

1           **A NOVEL BACKTRACKING APPROACH FOR TWO-AXIS SOLAR PV**  
2   **TRACKING PLANTS**

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20  
21   **ABSTRACT**

22   Solar tracking is a technique required to increase energy production in multiple  
23   photovoltaic (PV) facilities. In these plants, during low-elevation solar angle hours,  
24   shadows appear between the collectors causing a dramatic decrease in production. This  
25   paper presents a novel optimal tracking strategy to prevent the creation of these  
26   shadows. The presented method determines whether or not there is shading between  
27   collectors. Thus, when the collectors are not shaded, a tracking trajectory for maximum  
28   irradiance on the collectors is suggested. However, when the collectors are shaded,  
29   backtracking is proposed. Therefore, energy production in plants with this novel  
30   tracking method can be 1.31 % higher than that in PV installations with astronomical  
31   tracking. Moreover, this method allows the study of PV facilities for which there have  
32   been no published approaches, such as plants with non-rectangular collectors or those  
33   located on topographically heterogeneous surfaces.

34  
35   **KEYWORDS:** PV Solar Plants, Two-axis Solar Tracker, Power Losses by Shading in  
36   PV Plants, Backtracking.

## 39 1. INTRODUCTION

40 Technologies based on the use of solar energy have recently received more attention,  
41 and their development aims to respond to the growing need for renewable energy. In  
42 this context, scientific advances in the field of photovoltaics (PV) are allowing this  
43 technology to become an alternative sustainable energy source [1, 2]. However, these  
44 advances are not always properly applied to PV plant design and/or operation, and,  
45 consequently, the optimal development that these advances require for PV plants has  
46 not yet been achieved.

47 This is evident in the case of using solar tracking to increase the ability of PV plants to  
48 harness solar resources. Solar trackers can be classified as one- or two-axis trackers. In  
49 one-axis trackers, the collector's surface rotates around a fixed axis, while the surface  
50 moves around two fixed axes in two-axis trackers, which allows the collector plane to  
51 orientate towards any direction of the celestial sphere [3]. In this research area, authors  
52 [4, 5, 6] have analysed the effects of the type of tracking on energy production at  
53 different latitudes, and their results show that, in any case, the higher the latitude, the  
54 more effective the tracking, with differences reaching 57% [7].

55 In solar tracking PV plants, the collector's orientation is commonly governed by  
56 equations based on the astronomical movement of the Sun, which can predict the  
57 position of the Sun in the celestial sphere with an accuracy of an order of mrad [8-10].  
58 In this field, mathematical equations based on applying spherical trigonometry to solar  
59 movement have been developed to calculate the elevation and azimuth position for one-  
60 and two-axis trackers for each moment [7,10-14]. Recently, in contrast to this method, it  
61 is possible to deduce all of the astronomical factors governing the movement of the Sun  
62 and the orientation of solar tracking systems from the definition of 'solar vector' (unit  
63 vector along the direction towards the centre of the solar disk) and applying vector  
64 algebra [15-19].

65 Applying the astronomical model to solar tracking means that the angle formed between  
66 the direct solar rays and the normal angle to the collector's surface  $\theta$  must be as low as  
67 possible. With astronomical tracking, the value of the direct irradiance component is  
68 maximized, which is appropriate for applications focused on this component (such as  
69 concentration technologies). However, in PV, all irradiance components (direct, diffuse,  
70 and reflected irradiance) are usable. Therefore, this type of tracking is not the most  
71 suitable. As Duffie and Beckman [11] and Mousazadeh et al. [2] noted, on cloudy days,  
72 when the solar disk is not visible and direct radiation does not reach the collectors,

73 collectors located on a fixed horizontal position would collect more energy than those  
74 with astronomical tracking. Despite this, no work has been conducted to determine the  
75 appropriate equations for optimising solar tracking on cloudy days. Thus, it is necessary  
76 to study the influence of diffuse and reflected components on solar tracking in greater  
77 depth to determine the equations that can allow maximum radiative capture.

78 Additionally, one of the most important aspects to consider in plants with astronomical  
79 solar tracking is shading between the modules, which mainly occurs during the first and  
80 last hours of the day and causes production losses, as well as the appearance of hot spots  
81 in the modules [7].

