\}$, a polygonal approximation algorithm looks for a subset of points $P = \{(x'_i, y'_i) | i \in \{1, \dots, N_p\}\}$, where $P \subseteq C$ and $N_p \leq N$, so that P represents the entire contour C properly. The points of the polygonal approximation are usually called *dominant points* (DP). The problem of the generation of polygonal approximations of a contour can be approached in two different ways [9]: (1) *minimum-distortion problem* or (2) *minimum-rate problem*. The algorithms based on the minimum-distortion problem or *Min* $- \epsilon$ *problem* consider a predefined number d of dominant points and try to generate the *optimal* polygonal approximation with the minimum error from the contour among all the approximations with d dominant points [10]. Instead, the algorithms focused on the minimum-rate problem or *Min* $- \#$ *problem* try to generate the *optimal* polygonal approximation with the minimum number of dominant points, so that its adjustment error from the contour is less than a predefined error ϵ [11]. The polygonal approximations algorithms can be classified as (1) optimal or non-optimal and (2) supervised or unsupervised algorithms [12]. The optimal algorithms are based on an optimization criterion but have two main drawbacks [13]: the optimum depends on the applied criterion and requires a very high time complexity. On the other hand, the non-optimal algorithms are not designed to guarantee any kind of optimum, but can find reasonable polygonal approximations for real-time applications. For both optimal and non-optimal approaches, the supervised algorithms take into account

* Corresponding author.
E-mail address: ma1fegan@uco.es (N.L. Fernández-García).

https://doi.org/10.1016/j.patcog.2023.109396

one or more parameters to generate the polygonal approximations and, therefore, this is their main drawback, because these parameters must be tuned [14]. On the contrary, the unsupervised algorithms generate the polygonal approximations without using any kind of parameters [12].

Many measurements were proposed to evaluate the quality of the polygonal approximations, but all of these measurements suffer from very serious deficiencies or weaknesses (Section 2). Owing to this reason, a new evaluation measurement, called *normalized compression ratio and adjustment error* (NCA), to provide a fair evaluation of the performance of the polygonal approximation algorithms is proposed (Section 3). In order to evaluate the performance of the measurements for polygonal approximations, including the new proposal NCA, a new evaluation methodology, based on the *optimal quality curve* concept, is also proposed (Section 4) and used in the experiments (Section 5). The present paper is arranged as follows. Section 2 describes the related work. Section 3 explains the new measurement NCA for assessing polygonal approximations. Section 4 describes the new methodology, based on the *optimal quality curve* concept, to evaluate the quality of polygonal approximations. The experiments and results are detailed in Section 5. Finally, the main conclusions and future work are summarized in Section 6.

2. Related work

Two approaches are used to evaluate the quality of a polygonal approximation: *subjective* and *objective* [15]. The subjective approach is a qualitative evaluation in which several human observers visually compare the original contour with the polygonal approximation [16]. This approach is easy to apply, but cannot be automated and depends on the particular criteria of the observers. On the other hand, the objective approach is a quantitative evaluation in which two criteria must be taken into account [15]: (1) the *number of points* of the polygonal approximation and (2) its *adjustment error* to the contour. The objective approach can be automated but has a main drawback: the two criteria on which it is based are opposed to each other. Table 1 classifies the measurements for quantitative evaluation into categories according to the number of points of the polygonal approximation and the adjustment errors of distance, area or length. Two main categories are considered: (1) uncombined measurements and (2) combined measurements. The uncombined measurements are based solely on the number of points or on a single type of error, while the combined measurements use two or more of them.

The most common measurement to evaluate the number of points of the polygonal approximation is *compression ratio*: $\mathrm{CR} = \frac{N}{N_p}$, where N is the number of contour points and N_p is the number of points of the polygonal approximation or *dominant points* (DP). If DP decreases then CR increases, and vice versa. Similar measurements to CR are: (1) the *percentage of data reduction* [17] or (2) the *compression percentage* [18].

Different measurements were proposed to evaluate the adjustment error, such as (Table 1): (1) the *integral square error*: $\mathrm{ISE} = \mathrm{E}_2 = \sum_{i=1}^{N} e_i^2$, where e_i is the distance of the contour point P_i from the polygonal approximation; or (2) the *maximum deviation error*: $E_\infty = \max_{i\in\{1,\ldots,N\}}\{e_i\}$. Similar measurements to E_∞ were the *parallel strip* and the *minimum width* [19]. Other approaches considered the area deviation error [20], but these measurements cannot truly judge the quality of approximation as the error area outside the curve is balanced by an inside error area [21]. The length was also used to define new measurements [22]. Some measurements combined the adjustment errors and the length [23].

In general, if the number of points is reduced, then the error increases; otherwise, if the number of points is increased, then the error decreases. Because of this, the aim of the objective evaluation must be to achieve the best *trade-off* between the *number of points* and the *adjustment error* of the polygonal approximation [15]. The *Figure of Merit* $\mathrm{FOM} = \frac{CR}{ISE}$ was proposed [24] to make the *trade-off* between the compression ratio and the total distortion caused [25]. However, the terms CR and ISE used by FOM are not balanced, causing the measure to be biased towards approximations with lower ISE and many dominant points. Hence, FOM is not the best measure for comparing contours with different numbers of dominant points [15]. Three versions of FOM were: (1) *weighted sum of squared error*: $WE_2 = \frac{ISE}{CR}$ [26], defined as the inverse of *FOM*, (2) *weighted maximum error*: WEmax or $WE_\infty = \frac{E_\infty}{CR}$ [26] and (3) *modified figure of merit* [27] or $\mathrm{MFOM}-3 = ISE\ E_\infty\ (N_p)^3$ [28]. Other versions of FOM were also proposed [29]. Technically, all these measurements are similar to FOM and, therefore, suffer similar problems [21]. The parametric version $\mathrm{FOM}_n = \frac{CR^n}{ISE}$, where $n \in \{1, 2, 3\}$, was introduced to control the contribution of the compression ratio to the overall result to reduce the imbalance between the two terms and was motivated by the observation that ISE changes more rapidly than CR for almost all test shapes [30]. The inverse of FOM_n, defined as $WE_2^n = \frac{ISE}{CR^n}$, where $n \in \{1, 2, 3\}$, was also used [31]. WE_2^2 makes a better *trade-off* between compression ratio and error, because WE_2^1 and WE_2^3 favor polygonal approximations with many or few dominant points, respectively [32].

Other measurements were proposed by Rosin [15] or Carmona [32] to take into account the comparisons with optimal polygonal approximations, but this is their main difficulty, because obtaining optimal solutions is computationally very expensive [33].

Rosin's Merit [15] was defined as : $Merit = \sqrt{Fidelity \times Efficiency}$, where $Fidelity = \frac{E_{opt}}{E_{appr}} \times 100$ and $Efficiency = \frac{N_{opt}}{N_{appr}} \times 100$ where E_{appr} and N_{appr} are the error and the number of dominant points of the sub-optimal polygonal approximation, E_{opt} is the error produced by the optimal algorithm with the same number of dominant points and N_{opt} represents the number of dominant points that would require an optimal algorithm to produce the same error. *Rosin's Merit* can compare polygonal approximations with different number of dominant points. Nevertheless, this measurement also suffers a few weaknesses: the polygon that consists of just *break points* (points where the boundary takes a turn) will produce $Fidelity = 100$, $Efficiency = 100$ and $Merit = 100$. It means that the set of break points taken as dominant points will produce a perfect approximation, but this type of approximation is of no practical use since its compression ratio is very low [25] and most of break points do not provide relevant information. On the other hand, the optimal polygonal approximation with only three dominant points would also have the best value of *Rosin's Merit*, but would hardly resemble the contour [32], unless it was triangular in shape.

Carmona's Merit [32] is another measurement for optimal polygonal evaluation. If C is the original contour and P is the polygonal approximation to be evaluated, then the following steps must be carried out: (1) the optimal polygonal approximation for i points, where $i \in \{3, \ldots, n_b\}$ and n_b is the number of break points of C, must be obtained using an optimal algorithm [34]; (2) the value of $F_2 = \frac{ISE}{CR^2}$ must be computed for every optimal polygonal approximation; (3) an unimodal thresholding algorithm [35] is applied to the values of F_2 to select a *reference* optimal polygonal approximation; and, finally (4) this *reference* is used to evaluate the polygonal approximation P using the values of the fidelity and efficiency. This measurement has several and important drawbacks: (1) it has very high computational complexity, because many optimal polygonal approximations must be computed; (2) the *reference* optimal polygonal approximation is based on the values of F_2, but this measurement, which is called WE_2^2 in this document, does not always allow a quality *reference* polygonal approximation to be obtained. See the experiments in Section 5 for details.

Table 1
Measurements for Quantitative evaluation: classification of measurements into categories according to the number of polygonal points and distance, area or length errors.

Measurement [refs]	Number of points	Distance error		Area error	Length error
		Sum	Maximum		
Uncombined measurements					
Compression percentage: $\tau\% = 100 \times \frac{N_p}{N}$ [18]	X				
Compression ratio: $\mathrm{CR} = \frac{N}{N_p}$	X				
Percentage of data reduction: $100 \times \frac{N-N_p}{N}$ [17]	X				
$E_1 = \sum_{i=1}^{N} e_i$ [36]		X			
Integral square error: $\mathrm{ISE} = \mathrm{E}_2 = \sum_{i=1}^{N} e_i^2$		X			
Maximum deviation error: $E_\infty = \mathrm{Emax} = \max_{i\in\{1,\ldots,N\}}\{e_i\}$			X		
Minimum width [19]			X		
Parallel strip or infinite beam [19]			X		
Area between approximation polygon and curve $E_1 = \lvert A - A_p \rvert$ [37]				X	
Average (normalized) area deviation: $\frac{Sum}{N} = \frac{1}{N}\sum_{i=1}^{N} A_i$ [38]				X	
Mean area error: $\mathrm{MAE} = \frac{1}{N_p}\sum_{i=1}^{N_p} A_i$ [38]				X	
Normalized mean area error: $\mathrm{NMAE} = \frac{MAE}{T_d^2}$ [38]				X	
Relative area deviation: $E_A = 100 \times \frac{\lvert A-A_p \rvert}{A}$ [20]				X	
Approximate error [39]: $\epsilon = \frac{L-L_p}{L}$					X
Difference between the lengths: $\lvert L - L_p \rvert$ [40]					X
Efficiency of approximation: $100 \times \frac{L_p}{L_{optimal}}$ [22]					X
Length ratio: $R = \frac{L}{L_p}$ [41]					X
Length ratio: $LR = \frac{L_p}{L}$ [42]					X
Combined measurements					
Additive cost function $C = ISE + \lambda \times N_p$, [43]	X	X			
Average (normalized) integral square error: $\frac{E_{intsq}}{N} = \frac{ISE}{N}$ [44]	X	X			
Compression-pertinence ratio 1 $C\,Pr1 = 100 \times \frac{Pr^A}{CR}$ [36]	X	X			
Compression-pertinence ratio 2: $C\,Pr2 = 100 \times \frac{Pr^B}{CR}$ [36]	X	X			
Compromise ratio: $100 \times \frac{ISE}{CR}$ [45]	X	X			
Figure of Merit: $\mathrm{FOM} = \frac{CR}{ISE}$ [24]	X	X			
Inverse of Parametric Figure of Merit: $WE_2^n = \frac{ISE}{CR^n}$, $n \in \{1, 2, 3\}$ [31]	X	X			
Merit $= \sqrt{Fidelity \times Efficiency}$ [15]	X	X			
NCA $= \frac{1}{2} \times (\frac{1}{CR} + \frac{2}{1+e^{-\frac{\sqrt{ISE}}{D}}} - 1)$ (**proposal**)	X	X			
New parametric figure of merit: $\mathrm{FOM} - \mathrm{a} = ISE\ (N_p)^a$ [43]	X	X			
Optimization error: $E_0 = \frac{ISE \times N_p}{N^2} = \frac{ISE}{CR \times N}$ [18]	X	X			
Parametric Figure of Merit: $\mathrm{FOM}_n = \frac{CR^n}{ISE}$, $n \in \{1, 2, 3\}$ [30]	X	X			
Percentage relative difference: $PRD = \frac{E_{approx} - E_{opt}}{E_{opt}} \times 100$ [46]	X	X			
Reference polygonal approximation [32]	X	X			
Relative error $E_r = \frac{\sqrt{ISE}}{CR}$ [47]	X	X			
Root mean square error: $\mathrm{RMS} = \frac{\sqrt{ISE}}{N}$ [48]	X	X			
Root mean square error: $\mathrm{RMSE} = \sqrt{\frac{ISE}{N}}$ [49]	X	X			
Weighted sum of squared error: $WE_2 = \frac{ISE}{CR}$ [26]	X	X			
Modified figure of merit: $\mathrm{modifiedFOM} = \frac{CR^2}{ISE} \times \frac{CR}{E_\infty} = \frac{CR^3}{ISE \times E_\infty}$ [27]	X	X	X		
Modified figure of merit: $\mathrm{MFOM} - 3 = ISE\ E_\infty\ (N_p)^3$ [28]	X	X	X		
Average max error: $\frac{1}{N_p}\sum_{i=1}^{N_p} \mathrm{MaxErr}_i$ [29]	X		X		
$\mathrm{Merit}_\infty = \sqrt{Fidelity_\infty \times Efficiency_\infty}$ [50] (based on E_∞)	X		X		
New parametric figure of merit: $\mathrm{FOM} - \mathrm{a} = E_\infty (N_p)^a$ [28]	X		X		
Weighted maximum error: WEmax or $WE_\infty = \frac{E_\infty}{CR}$ [51]	X		X		
Deviation ratio: $r = \frac{ISE}{L}$ [23]		X			X
Ratio of the length and maximum error: $\frac{L_p}{E_\infty}$ [6]			X		X
Relative error [46] or normalized maximum error [3]: $\frac{E_\infty}{L}$			X		X
Sum of normalized maximum deviations: $\sum_{k=1}^{N_p} \frac{max_{k<i<k1} e_i}{L}$ [52]			X		X
Normalized area deviation: $\frac{E_1}{L}$ [37]				X	X
Standard area deviation: $\frac{\lvert A-A_p \rvert}{L}$ [37]				X	X

In summary, many measurements were proposed to evaluate polygonal approximations, but none of them is considered unanimously the best one. Furthermore, it is unknown that a comparative study of measurements for assessing polygonal approximations has been carried out. Due to these reasons, a new assessment measurement (NCA) is proposed (Section 3) and its performance is compared with the most popular measurements (Section 5) using the new evaluation methodology based on the *optimal quality curve* concept (Section 4).

3. New assessment measurement

A new objective assessment measurement, called *normalized compression ratio and adjustment error* (NCA), is proposed to evaluate the quality of the polygonal approximations. Four criteria were considered to design NCA: (1) both the *number of points of the polygonal approximation* and the *adjustment error*, which are opposed to each other, must be taken into account; (2) the values of the measurement must be normalized between 0 (best value) and 1 (worst value); (3) its computational complexity must be linear ($\mathcal{O}(n)$), where n is the number of contour points; and (4) for each contour, only one polygonal approximation should be considered the best. NCA is defined as $\frac{1}{2} \times \left(\frac{1}{CR} + NISE\right)$ where $\frac{1}{CR}$ is the inverse of compression ratio and is computed as $\frac{1}{CR} = \frac{N_p}{N}$, where N and N_p are the contour points and the dominant points of the polygonal approximation, respectively. The inverse of compression ratio measures the number of points of the polygonal approximation and is normalized in the interval [0,1]. If the polygonal ap-

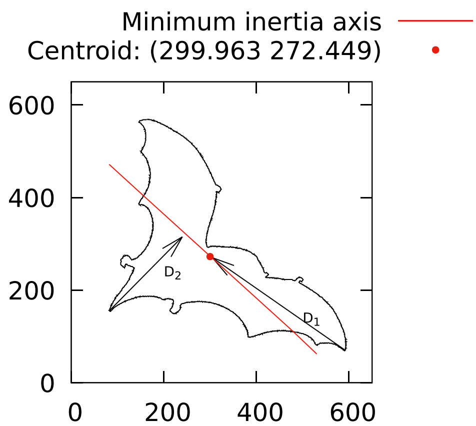


Fig. 1. Contour *bat-1* (N=1960). Maximum distance from the contour points to the centroid: $D_1 = 355.347$. Maximum distance from the contour points to the minimum inertia axis: $D_2 = 233.091$. $D_3 = \frac{D_1+D_2}{2} = 294.219$.

proximation consists of all the contour points, then the inverse of compression ratio takes the worst value: 1.0. NISE is the *normalized integral square error* and is defined as $\frac{2}{1+e^{-\frac{\sqrt{ISE}}{D}}} - 1$. NISE measures the adjustment error of the polygonal approximation to the contour. A sigmoid function is used to normalize the values of NISE in the interval [0.0,1.0). The square root and the division by the constant D are used to reduce the magnitude of ISE. D is a user-defined constant greater than 0.0. In the experiments, the value of D was defined as: $D = \frac{D_1+D_2}{2}$, where D_1 was the maximum distance from the contour points to the centroid, and D_2 was the maximum distance from the contour points to the minimum inertia axis (Figure 1).

The definition of NCA satisfies the first three criteria listed above: (1) both the number of points and the adjustment error are used; (2) the values of the measurement are normalized between 0 and 1; and (3) its computational complexity is linear. The fourth criterion, related to the best polygonal approximation for each contour, is also satisfied, because NCA always allows to obtain the global optimum minimum value. A modeling process is done to verify this fact, in which NCA can be modeled using the number of the points of the polygonal approximation. Without loss of generality, a contour with $N = 100$ points and a polygonal approximation with n dominant points are considered. In this case, $\frac{1}{CR} = \frac{n}{100}$. The value of NISE is computed using ISE, which decreases when the value of n increases and vice versa. Owing to this, *ISE* can be *roughly* modeled by the inverse of the number of dominant points n: $ISE \approx \frac{1}{n}$. Taking into account these two expressions, the value of NCA can be *roughly* modeled by the function: $f(n) = \frac{1}{2} \times (\frac{n}{100} + \frac{2}{1+e^{-\sqrt{\frac{1}{n}}}} - 1)$ This function is continuous and allows to obtain the global optimum minimum value, which corresponds to the best polygonal approximation (Fig. 2). The experiments also confirm that NCA fulfills this fourth criterion (Section 5).

4. Optimal quality curve

A new evaluation methodology, based on the *optimal quality curve* concept, is proposed to characterize the performance of the assessment measurements of the polygonal approximations. This methodology is inspired by the characterization of empirical discrepancy evaluation measures [53]. An *optimal quality curve* OQC(M, C) can be generated for every combination of a measurement (M) and a contour (C): $OQC(M,C) = \{(i, M(OPA(i)))|i \in \{3,\dots,N\}\}$, where OPA(i) is the global optimal polygonal approximation of the contour C with i dominant points, and $M(OPA(i))$ is the error of OPA(i) computed by the measurement M (Figs. 4 and 7). The global optimal polygonal approximation OPA(i) is calculated using an optimal algorithm so that the integral square error (ISE) is the global minimum, regardless of the initial point of the contour C [33].

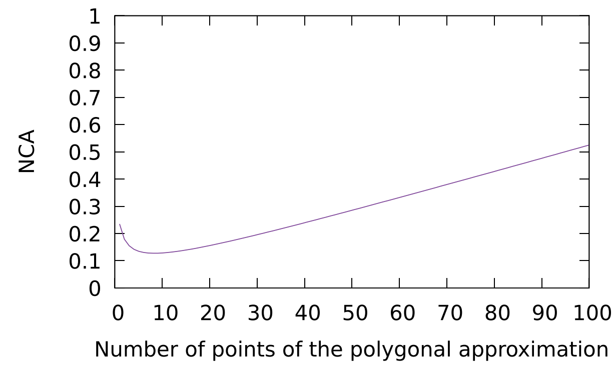


Fig. 2. Plot of the modeling of NCA curve: $f(n) = \frac{1}{2} \times (\frac{n}{100} + \frac{2}{1+e^{-\sqrt{\frac{1}{n}}}} - 1)$.

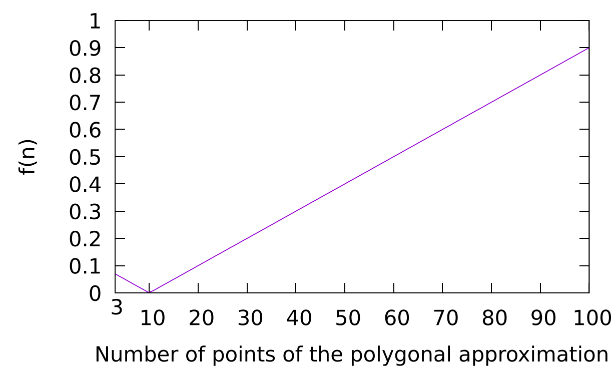


Fig. 3. Plot of an ideal *optimal quality curve*: $f(n) = \frac{|n-b|}{N}$, where $b = 10$ and $N = 100$.

A simple visual analysis of the *optimal quality curve* allows possible drawbacks or weaknesses of the measurement to be detected. If a measurement M assess the quality of the polygonal approximations in a decreasing[1] way, so that the best value is the lowest one, then the absolute minimum value of the optimal quality curve OQC(M, C) corresponds to the *best* global optimal polygonal approximation OPA(best), which gets the lowest value M(OPA(best)) for the measurement M among all global optimal polygonal approximations OPA(i) to the contour C for $i \in \{3,\dots,N\}$. The optimal quality curve should allow the best value M(OPA(best)) to be found easily. Besides, the optimal quality curve should not have local minimum values corresponding to very different number of dominant points. In short, the *optimal quality curve* should have a *global minimum* value and its first derivative must go from very negative values to very positive values. The desirable or ideal *optimal quality curve* would correspond to the function $f(x) = \frac{|x-b|}{N}$, where N is the number of contour points and b is the number of dominant points of the best polygonal approximation OPA(best), which obtains the absolute minimum value for the measurement M (Fig. 3). These properties must be true for every measurement and contour. On the other hand, if a measurement does not show these properties for a given contour then this measurement should not be used to evaluate the quality of the polygonal approximations of that contour because the measurement would be unfair with them.

[1] If a measurement M uses the increasing way then the "greatest" and the "maximum" must be considered.