82 To characterise and optimise the design of tracking plants, Diaz-Dorado et al. [20-21]  
83 developed a model that considers the arrangement of the cells within the photovoltaic  
84 modules, as well as the exact position of each module within the tracking surface, to  
85 determine the shading effects for all cells in the tracker. In this model, shading is  
86 characterised following a conventional tracking strategy to achieve perpendicularity  
87 between the direct solar rays and the collector's surface [20-21].

88 To estimate power losses caused by shading, Martinez-Moreno et al. [22] have  
89 proposed a predictive model that does not require any specific information regarding the  
90 connections between the cells and modules. This model has been validated by different  
91 authors [23, 24] who have developed more extensive models based on Martinez-  
92 Moreno's model to determine the productivity of PV plants. Similarly, Perpiñan [25]  
93 developed a method for estimating and optimising energy costs based on plant design  
94 parameters, specifically the ground cover ratio (GCR, which is the ratio between the PV  
95 module area and the terrain occupied by the PV plant). For this, the method uses  
96 Gordon and Wenger's hypothesis [26] when determining energy losses due to shading,  
97 which considers the losses proportional to the percentage of the shaded area. Navarte  
98 and Lorenzo [27] studied the productivity of a PV plant considering different types of  
99 solar tracking and three simple hypotheses for estimating losses by shading.

100 Panico et al. [28] proposed backtracking as an approach to minimise the effects of  
101 shading. This technique involves deviating the direction of the solar trackers from the  
102 solar position to avoid shading between the collectors when necessary. Different authors  
103 [7, 28, 29] have demonstrated the advantages of backtracking, as follows:

104 A. Advantages of land use: By avoiding the effects of shading, the distances  
105 between trackers can be reduced, resulting in greater GCR.

106 B. Operating advantages: The work conducted by Lorenzo and Navarte [7]  
107 indicates that, in all cases, energy balance is more favourable in plants with  
108 backtracking than in those allowing shading between collectors.

109 C. Design advantages: The absence of shading and, therefore, of hot spots, suggests  
110 lower maintenance costs.

111 Therefore, the reliability of plants with backtracking is greater than that in plants that  
112 allow shading [7,29]. Consequently, many technological solutions to implement  
113 backtracking are being developed [30-32].

114 To determine the orientation of the collectors during backtracking, different methods  
115 based on the geometric determination of shadows between polygons have been  
116 proposed [4,7,33]. However, these methods are often limited to simple geometric  
117 situations such as:

- 118 i. Exclusively rectangular collectors.
- 119 ii. Regular ground layouts where only the shading between contiguous collectors is  
120 considered.
- 121 iii. Flat topographic surfaces.
- 122 iv. Horizontal topographic surfaces.
- 123 v. Movement around the azimuthal and elevational axes without considering other  
124 combinations of axes that entail the rotation of the collector around the normal  
125 axis to the collector's surface.

126 Considering the aforementioned limitations, this study presents a simple and more  
127 generic backtracking method to avoid shadows and optimise solar energy collection.

128 The method is based on the vector treatment of the geometry of the Sun-Earth position,  
129 as well as the implicit geometry of solar tracking plants. Furthermore, this method does  
130 not *a priori* assume the astronomical tracking hypothesis commonly assumed in the  
131 literature, which aims to maintain the position of the collector's surface perpendicular to  
132 the direct solar rays [15]. Thus, the proposed method allows the following, which are  
133 novelties in comparison to conventional methodologies:

- 134 1. The study of plants with non-rectangular surface collectors.
- 135 2. The analysis of facilities where collectors are not necessarily located at the  
136 regular nodes of a geometric grid.
- 137 3. The determination and comparison of the effects of different tracking modes.
- 138 4. The consideration of plants located on real topographical surfaces, and not only  
139 flat or horizontal surfaces.

140 5. The consideration of global irradiance on collectors, instead of being limited to  
 141 direct irradiance (typical for astronomical tracking).