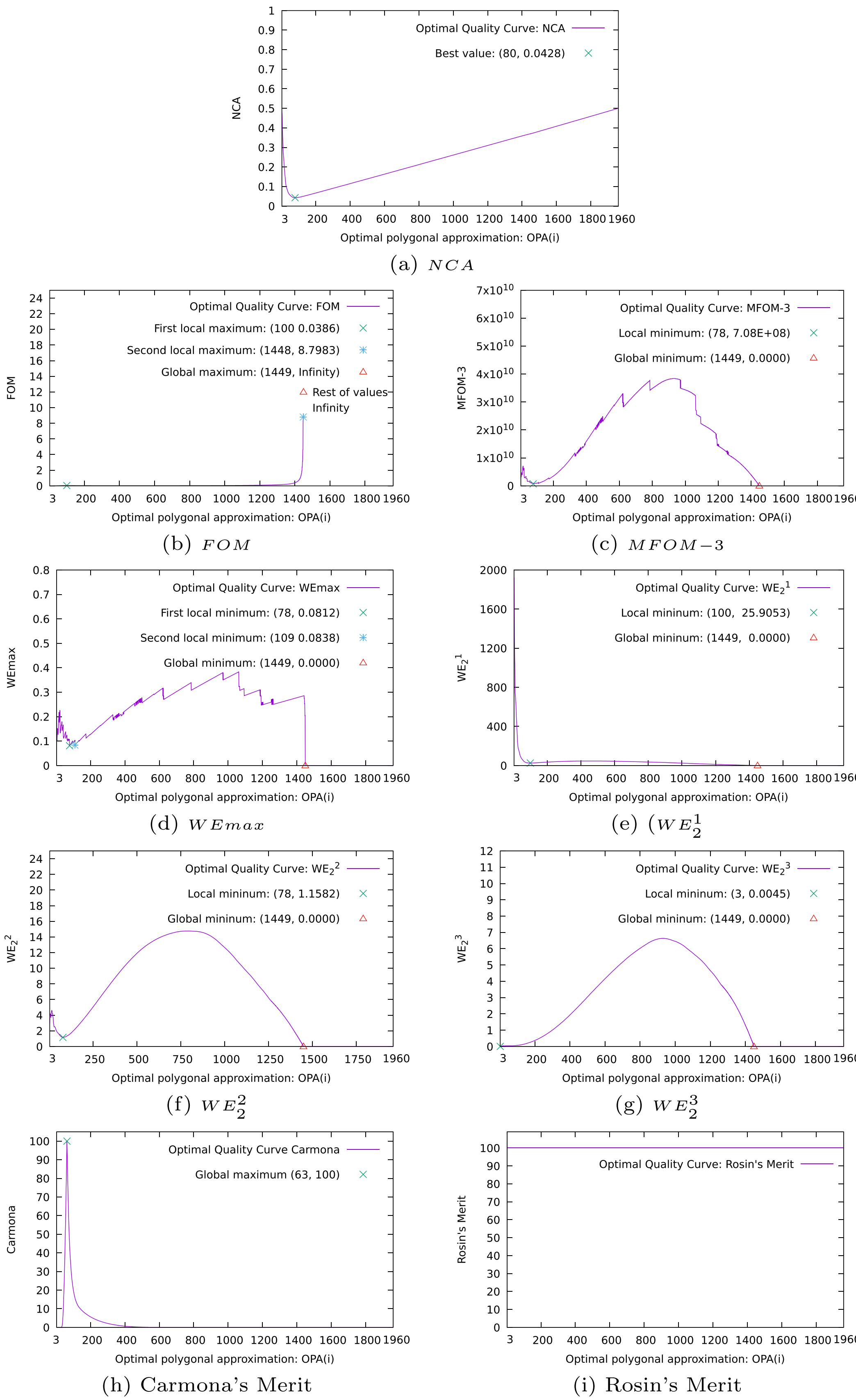


Fig. 4. Optimal quality curves generated by some measurements for contour *bat-1* ($N = 1960$; number of break points = 1449).

5. Experiments

In order to evaluate the performance of the most popular measurements for assessing polygonal approximations and the new proposal NCA (Section 3), the new experimental methodology (Section 4), was used in two experiments: (1) the first experiment showed how the *optimal quality curve* concept was applied to characterize the behaviour of the measurements when they were used to evaluate the quality of optimal polygonal approximations (Section 5.1); and (2) the second experiment compared the performance of NCA, WE_2^2, *Rosin's Merit* [15] and Carmona's Merit [32] using the well-known contour *semicircles* [54] (Section 5.2).

5.1. First experiment

The *optimal quality curve* concept (Section 4) was used to characterize the performance of the new proposal NCA (Section 3) and those of eight polygonal approximations measurements: FOM, WEmax, MFOM − 3, WE_2^1, WE_2^2, WE_2^3, *Rosin's Merit* [55] and *Carmona's Merit* [32] (Section 2), which were chosen owing to two reasons: (1) they combined the number of points and the error of the polygonal approximations; and (2) they were used frequently to compare polygonal approximations [12]. These measurements were used to evaluate the quality of the *optimal* polygonal approximations (regardless the initial point) of 70 shapes of "MPEG-7" database [56]. This database was used previously [12], is publicly available in [57] and contains 1400 images, classified into 70 categories, and each category includes 20 samples, with different rotation, size and position, and even image resolution [58]. Only the first shape contour of every category was used in the experiments. The *optimal quality curve* $\mathrm{OQC(M, C)}$ was generated for every combination of measurement M and contour C, that is to say, 9 $M \times$ 70 $C = 630$ $\mathrm{OQC(M, C)}$ were generated. Due to lack of space, only some *optimal quality curves* are plotted in Figs. 4, 7, 9, and 10. The full set of *optimal quality curves* is available in [59]. The best polygonal approximation proposed by some measurements for various contours are also shown in Figs. 6, 8, 9, and 10.

The analysis of the *optimal quality curves* of the measurements allows to highlight the following results. NCA generated *optimal quality curves*, where the best value (the minimum) can be found easily (Figs. 4(a), 7(a), 9(d), and 10(e)). Besides, the proposed *optimal* polygonal approximations have a reasonable number of dominant points (Figs. 6(d), 8(e), 9(f), and 10(f)).

FOM generated *optimal quality curves* in an increasing way, so that the best value (the maximum) is ∞, which was obtained for all *optimal* polygonal approximations with a number of points equal to or greater than the number of the *break points* of the contour (Figs. 4(b) and 7(b)). The optimal polygonal approximations proposed by FOM are shown in Figs. 6(e) and 8(f). As Rosin already pointed out [15], FOM is biased to favor polygonal approximations with a large number of points. MFOM − 3, WEmax, WE_2^1, WE_2^2 and WE_2^3 generated *optimal quality curves* in an decreasing way, so that the best value (the minimum) is 0.0, which was also obtained for all *optimal* polygonal approximations with a number of points equal to or greater than the number of the *break points* of the contour (Figs. 4 and 7). Obviously, a polygonal approximation with such a number of points cannot be considered the best (Figs. 6(g) and 8(f)), and, therefore, these measurements are not suitable for a fair polygonal approximation evaluation. A more detailed analysis shows that MFOM − 3 generated a very different *optimal quality curves*: a clear local minimum can be found for contour *bat-1* (Figs. 4(c) and 6(c)), but the local minimum for the contour *cellular_phone-1* is very close to the number of *break points* (Figs. 7(c) and 8(f)). WEmax generated a very irregular *optimal quality curves* with several local minimums (Figs. 4(d) and 7(d)), which corresponded to different polygonal approximations of the same contour (Fig. 6(c), (f) for *bat-1*, and Fig. 8(b) and (d) for *cellular-phone-1*). WE_2^2 generated *optimal quality curves* very different from each other. For instance, a local minimum can be found easily for contour *bat-1* (Figs. 4(f) and 6(c)), but this does not happened for other contours, like *cellular_phone-1* (Figs. 7(f) and 8(e)) or *truck-01* (Fig. 9(a), (b) and (e)). In some cases, WE_2^2 found a local minimum for an *optimal* polygonal approximation with few dominant points (Fig. 9(a) and (e) for contour *truck-01*) or with many points (Fig. 10(c) and (d) for contour *carriage-01*).

Table 2
Comparison of optimal polygonal approximations generated by *NCA* and WE_2^2 for contours *bat-1* ($N = 1960$), *carriage-01* ($N = 729$), *cellular_phone-1* ($N = 1102$) and *truck-01* in Fig. 5.

Contours (points)	NCA value	NCA DP	$\mathbf{WE_2^2}$ value	$\mathbf{WE_2^2}$ DP
bat-1 ($N = 1960$)	0.0428	80	1.1582	78
carriage-01 ($N = 729$)	0.0685	63	0.3513	93
cellular_phone-1 ($N = 1102$)	0.0295	32	0.0813	32
truck-01 ($N = 270$)	0.0948	26	0.7582	4
Average	**0.0589**	50.2500	0.5873	517,500
Weighted Average	**0.0473**	60.3327	0.6945	632,901

The Fig. 5 shows a more complex example which has multiple objects but disconnected: the optimal polygonal approximations (OPA) generated by NCA and WE_2^2 for four contours are shown. The Table 2 shows the values provided by NCA and WE_2^2 and the number of the dominant points (DP) of these optimal polygonal approximations. The values of $Average = \sum_{i=1}^{k} \frac{M(i)}{k}$ and $Weighted\ Average = \frac{\sum_{i=1}^{k} M(i) \times n_i}{\sum_{i=1}^{k} n_i}$ are also computed, where $M(i)$ is the value of measurement M (NCA or WE_2^2) for contour i; n_i is the number of points of contour i, and k is the number of contours. As stated above, WE_2^2 provides optimal polygonal approximations with many dominant points (contour *carriage-01*) or with few points (contour *truck-01*) in comparison with the new proposal NCA.

In short, this first experiment showed that the new proposal NCA provides better polygonal approximations than the other measurements evaluated in this comparative study based on the analysis of the *optimal quality curves*.

Carmona's Merit [32] generated *optimal quality curves* in an increasing way form (0% - 100%) with uni-modal shape (Figs. 4(e), 7(e), 9(a) and 10(a)), but this measurement favors polygonal approximations with very few points that do not resemble the original contours (Fig. 9(c) and (e) for contour *truck-01*). The *optimal quality curves* generated by *Rosin's Merit* showed that this measurement cannot be used to evaluate the quality of the *optimal* polygonal approximations, because always assign the best value (100%), regardless of the number of dominant points (Fig. 11(a)). It should be remembered that *Carmona's Merit* and *Rosin's Merit* have a very high computational complexity.

5.2. Second experiment

The second experiment compared the performances of NCA, WE_2^2, *Rosin's Merit* [55], and *Carmona's Merit* [32] using the well-known contour *semicircles* [54] (Fig. 12) and 36 polygonal approximation algorithms. At first, three notes should be highlighted: (1) this experiment was not intended to evaluate the quality of the algorithms, but the performance of the quality measurements; (2) the contour *semicircles* was chosen because it was used for the last decades to study the performance of many polygonal approximation algorithms; and (3) WE_2^2, *Rosin's Merit* and *Carmona's Merit* were chosen because, despite its drawbacks (Sections 2 and 5.1), they are still used to evaluate the quality of the polygonal approximations [12].

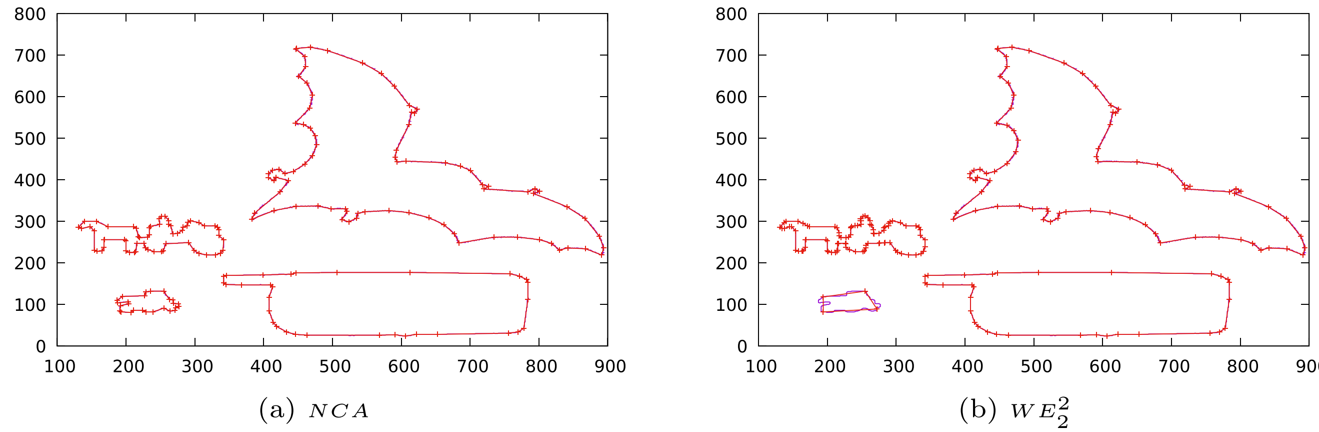


Fig. 5. Optimal polygonal approximations (OPA) for contours *bat-1* ($N = 1960$), *carriage-01* ($N = 729$), *cellular_phone-1* ($N = 1102$) and *truck-01* ($N = 270$) generated by (a) *NCA* and (b) WE_2^2.

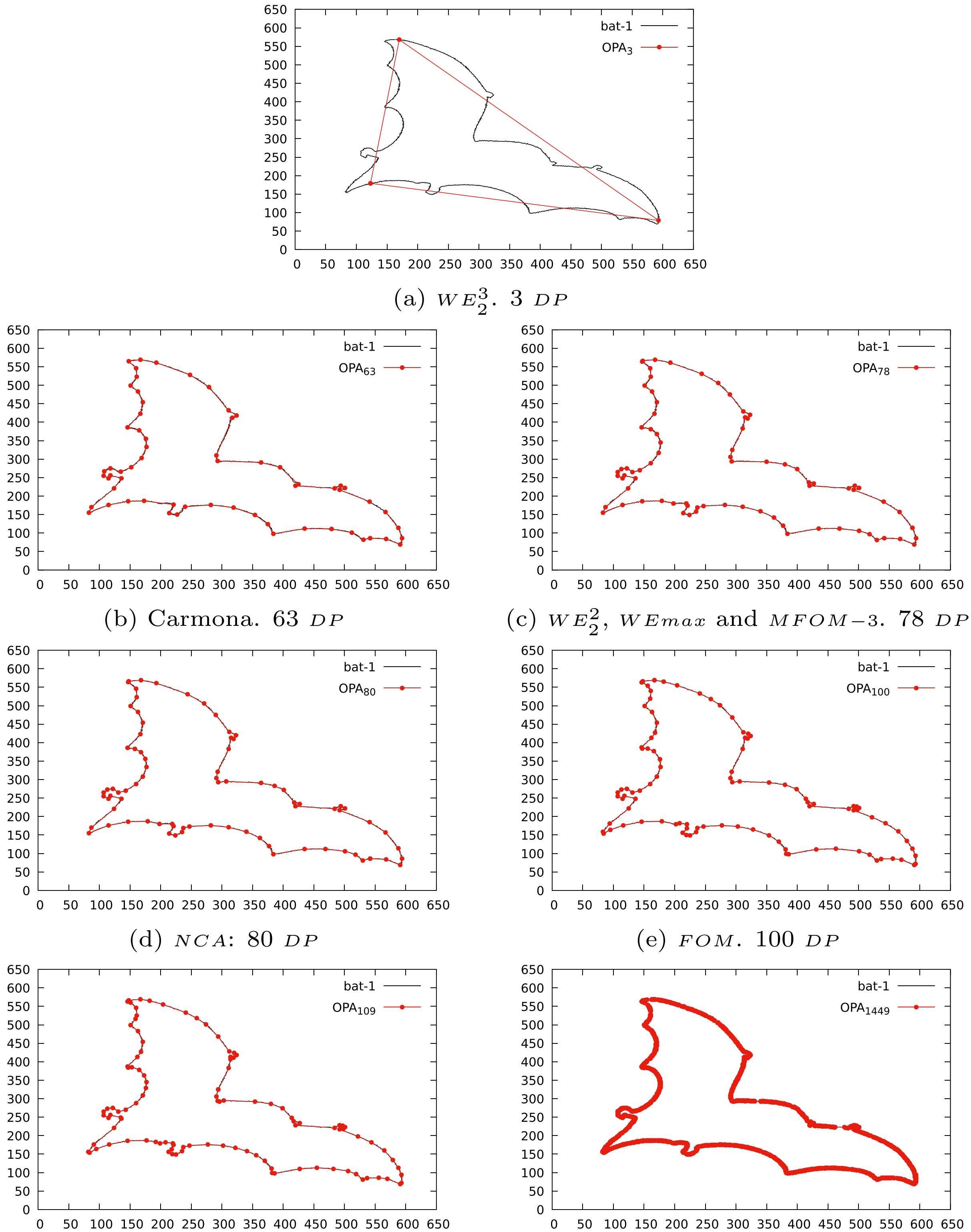
Table 3
Second experiment. Comparison of polygonal approximation algorithms using the contour *semicircles* and *Rosin's Merit*, (b) WE_2^2 and (c) *NCA*.

Authors (year): method [refs]	DP	CR	ISE	NCA value	NCA rank	WE_2^2 value	WE_2^2 rank	Merit value	Merit rank
Horng and Li (2002) [13]	15	6.8000	14.3994	**0.1269**	**1**	0.3114	2	**100.0000**	**1**
Zhou & Lu (2010) [63]	14	7.2857	17.3855	0.1273	2	0.3276	3	**100.0000**	**1**
Lowe (1987) [15]	13	7.8462	21.6600	0.1291	3	0.3518	5	97.1305	6
Wang et al.**(2014)** [62]	12	8.5000	26.0045	0.1304	4	0.3599	6	**100.0000**	**1**
Yin (2004): HDPSO2 [61]	10	10.2000	38.9200	0.1363	5	0.3741	7	**100.0000**	**1**
Sarkar (1993): 2-points-method [15]	20	5.1000	13.6500	0.1500	6	0.5248	10	72.1445	9
Banerjee et al. (1996) [15]	13	7.8462	39.3600	0.1514	7	0.6394	13	63.5739	13
Sarkar (1993): 1-points-method [15]	19	5.3684	17.3770	0.1518	8	0.6030	12	65.2429	11
Chung et al. (1994) [15]	22	4.6364	12.3600	0.1573	9	0.5750	11	65.9778	10
Held et al. (1994) [15]	17	6.0000	28.5000	0.1582	10	0.7917	17	54.0854	16
Ramer (1972) [15]	26	3.9231	5.2700	0.1598	11	0.3424	4	84.3933	8
Freeman & Davis (1997) [15]	19	5.3684	23.3100	0.1609	12	0.8088	19	53.2439	17
Chung et al (1994) [15]	14	7.2857	45.6000	0.1629	13	0.8591	20	51.4494	18
Douglas & Peuker (1973) [15]	16	6.3750	37.1200	0.1637	14	0.9134	21	48.1895	20
Arcelli & Ramella (1993) [15]	10	10.2000	75.1000	0.1691	15	0.7218	14	64.5096	12
Teh & Chin (1989) [15]	22	4.6364	20.6100	0.1716	16	0.9588	23	44.8868	22
Anderson & Bezdek (1984) [15]	18	5.6667	36.1400	0.1724	17	1.1255	26	42.3372	25
Rosenfeld & Wezka (1975) [15]	14	7.2857	59.1200	0.1756	18	1.1138	25	43.8402	24
Anderson & Bezdek (1984) [15]	29	3.5172	6.4300	0.1779	19	0.5198	8	60.3693	14
Ray & Ray (1992) [15]: (2)	27	3.7778	11.5000	0.1801	20	0.8058	18	45.9324	21
Chung et al. (1994) [15]	28	3.6429	9.6900	0.1811	21	0.7302	15	48.8511	19
Rosenfeld & Johnston (1973) [15]	30	3.4000	8.8500	0.1890	22	0.7656	16	44.7832	23
Ray & Ray (1992) [15]: (1)	29	3.5172	11.8180	0.1906	23	0.9553	22	38.9853	28
Rosenfeld & Johnston (1973) [15]	12	8.5000	92.3700	0.1914	24	1.2785	27	40.6401	27
Banerjee et al. (1996) [15]	27	3.7778	19.4000	0.1943	25	1.3593	29	30.7716	30
Banerjee et al. (1996) [15])	6	17.0000	150.5300	0.1961	26	0.5209	9	95.9549	7
Ansari & Huang (1991) [15]	28	3.6429	17.8300	0.1966	27	1.3436	28	30.4967	31
Rattarnangsi & Chin (1992) [15]	9	11.3333	130.1300	0.1999	28	1.0131	24	57.6519	15
Deguchi (1990) [15]	13	7.8462	99.0400	0.2007	29	1.6088	30	33.2517	29
Freeman & Davis (1997) [15]	17	6.0000	79.5300	0.2067	30	2.2092	33	26.4984	33
Melen & Ozanian (1993) [15]	13	7.8462	122.4400	0.2151	31	1.9889	32	28.8197	32
Rosenfeld & Wezka (1975) [15]	34	3.0000	15.4000	0.2219	32	1.7111	31	23.2544	34
Prasad (2013) [64]: PRO 0.2	44	2.3182	0.615385	0.2268	33	**0.1145**	**1**	**100.0000**	**1**
Phillips & Ronsenfeld (1987) [15]	14	7.2857	184.0900	0.2514	34	3.4681	35	19.5399	35
Sankar & Sharma (1978) [15]	10	10.2000	769.5300	0.3765	35	7.3965	36	14.4876	36
Williams (1978) [15]	5	20.4000	1191.6800	0.4000	36	2.8635	34	41.4353	36

Features of semicircles. $N = 102$. Number of break points (BP) = 52. Length = 117.74. Maximum distance to centroid = 17.9227. Maximum distance to the axis of minimum inertia = 17.4706.