142 Therefore, the method presented here will be useful for optimising the design of new  
 143 photovoltaic two-axis tracker plants, as well as for controlling the movement of current  
 144 plants by improving and optimising their electrical production.

145

## 146 2. MATERIALS AND METHODS

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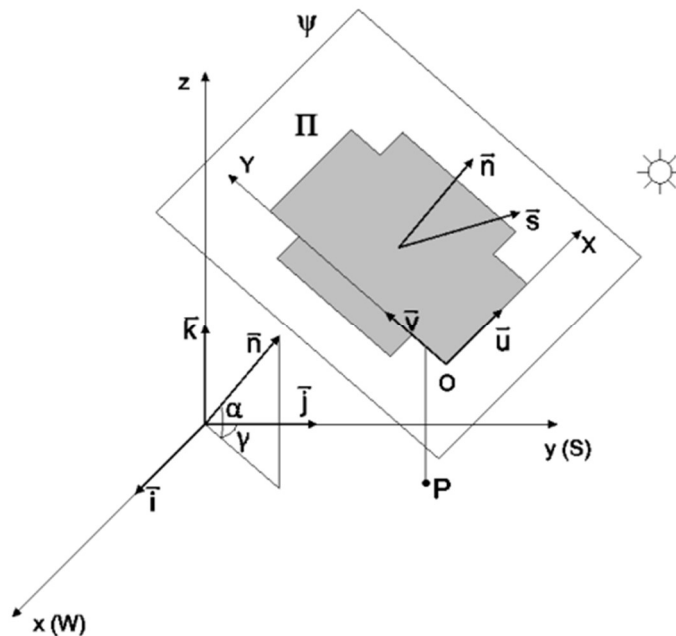
### 148 2.1 Astronomical and Vector Fundamentals

149 To optimise the trajectory of solar trackers in PV plants, this work is based on the  
 150 definition of the solar vector  $\vec{s}$  in an Earth reference system, where the Ox axis is  
 151 oriented to the West, the Oy axis is oriented to the South, and the Oz axis is oriented to  
 152 the zenithal direction, with  $\vec{i}, \vec{j},$  and  $\vec{k}$  the respective unit vectors (Figure 1). The solar  
 153 vector is given by equation (1), where  $\delta$  is the solar declination [16] and  $\Omega t$  is the hourly  
 154 angle, which is defined as the product of the Earth's rotation speed ( $\Omega = 2\pi/24 \text{ rad/h}$ )  
 155 and the time elapsed since the solar noon.

156 
$$\vec{s} = s_x \vec{i} + s_y \vec{j} + s_z \vec{k} = \sin \Omega t \cos \delta \vec{i} + (\cos \Omega t \cos \delta \sin \varphi - \sin \delta \cos \varphi) \vec{j} +$$

$$+ (\cos \Omega t \cos \delta \cos \varphi + \sin \delta \sin \varphi) \vec{k}$$
 (1)

157



158 *Figure 1. Representation of the collector's surface in the Earth reference system*

159

160 Figure 1 also shows the polygon  $\Pi$ , which represents the perimeter of the collector's  
 161 surface, and the vector normal to that surface,  $\vec{n}$ . The components of the vector  $\vec{n}$  in the  
 162 Earth reference system, depending on the azimuth ( $\gamma$ ) and elevation ( $\alpha$ ) angles of the  
 163 collectors, are given by equation (2).

$$164 \quad \vec{n} = \cos \alpha \cdot \sin \gamma \vec{i} + \cos \alpha \cdot \cos \gamma \vec{j} + \sin \alpha \vec{k} \quad (2)$$

165 Additionally, the projection plane is considered as the plane that contains the collector's  
 166 surface. A flat coordinate system associated with this plane is defined (OXY) with unit  
 167 vectors  $\vec{u}$  and  $\vec{v}$ . During tracking, the system will move while rigidly attached to the  
 168 collector polygon. As a result, the mathematical expressions for  $\vec{u}$  and  $\vec{v}$  will depend on  
 169 the collector's orientation at every moment, given by  $\alpha$  and  $\gamma$ , and be conditioned by the  
 170 type of tracking. Equations (3)-(16) present the expressions for the most frequent  
 171 tracking typologies (shown in Figure 2).