The following methodology was developed: (1) the *optimal quality curves* for the contour *semicircles* with NCA, WE_2^2, *Rosin's Merit*, and *Carmona's Merit* were generated (Fig. 11); (2) the best polygonal approximations proposed by NCA, WE_2^2, *Rosin's Merit*, and *Carmona's Merit* were compared each other (Fig. 12); and, finally, (3) the quality of the polygonal approximations of the contour *semicircles* generated by 43 algorithms was evaluated using NCA, WE_2^2 and *Rosin's Merit* (Table 3);

The analysis of the *optimal quality curves* of the contour *semicircles* generated by these measurements allows the following results to be drawn: (1) as indicated in the first experiment, the *optimal quality curve* generated by the *Rosin's Merit* always assigns the best value (the maximum = 100%) to all *optimal* polygonal approximations; therefore, this measurement cannot allow to select the best polygonal approximation among all (Figs. 11(a) and 12); (2) the *optimal quality curve* generated by *Carmona's Merit* assigns



(a) WE_2^3. 3 DP

(b) Carmona. 63 DP

(c) WE_2^2, $WEmax$ and $MFOM-3$. 78 DP

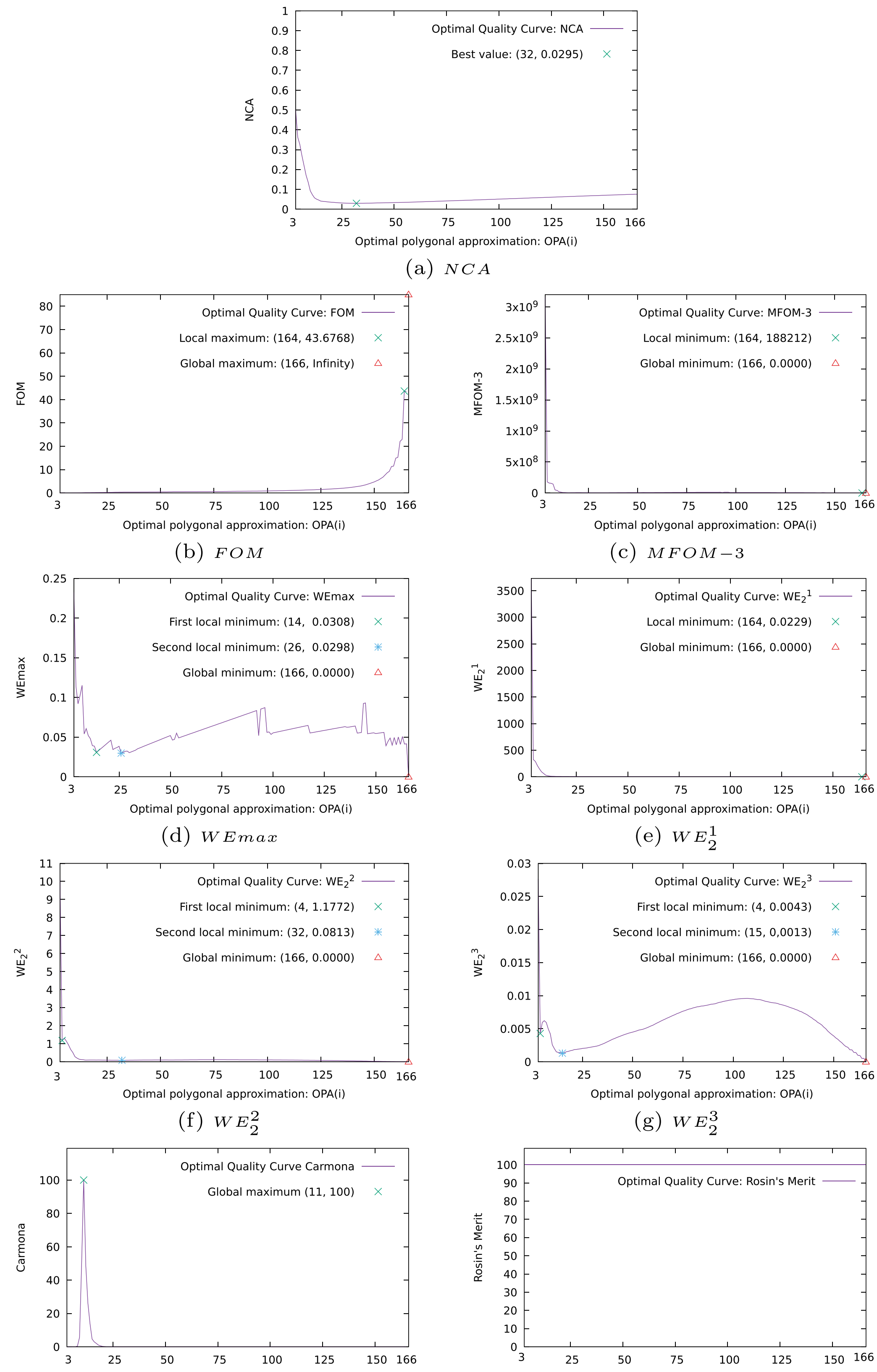
(d) NCA: 80 DP

(e) FOM. 100 DP

(f) $WEmax$. 109 DP

(g) WE_2^1, WE_2^2, WE_2^3, FOM, $WEmax$, and $MFOM-3$. 1449 DP

Fig. 6. Optimal polygonal approximations (OPA) for contour *bat-1* ($N = 1960$; number of break points = 1449) proposed by some measurements and their number of dominant points.



(a) NCA (b) FOM (c) $MFOM-3$ (d) $WEmax$ (e) WE_2^1 (f) WE_2^2 (g) WE_2^3 (h) Carmona's Merit (i) Rosin's Merit

Fig. 7. Optimal quality curves generated by some measurements for contour *cellular_phone-1* ($N = 1102$). Note: in the X axis, only are plotted the values from 3 to 166 (number of break points).

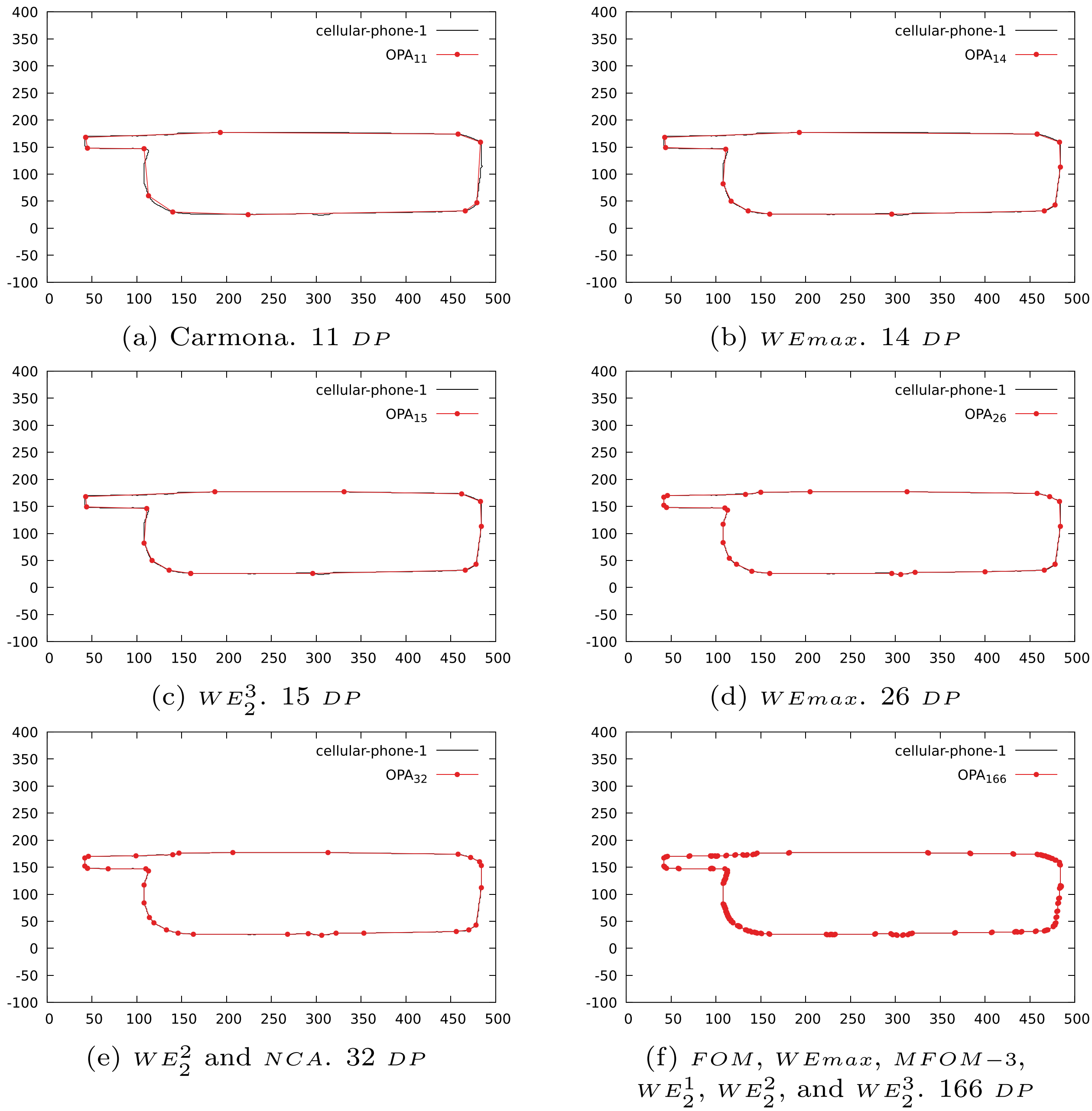
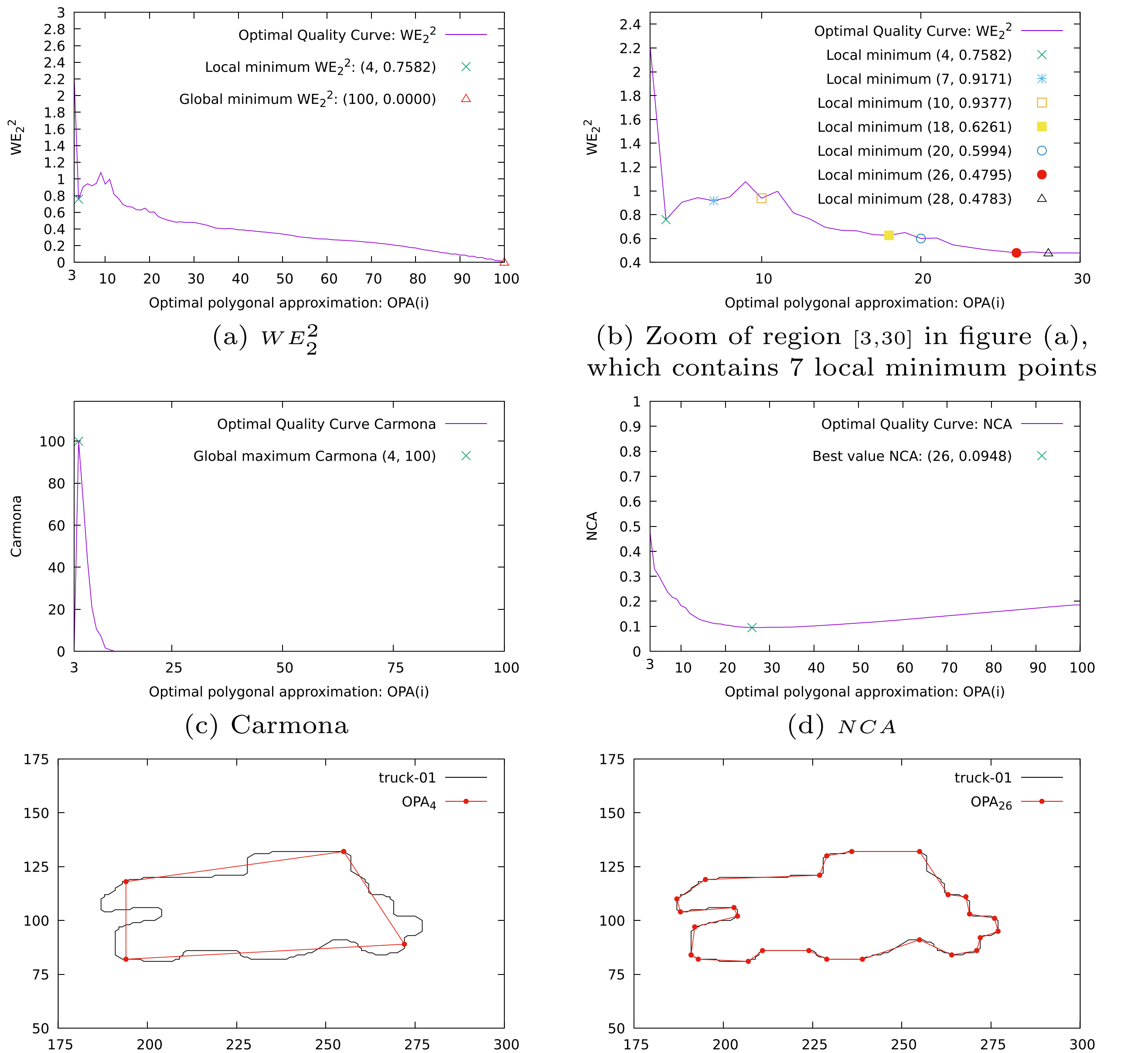


Fig. 8. Optimal polygonal approximations (OPA) for contour *cellular_phone-1* ($N = 1102$) proposed by some measurements and their number of dominant points (DP).

the best value to the *optimal* polygonal approximation with *only* 6 dominant points (Figs. 11(c) and 12(a)); (3) the *optimal quality curve* of WE_2^2 proposed several local minimums, but reached the global minimum (value = 0.0) when the *optimal* polygonal approximation was composed of the all 52 *break points* of the contour *semicircles* (Fig. 11 (b)), but this type of approximation is of no practical use, as it was stated previously [25]; and (4) the *optimal quality curve* of NCA allows to identify the best value (the minimum) with the optimal polygonal approximation composed of 15 dominant points (Figs. 11(c) and 12(e)). Therefore, only NCA allows to identify clearly the best polygonal approximation of the contour *semicircles*.

In addition, *Rosin's Merit*, WE_2^2 and NCA were used to evaluate the polygonal approximations of 36 polygonal approximation algorithms that were applied to the contour *semicircles*. Table 3) shows the results: DP, CR, NCA's values, NCA's rank, WE_2^2's values, WE_2^2's rank, *Merit*'s values, *Merit*'s rank, and *Rosin's* rank [15]. The rankings of NCA, WE_2^2 and *Merit* indicate the global position of each algorithm among all 36 algorithms. The algorithms were ranked according to the *NCA*'s values in Table 3. The best values were highlighted in **bold**.

The analysis of the results makes it possible to highlight the following facts. The *Rosin's Merit* proposed several polygonal approximations as the best but with different number of dominant points (*DP*): with 10 *DP*: Yin (2013): HDPSO1 [60,61]; with 12 *DP*: Wang et al. (2014) [62]; with 14 *DP*: Zhou & Lu (2010) [63]; with 15 *DP*: Horng & Li (2002) [13]; or with 44 *DP*: Prasad (2013): PRO 0.2 [64]. Fig. 11(a) shows how all these polygonal approximations achieved the best value in the *optimal quality curve*. These polygonal approximations are shown in Fig. 12. WE_2^2 proposed a polygonal approximation with 44 *DP*, generated by Prasad (2013): PRO 0.2 [64], as the best. Obviously, this polygonal approximation has too many and redundant dominant points to be considered as the best (Figs. 11(b) and 12(f)). NCA proposed the polygonal approxi-



(a) WE_2^2 (b) Zoom of region [3,30] in figure (a), which contains 7 local minimum points (c) Carmona (d) NCA (e) Carmona and WE_2^2. 4 DP (f) NCA. 26 DP

Fig. 9. Optimal quality curves for contour *truck-01* ($N = 270$) generated by (a) WE_2^2; (b) zoom of region [3, 30] in (a), which contains 7 local minimum points for WE_2^2; (c) Carmona and (d) *NCA*. Optimal polygonal approximations (OPA) for contour *truck-01* ($N = 270$) proposed by (e) Carmona and WE_2^2, and (f) *NCA* and their number of dominant points.

mation generated by Horng & Li (2002) [13] as the best, with 15 *DP* (Table 3 and Figs. 11(d) and 12(e)). Furthermore, this polygonal approximation is symmetric like contour *semicircles*. This experiment clearly showed that the new proposal NCA can fairly assess the quality of polygonal approximations algorithms.

6. Conclusions and future work

Two proposals were presented in this work: (1) a new assessment measurement, called *normalized compression ratio and adjustment error* (NCA), for a fair evaluation of polygonal approximation algorithms (Section 3), and (2) a new methodology for evaluation of measurements for assessing polygonal approximations (Section 4). The new assessment measurement NCA has four very important properties: (1) it takes into account both the number of points and the error of the polygonal approximation; (2) it is normalized from 0 (best value) to 1 (worst value); (3) its computational complexity is linear ($\mathcal{O}(n)$), where n is the number of contour points; and (4) for each contour, only a single polygonal approximation is considered the best. On the other hand, the new evaluation methodology is based on the *optimal quality curve* concept, which can characterize the performance of the assessment measurements for polygonal approximations. A simple visual analysis of the *optimal quality curve* allows possible drawbacks or weaknesses of the evaluation measurement to be detected.

Two experiments were developed in a comparative study to evaluate the quality of the most popular measurements and the new proposal NCA. In the first experiment (Section 5.1), the analysis of the *optimal quality curves* of eight well-known measurements (FOM, WEmax, MFOM − 3, WE_2^1, WE_2^2, WE_2^3,*Rosin's Merit*, and *Carmona's Merit*) showed that all of them have very important weaknesses, whereas the *optimal quality curves* of NCA showed

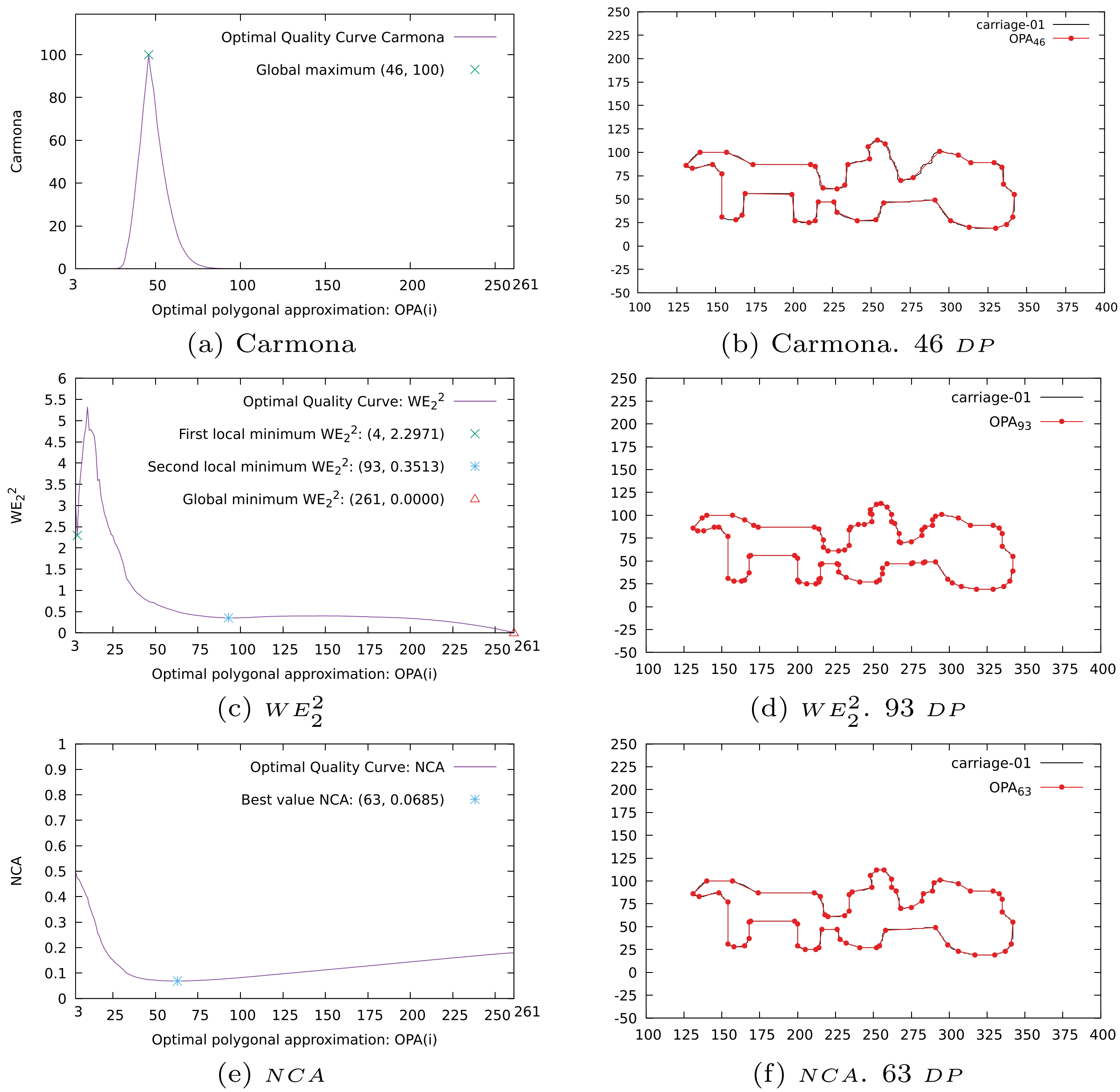


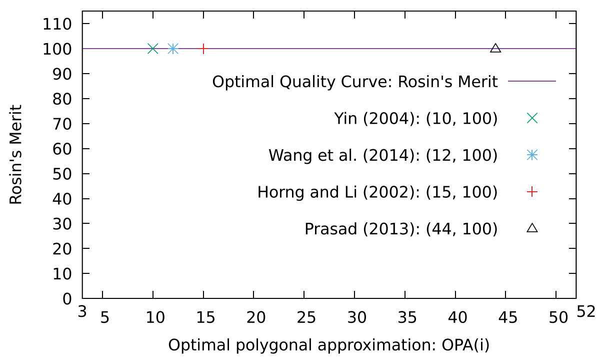
Fig. 10. Optimal quality curves for contour *carriage-01* ($N = 729$) generated by (a) Carmona, (c) WE_2^2 and (e) *NCA*. Note: in the X axis, only are plotted the values from 3 to 261 (number of break points). Optimal polygonal approximations (OPA) for contour *carriage-01* ($N = 729$) proposed by (b) Carmona; (d) WE_2^2, and (f) *NCA* and their number of dominant points.

that a better *optimal* polygonal approximation could be found easily.

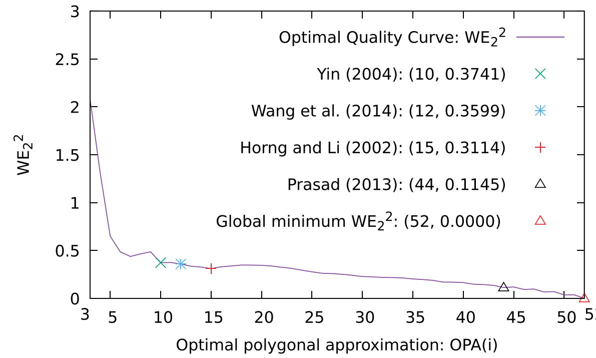
In the second experiment (Section 5.2, the new proposal NCA and WE_2^2, *Rosin's Merit* and *Carmona's Merit* were used to evaluate the quality of the polygonal approximations of the well-known contour *semicircles* generated by 36 algorithms. This experiment showed that NCA clearly outperformed to WE_2^2, *Rosin's Merit* and *Camona's Merit*, because it was able to find the best polygonal approximation among all possible ones.

In summary, (1) the proposed measurement NCA can be used to fairly evaluate and compare polygonal approximation algorithms of 2D closed curves or *contours*; (2) NCA takes into account both the *number of points of the polygonal approximation* and the *adjustment error*; (3) NCA is normalized between 0 (best value) and 1 (worst value); (4) its computational complexity is linear ($\mathcal{O}(n)$), where n is the number of contour points. Furthermore, the new evaluation methodology, based on the *optimal quality curve* concept, can be used to characterize the performance of the measurements for assessing polygonal approximations.