172 - Azimuth-elevation tracking (A-E)

$$173 \quad \vec{u} = -\cos \gamma \vec{i} + \sin \gamma \vec{j} \quad (3)$$

$$174 \quad \vec{v} = \sin \alpha \cdot \cos \gamma \vec{i} - \sin \alpha \cdot \cos \gamma \vec{j} + \cos \alpha \vec{k} \quad (4)$$

175 - Equatorial tracking (EQ)

$$176 \quad \vec{u} = -\cos \theta_1 \vec{i} + \sin \theta_1 \cdot \cos \varphi \vec{j} + \sin \theta_1 \cdot \sin \varphi \vec{k} \quad (5)$$

$$177 \quad \vec{v} = -\sin \theta_2 \cdot \sin \theta_1 \vec{i} - (\cos \theta_1 \cdot \sin \theta_2 \cdot \cos \varphi + \cos \theta_2 \cdot \sin \varphi) \vec{j} - (\cos \theta_1 \cdot \sin \theta_2 \cdot \sin \varphi -$$

$$178 \quad \cos \theta_2 \cdot \sin \theta_1) \vec{k} \quad (6)$$

179 where

$$180 \quad \theta_1 = \tan^{-1} \left( \frac{\cos \alpha \cdot \sin \gamma}{\cos \alpha \cdot \cos \gamma \cdot \cos \varphi - \sin \alpha \cdot \sin \varphi} \right) \quad (7)$$

$$181 \quad \theta_2 = \sin^{-1} (\cos \alpha \cdot \cos \gamma \cdot \sin \varphi + \sin \alpha \cdot \cos \varphi) \quad (8)$$

182 - Elevation-Rolling tracking (E-R)

$$183 \quad \vec{u} = -\cos \theta_2 \vec{i} + \sin \theta_1 \cdot \sin \theta_2 \vec{j} + \cos \theta_1 \cdot \sin \theta_2 \vec{k} \quad (9)$$

$$184 \quad \vec{v} = -\cos \theta_1 \vec{j} + \sin \theta_1 \vec{k} \quad (10)$$

185 where

$$186 \quad \theta_1 = \tan^{-1} (\cos \gamma \cdot \cot \alpha) \quad (11)$$

$$187 \quad \theta_2 = \sin^{-1} (\cos \alpha \cdot \sin \gamma) \quad (12)$$

188 - Rolling-Elevation tracking (R-E)

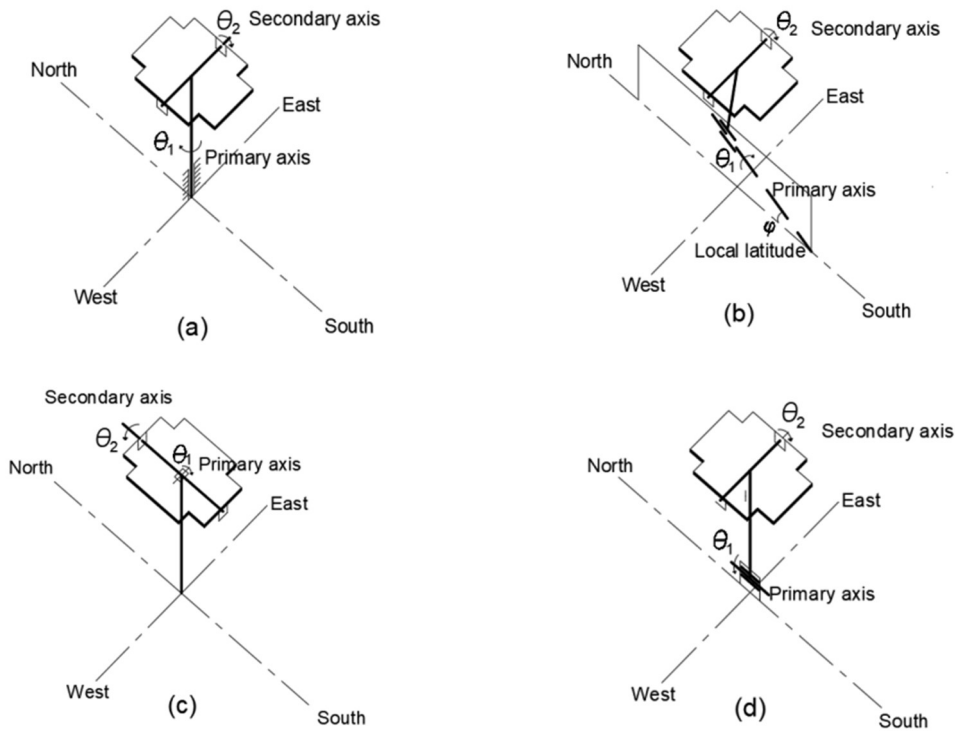
$$189 \quad \vec{u} = -\cos \theta_1 \vec{i} + \sin \theta_1 \vec{k} \quad (13)$$