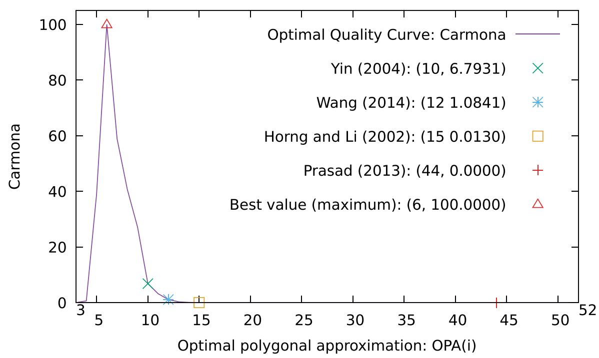
Finally, future work should be aimed at five different ways: (1) study of the influence of the parameter D on the performance of NCA: in the experiments, the values of the maximum distance from the contour points to the centroid (D_1), and the maximum distance from the contour points to the minimum inertia axis (D_2) were also used, but, in some cases, D_1 proposed polygonal approximations with few points, whereas D_2 proposed polygonal approximations with many points, whereas $D = \frac{D_1+D_2}{2}$ obtained better results; (2) fair comparison of different polygonal approximation algorithms using NCA and a large database of contours to choose the best one; (3) design a new polygonal approximation algorithm using NCA as an objective function; (4), adaption of NCA to evaluate polygonal approximation of 2D *open* curves, such as time series [65]; and (5) study the combination of these algorithms with Machine Learning approaches to improve their performances.



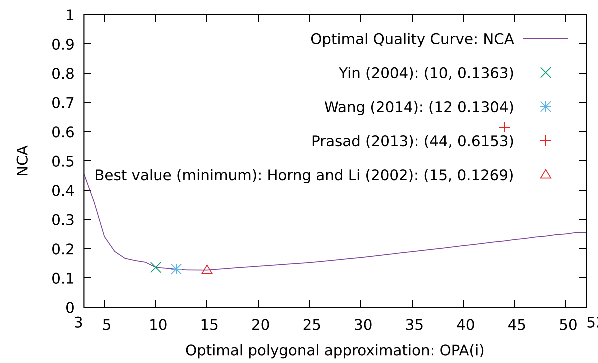
(a) *Rosin's Merit*



(b) WE_2^2

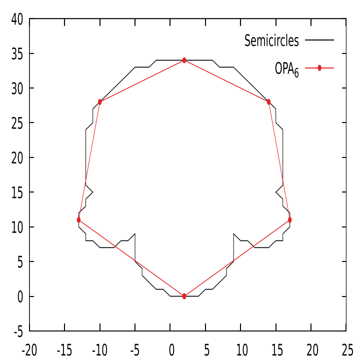


(c) Carmona

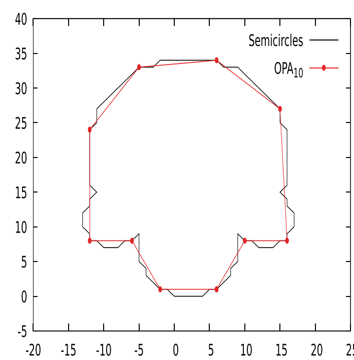


(d) NCA

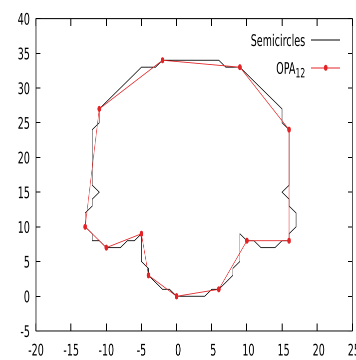
Fig. 11. Optimal quality curves for contour *semicircles* ($N = 105$) generated by (a) *Rosin's Merit*, (b) WE_2^2 and (c) *NCA*. Note: in the X axis, only are plotted the values from 3 to 52 (number of break points).



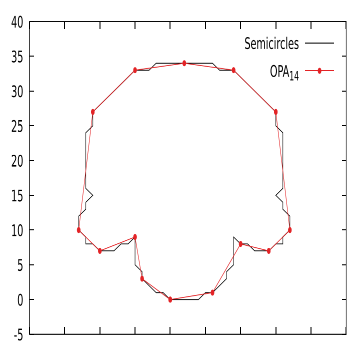
(a) DP=6; $Carmona = 100\%$ **$Rosin = 100\%$**;



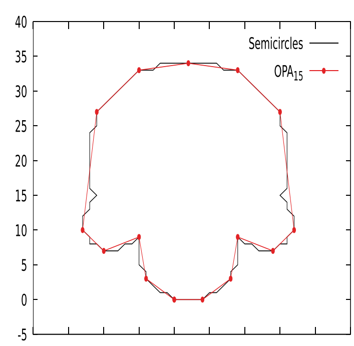
(b) DP=10; $Rosin = 100\%$;



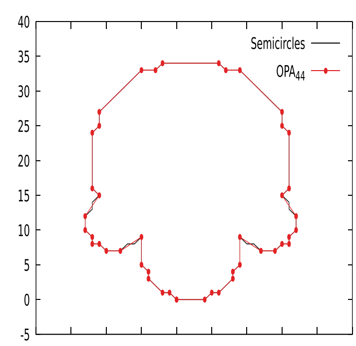
(c) DP=12; $Rosin = 100\%$;



(d) DP=14; $Rosin = 100\%$



(e) DP=15; NCA=0.1269 ; $Rosin = 100\%$



(f) DP=44; WE_2^2=0.1145 ; $Rosin = 100\%$

Fig. 12. Some *optimal* polygonal approximations (OPA_n) for contour *semicircles* ($N = 105$; number of break points = 52) proposed by *Carmona's Merit*, *Rosin's Merit*, WE_2^2 and NCA.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgment

This work was developed with the support of the Research Project TIN2016-75279-P of Spain Ministry of Economy, Industry, and Competitiveness, and FEDER. Funding for open access charge: Universidad de Córdoba / CBUA.