$$190 \quad \vec{v} = \sin \theta_1 \cdot \cos \theta_2 \vec{i} + \sin \theta_2 \vec{j} + \cos \theta_1 \cdot \cos \theta_2 \vec{k} \quad (14)$$

191 where

192  $\theta_1 = \tan^{-1}(\sin\gamma \cdot \cos\alpha)$  (15)

193  $\theta_2 = \cos^{-1}(\cos\gamma \cdot \cos\alpha)$  (16)



194 *Figure 2. Common tracking strategies: a) Azimuth-elevation tracking (A-E), b)*  
 195 *Equatorial tracking (EQ), c) Elevation-Rolling tracking (E-R), and d) Rolling-Elevation*  
 196 *tracking (R-E)*

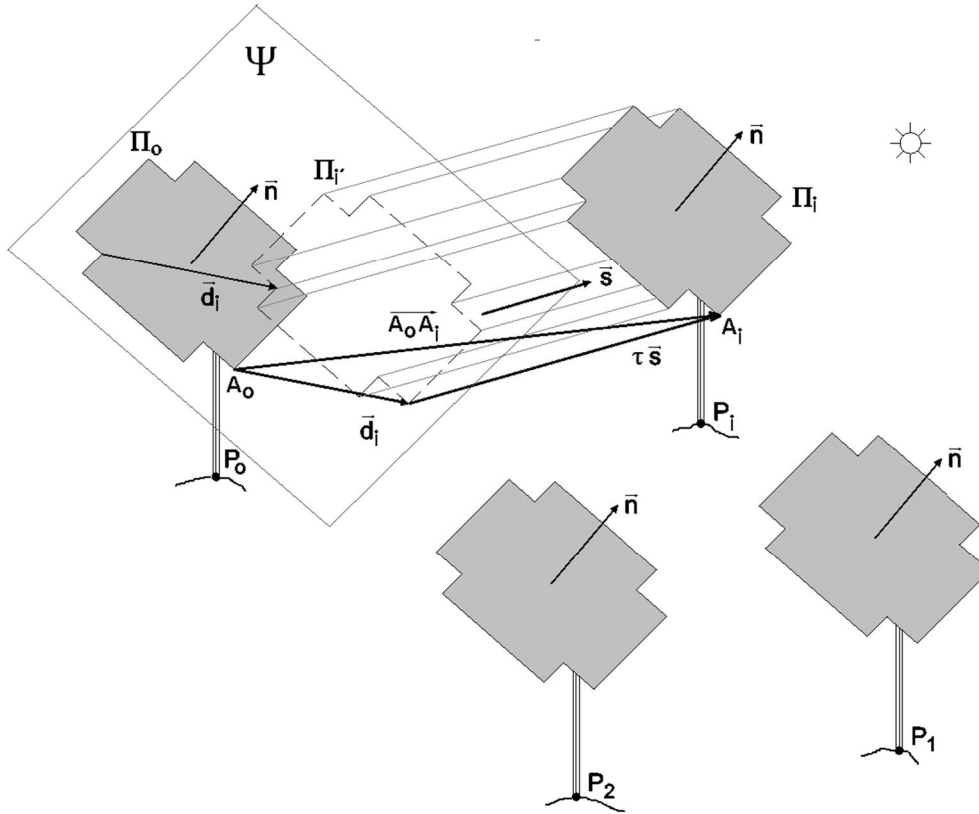
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## 198 2.2 Geometrical Methodology