References

[1] R. Haralick, Dialogue: performance characterization in computer vision, CVGIP Image Understanding 60 (2) (1994) 245–265, doi:10.1006/ciun.1994.1050.
[2] R.C. Gonzalez, R.E. Woods, Digital Image Processing, Addison-Wesley, 1992, pp. 1–19.
[3] H. Zhang, J. Guo, Optimal polygonal approximation of digital planar curves using meta heuristics, Pattern Recognit. 34 (2001) 1429–1436, doi:10.1016/S0031-3203(00)00097-2.
[4] S. Loncaric, A survey of shape analysis techniques, Pattern Recongnit. 31 (1998) 983–1001, doi:10.1016/S0031-2023(97)00122-2.
[5] S. Lee, Y. Jeong, J. Kwak, D. Park, K.H. Park, Advanced real-time dynamic programming in the polygonal approximation of ecg signals for a lightweight embedded device, IEEE Access 7 (2019) 162850–162861, doi:10.1109/ACCESS.2019.2952399.
[6] D.G. Lowe, Three dimensional object recognition from single two dimensional images,, Artif. Intell. 31 (1987) (1987) 355–395, doi:10.1016/0004-3702(87)90070-1.
[7] X. Wang, B. Feng, X. Bai, W. Liu, L. Jan Latecki, Bag of contour fragments for robust shape classification, Pattern Recognit. 47 (6) (2014) 2116–2125, doi:10.1016/j.patcog.2013.12.008.
[8] I. Sharif, D. Chaudhuri, N.K. Kushwaha, A. Samal, B.M. Singh, An efficient algorithm to decompose a compound rectilinear shape into simple rectilinear shapes, Turkish J. Electr. Eng.Comput. Sci. 26 (2018) 150–161, doi:10.3906/elk-1608-50.
[9] H. Imai, M. Iri, Polygonal approximations of a curve formulations and algorithms, in: G.T. Toussaint (Ed.), Computational Morphology, Machine Intelligence and Pattern Recognition, Vol. 6, North-Holland, 1988, pp. 71–86, doi:10.1016/B978-0-444-70467-2.50011-4.
[10] E. Aguilera-Aguilera, A. Carmona-Poyato, F. Madrid-Cuevas, R. Muoz-Salinas, Novel method to obtain the optimal polygonal approximation of digital planar curves based on mixed integer programming, J. Vis. Commun. Image R. 30 (2015) 106–116, doi:10.1016/j.jvcir.2015.03.007.
[11] M. Salotti, Optimal polygonal approximation of digitized curves using the sum of square deviations criterion, Pattern Recognit. 35 (2002) 435–443, doi:10.1016/S0031-3203(01)00051-6.
[12] N.-L. Fernández-García, L. Del-Moral-Martínez, A. Carmona-Poyato, F. Madrid-Cuevas, R. Medina-Carnicer, A new thresholding approach for automatic generation of polygonal approximations, Pattern Recognit. Lett. 135 (2020) 138–145, doi:10.1016/j.patrec.2020.04.014.
[13] J.-H. Horng, J.T. Li, An automatic and efficient dynamic programming algorithm for polygonal approximation of digital curves, Pattern Recognit. Lett. 23 (2002) 171–182, doi:10.1016/S0167-8655(01)00098-8.
[14] A. Carmona-Poyato, F. Madrid-Cuevas, R. Medina-Carnicer, R. Muoz-Salinas, Polygonal approximation of digital planar curves through break point suppression, Pattern Recognit. 43 (2010) 14–25, doi:10.1016/j.patcog.2009.06.010.
[15] P.L. Rosin, Techniques for assessing polygonal approximations of curves, IEEE Trans. Pattern Anal. Mach.Intell. 19 (1997) 659–666, doi:10.1109/34.601253.
[16] M.A. Fischler, H.C. Wolf, Locating perceptually salient points on planar curves, IEEE Trans. Pattern Anal. Mach. Intell. 16 (2) (1994) 113–129, doi:10.1109/34.273737.
[17] G. Papakonstantinou, Optimal polygonal approximation of digital curves, Signal Processs. 8 (1) (1985) 131–135, doi:10.1016/0165-1684(85)90094-5.
[18] J.M. Iesta, M. Buenda, M.A. Sarti, Reliable polygonal approximation of imaged real objects through dominant point detection, Pattern Recognit. 31 (6) (1998) 685–697, doi:10.1016/S0031-3203(97)00081-2.
[19] H. Imai, M. Iri, Computational-geometric methods for polygonal approximations of a curve, Comput. Vis. Graph. 36 (1986) 31–41, doi:10.1016/S0734-189X(86)80027-5.
[20] J. Liu, J. Zhang, F. Xu, Z. Huang, Y. Li, Adaptive algorithm for automated polygonal approximation of high spatial resolution remote sensing imagery segmentation contours, IEEE Trans. Geosci. Remote Sens. 52 (2) (2014) 1099–1106, doi:10.1109/TGRS.2013.2247407.
[21] A. Masood, A. Haq, A novel approach to polygonal approximation of digital curves, J. Vis. Commun. Image Represent. 18 (2007) 264–274, doi:10.1016/j.jvcir.2006.12.002.
[22] A. Kolesnikov, Segmentation and multi-model approximation of digital curves, Pattern Recognit. Lett. 33 (2012) 1171–1179.
[23] Z. Liu, J. Watson, A. Allen, A polygonal approximation of shape boundaries of marine plankton based-on genetic algorithms, J. Vis. Commun. Image R. 41 (2016) 305–313, doi:10.1016/j.jvcir.2016.10.010.
[24] D. Sarkar, A simple algorithm for detection of significant vertices for polygonal approximation of chain-coded curves, Pattern Representation 14 (1993) 959–964, doi:10.1016/0167-8655(93)90004-W.
[25] A. Masood, Dominant point detection by reverse polygonization of digital curves, Image Vis. Comput. 26 (2008) 703–715, doi:10.1016/j.imavis.2007.08.006.
[26] W.Y. Wu, An adaptive method for detecting dominant points, Pattern Recognit. 36 (2003) 2231–2237, doi:10.1016/S0031-3203(03)00087-6.
[27] T. Nguyen, I. Debled-Rennesson, Parameter-free method for polygonal representation of noisy curves, in: Proceedings of International Workshop on Combinatorial Image Analysis IWCIA, 2009, pp. 65–78. https://hal.archives-ouvertes.fr/hal-00437309/
[28] A. Kolesnikov, T. Kauranne, Unsupervised segmentation and approximation of digital curves with rate-distortion curves modeling, Pattern Recognit. 47 (2014) 623–633, doi:10.1016/j.patcog.2013.09.002.
[29] S. Saha, S. Goswani, P.R.S. Mahapatra, A heuristic strategy for sub-optimal thick-edged polygonal approximation of 2-D planar shape, Int. J. Image Graph. Signal Process. 4 (2018) 48–58.
[30] M. Marji, P. Siy, Polygonal representation of digital planar curves through dominant point detection - a nonparametric algorithm, Pattern Recognit. 37 (2004) 2113–2130, doi:10.1016/j.patcog.2004.03.004.
[31] A. Carmona-Poyato, N. Fernández-García, R. Medina-Carnicer, F. Madrid-Cuevas, Dominant point detection: a new proposal, Image Vis. Comput. 23 (2005) 1226–1236, doi:10.1016/j.patcog.2009.06.010.
[32] A. Carmona-Poyato, R. Medina-Carnicer, F.J. Madrid-Cuevas, R. Muoz-Salinas, N.L. Fernández-García, A new measurement for assessing polygonal approximation of curves, Pattern Recognit. 4 (2011) 44–54, doi:10.1016/j.patcog.2010.07.029.
[33] A. Carmona-Poyato, E.J. Aguilera-Aguilera, F.J. Madrid-Cuevas, M.J. Marn-Jimnez, N.L. Fernández-García, New method for obtaining optimal polygonal approximations to solve the min-ϵ problem, Neural Comput. Appl. 28 (2017) 2239–2383, doi:10.1007/s00521-016-2198-7.
[34] J.C. Perez, E. Vidal, Optimum polygonal approximation of digitized curves, Pattern Recognit. Lett. 15 (1994) 743–750, doi:10.1016/0167-8655(94)90002-7.
[35] P.L. Rosin, Unimodal thresholding, Pattern Recognit. 34 (2001) 2083–2096. 10.1.1.159.9940
[36] R. Neumann, G. Teisseron, Extraction of dominant points by estimation of the contour fluctuations, Pattern Recognit. 35 (2002) 1447–1462, doi:10.1016/S0031-3203(01)00145-5.
[37] K. Wall, P.E. Danielsson, A fast sequential method for polygonal approximation of digitized curves, Comput. Vis. Graph. Image Process. 28 (1984) 220–227, doi:10.1016/S0734-189X(84)80023-7.
[38] W.-W. Wu, M.-J.J. Wang, Detecting dominant points by the curvature-based polygonal approximation, Graph. Models Image Processs. 55 (2) (1993) 79–88, doi:10.1006/cgip.1993.1006.
[39] Y. Sato, Piecewise linear approximation by plane curves by perimeter optimization, Pattern Recognit. 25 (1992) 1535–1543, doi:10.1016/0031-3203(92)90126-4.
[40] C.-C. Tseng, C.-J. Juan, H.-S. Chan, J.-F. Lin, An optimal line segment extraction algorithm for online chinese character recognition using dynamic programming, Pattern Recognit. Lett. 19 (1998) 953–961, doi:10.1016/S0167-8655(98)00071-3.
[41] H.-C. Lin, L.-L. Wang, S.-H. Yang, Fast heuristics for polygonal approximation of a 2D shape boundary, Signal Process. 60 (1997) 235–241, doi:10.1016/S0165-1684(97)80008-4.
[42] M.T. Parvez, S.A. Mahmoud, Polygonal approximation of digital planar curves through adaptive optimizations, Pattern Recognit. Lett. 31 (2010) 1997–2005, doi:10.1016/j.patrec.2010.06.007.
[43] A. Kolesnikov, Non parametric polygonal and multimodel approximation of digital curves with rate-distortion curve modeling, in: 18th IEEE International Conference of Image Processing, Vol. ICIP 2011, 2011, p. 2889, doi:10.1109/ICIP.2011.6116152.
[44] J.-H. Horng, J. Li, An automatic and efficient dynamic programming algorithm for polygonal approximation of digital curves, Pattern Recognit. Lett. 23 (2002) 171–182, doi:10.1016/S0167-8655(01)00098-8.
[45] R. Dinesh, D.S. Guru, Finite automata inspired model for dominant point detection: a non-parametric approach, in: Proceedings of the International Conference on Computing: Theory and Applications, Vol. (ICCTA'07), Kolkata, 2007, pp. 579–583, doi:10.1109/ICCTA.2007.63.
[46] J. Ventura, J. Chen, Segmentation of two-dimensional curve contours, Pattern Recognit. 25 (1992) 1129–1140, doi:10.1016/0031-3203(92)90016-C.
[47] A. Held, K. Abe, C. Arcelli, Towards a hierarchical contour description via dominant point detection, IEEE Trans. Sys. Man Cybern. 24 (1994) 942–949, doi:10.1109/21.293514.
[48] P.S. Heckbert, M. Garland, Survey of Polygonal Surface Simplification Algorithms, Technical Report, Carnegie Mellon University, 1997.

[49] A. Kolesnikov, Fast algorithm for ISE-bounded polygonal approximation, in: In: Proc. IEEE Internat. Conf. on Image Processing, Vol. ICIP 2008, 2008, pp. 1013–1015. San Diego, USA
[50] X. Gao, F. Sattar, A. Quddus, R. Venkateswarlu, Multiscale contour corner detection based on local natural scale and wavelet transform, Image Vis. Comput. 25 (2007) 890–898, doi:10.1016/j.imavis.2006.07.002.
[51] W.Y. Wu, Dominant point detection using bending value, Image Vis. Comput. 21 (2003) 517–525, doi:10.1016/S0262-8856(03)00031-3.
[52] P. Rosin, G. West, Nonparametric segmentation of curves into various representations, IEEE Trans. Pattern Anal. Mach.Intell. 17 (1995) 1140–1153, doi:10.1109/34.476507.
[53] N.L. Fernández-García, R. Medina-Carnicer, A. Carmona-Poyato, F.J. Madrid-Cuevas, M. Prieto-Villegas, Characterization of empirical discrepancy evaluation measures, Pattern Recognit. Lett. 25 (1) (2004) 35–47, doi:10.1016/j.patrec.2003.08.011.
[54] C. Teh, R. Chin, On the detection of dominant points in digital curves, Trans. Pattern Anal. Mach. Intell. 11 (1989) 859–872.
[55] P.L. Rosin, Assessing the behaviour of polygonal approximation algorithms, in: British Machine Vision Conference, Vol. BMVC 1998, 1998, doi:10.5244/C.12.67.
[56] S. Jeannin, M. Bober, Description of core experiments for MPEG-7 motion/shape, MPEG-7, ISO/IEC/JTC1/SC29/WG11/MPEG99/N2690, Vol. 2690, 1999.
[57] MPEG-7 core experiment CE-shape-1 test set (part b), benchmarking image database for shape recognition techniques, 1999, (http://www.cis.temple.edu/-latecki/TestData/mpeg7shapeB.tar.gz),Accessed: 2021-06-23.
[58] L. Latecki, R. Lakmper, U. Eckhardt, Shape descriptors for non-rigid shapes with a single closed contour, in: IEEE Conf. on Computer Vision and Pattern Recognition (CVPR), 2000, pp. 424–429, doi:10.1109/CVPR.2000.855850.
[59] Images used in the experiments, 2022, (http://www.uco.es/users/ma1fegan/Comunes/investigacion/imagenes.html),Accessed: 2022-11-23.
[60] P.-Y. Yin, A discrete particle swarm algorithm for optimal polygonal approximation of digital curves, J. Vis. Commun. Image Representation 15 (2004) 241–260, doi:10.1016/j.jvcir.2003.12.001.
[61] A.R. Backes, O.M. Bruno, Polygonal approximation of digital planar curves through vertex betweenness, Inf. Sci. 22 (2013) 795–804.
[62] B. Wang, D. Brown, X. Zhang, H. Li, Y. Gao, J. Cao, Polygonal approximation using integer particle swarm optimization, Inf. Sci. 278 (2014) 311–326, doi:10.1016/j.ins.2014.03.055.
[63] X. Zhou, Y. Lu, Efficient polygonal approximation of digital curves via Monte Carlo optimization, in: 20th International Conference on Pattern Recognition, Istanbul, 2010, pp. 3513–3516, doi:10.1109/ICPR.2010.857.
[64] D.K. Prasad, PRO: a novel approach to precision and reliability optimization based dominant point detection, J. Optim. 2013 (Article ID 345287) (2013) 1–15, doi:10.1155/2013/345287.
[65] A. Carmona-Poyato, N.L. Fernndez-Garca, F.J. Madrid-Cuevas, A.M. Durn-Rosal, A new approach for optimal time-series segmentation, Pattern Recognit. Lett. 135 (2020) 153–159, doi:10.1016/j.patrec.2020.04.006.

N.L. Fernández-García received the bachelor degree in Mathematics from Complutense University of Madrid (Spain) in 1988. He received the PhD in Computer Science from the Polytechnic University of Madrid (Spain) in 2002. Since 1990, he has been working with the Department of Computing and Numerical Analysis at Cordoba University. He is a lecturer of Computer Science and Artificial Intelligence at Cordoba University. His research is focused on edge detection, 2-D object recognition, and evaluation of computer vision algorithms.

L. Del-Moral Martínez is an engineer in computer science from Cordoba University (2011) and has completed the university master of Intelligent Systems at Cordoba University (2012). He received the PhD degree in Computer Science at Cordoba University in 2017. He is a high school teacher of Computer Science. His research is focused on 2-D object recognition and evaluation of computer vision algorithms.

A. Carmona-Poyato received his title of Agronomic Engineering and PhD degree from the University of Cordoba(Spain), in 1986 and 1989, respectively. Since 1990 he has been working with the Department of Computing and Numerical Analysis of the University of Cordoba as lecturer. His research is focused on image processing and 2-D object recognition.

F. J. Madrid-Cuevas received the Bachelor degree in Computer Science from Malaga University (Spain) and the PhD degree from Polytechnic University of Madrid (Spain), in 1995 and 2003, respectively. Since 1996 he has been working with the Department of Computing and Numerical Analysis of Cordoba University, currently he is an assistant professor. His research is focused mainly on image segmentation, 2-D object recognition and evaluation of computer vision algorithms.

R. Medina-Carnicer received the Bachelor degree in Mathematics from University of Sevilla (Spain). He received the PhD in Computer Science from the Polytechnic University of Madrid (Spain) in 1992. Actually he is Professor of Computer Vision at Cordoba University (Spain). His research is focused on edge detection, evaluation of computer vision algorithms and pattern recognition.