199 Based on the geometric fundamentals defined in the previous section, this work studies  
 200 shadows to dichotomously determine whether there is an intersection between the PV  
 201 collectors at a specific time, rather than to quantify the shape and size of the shaded  
 202 polygons. Therefore, by calculating the irradiance received by the collector's surfaces  
 203 for a given hour at different positions when there is no shading, the maximum irradiance  
 204 position can be elucidated. Moreover, by conducting this study over a certain period, it  
 205 is possible to define the trajectory of the collectors that optimises energy capture by a  
 206 PV plant for each day of the year.

207 In this study, it is considered that all collectors have the same geometric shape and  
 208 move in the same manner. Considering this, it can be stated that the planes that contain  
 209 the collector surfaces are always parallel. Therefore, regardless of the solar position  
 210 with respect to the collectors, the solar projection  $\Pi'_i$  of any collector  $\Pi_i$  on plane  $\Psi$

211 containing the reference tracker  $\Pi_0$  will produce a polygon with the same shape and  
 212 dimensions as the collector polygon  $i$  (Figure 3). From this projection, it can be  
 213 concluded that  $\Pi_i$  shades  $\Pi_0$  if polygons  $\Pi_0$  and  $\Pi'_i$  intersect.  
 214



215  
 216  
 217

Figure 3. Geometry of the set of trackers

218 As all collectors are considered equal and the perimeters of the projected collectors in  
 219 the solar direction maintain their geometry, polygon  $\Pi'_i$  could be considered as a  
 220 translation of the reference collector  $\Pi_0$  contained on plane  $\Psi$ , with  $\vec{d}_i$  as the translation  
 221 vector. Similarly, as the collectors remain parallel, the distance between any two  
 222 equivalent points ( $A_i$  and  $A_0$ ) of collectors  $\Pi_i$  and  $\Pi_0$  is constant. That is,  $\overrightarrow{A_0A_i} = \overrightarrow{P_0P_i}$ .  
 223 Consequently, to determine  $\vec{d}_i$ , the parallelogram rule is applied to the vectors involved  
 224 in the described geometric problem (Figure 3), which produces equation (17).

$$225 \quad \overrightarrow{P_0P_i} = \overrightarrow{A_0A_i} = \vec{d}_i + \tau \vec{s} \quad (17)$$

226 Furthermore, to determine  $\tau$ , the scalar product between equation (17) and the vector  
 227 normal to plane  $\Psi$ ,  $\vec{n}$ , is calculated, which produces equation (18), where  $\vec{n} \cdot \vec{d}_i$  is zero as  
 228 both vectors are perpendicular.



229  $\vec{n} \cdot \overrightarrow{P_o P_i} = \vec{n} \cdot \vec{d}_i + \vec{n} \cdot \tau \cdot \vec{s}$  (18)

230 Consequently, scalar  $\tau$  is given by equation (19).

231  $\tau = \frac{\overrightarrow{P_o P_i} \cdot \vec{n}}{\vec{s} \cdot \vec{n}}$  (19)

232 Substituting (19) into (17), the translation vector of projection  $\Pi'_i$  with respect to  
 233 reference collector  $\Pi_0$  on plane  $\Psi$ ,  $\vec{d}_i$ , can be obtained (Equation 20).

234  $\vec{d}_i = \overrightarrow{P_o P_i} - \frac{\overrightarrow{P_o P_i} \cdot \vec{n}}{\vec{s} \cdot \vec{n}} \vec{s}$  (20)

235 Thus, expression (20) allows the components of  $\vec{d}_i$  in Earth reference system Oxyz to be  
 236 calculated. However, as  $\vec{d}_i$  belongs to the OXY plane, the Cartesian components in the  
 237 collector plane can be determined by equations (21) and (22).

238  $d_X = \vec{d} \cdot \vec{u}$  (21)

239  $d_Y = \vec{d} \cdot \vec{v}$  (22)

240 Once the projections have been obtained, a test based on Minkowski algebra [17-19] is  
 241 conducted to determine whether the polygons intersect and, therefore, whether there  
 242 would be shading. For this, all the feasible polygons on  $\Psi$  resulting from moving  $\Pi_0$  are  
 243 drawn so that any point on its perimeter matches the origin of the OXY reference system  
 244 associated with plane  $\Psi$  (Figure 4). Polygon  $\Sigma$  is defined as the envelope of this family  
 245 of polygons. Therefore, it is possible to affirm that  $\Pi_0$  and  $\Pi'_i$  intersect if the  
 246 representation of the corresponding  $\vec{d}_i$ , vector moved to the origin of the OXY reference  
 247 system, is fully included in  $\Sigma$  (Figure 4).

248 To ensure that reference collector  $\Pi_0$  is not shaded at a given time, it is necessary to  
 249 check that it is not shaded by any other collector in the PV plant. Given that envelope  $\Sigma$   
 250 is the same for any pair of collectors as they all exhibit the same geometry and remain  
 251 parallel, it would be sufficient to determine whether the  $\vec{d}_i$  vectors (for  $i=1, N-1$ , with N  
 252 being the number of PV panels in the plant) linked to each pair of collector surfaces,  $\Pi_0$ -  
 253  $\Pi_i$ , are included in envelope  $\Sigma$  for cases that meet the following conditions:

- 254 I. Collector  $\Pi_i$  is visible from the reference collector  $\Pi_0$ :  $\overrightarrow{P_o P_i} \cdot \vec{n} > 0$ .
- 255 II. The sun does not irradiate the rear side of the collectors:  $\vec{s} \cdot \vec{n} > 0$ .
- 256 III. It is a specific moment of the solar day:  $\vec{s} \cdot \vec{k} > 0$ .

257 Under these conditions, a single  $\vec{d}_i$  included in the  $\Sigma$  envelope will indicate that the  
 258 studied collector is shaded.

259

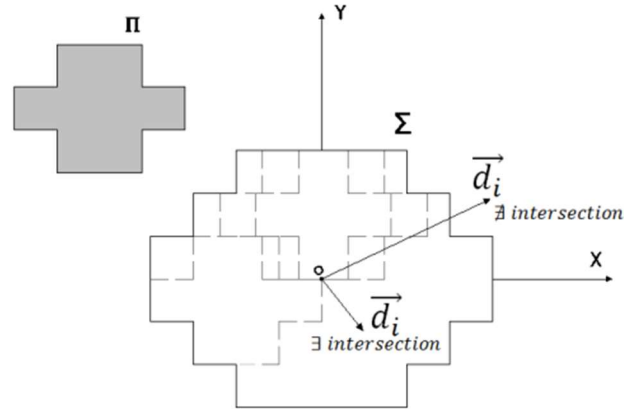


Figure 4. Obtaining enveloping polygon  $\Sigma$  from  $\Pi$

### 2.3 Optimisation of the Collector Position under the no Shading Hypothesis

According to the described method, for each moment in time, whether the reference collector is shaded or not for different collector orientations (given by its azimuth,  $\gamma$ , and elevation,  $\alpha$ ) can be analysed. Based on this analysis, for any specific moment in time, it is also possible to represent the delimitation of the two regions in a cylindrical chart ( $\gamma, \alpha$ ): one corresponding to shaded collectors and another corresponding to non-shaded collectors. In addition, as will be demonstrated in the application, the irradiance received by the collectors at each orientation can be also represented on the same chart using irradiance isovalue curves. From these two delimited areas and using the irradiance isovalue curves, the point with maximum irradiance for each moment in time, and, consequently, the optimum orientation of the solar trackers, can be selected. Repeating this process for different moments in time the same day can allow the optimal tracking trajectory (with maximum irradiance and without shading) to be defined.

### 3. RESULTS AND DISCUSSION

Once the proposed methodology has been described, the optimal trajectories for tracking and backtracking at the "El Molino" PV solar plant located in Cordoba, Spain, are obtained (latitude= $37.75492^\circ\text{N}$ ; longitude= $5.04548^\circ\text{W}$ ). This plant is an Azimuth Elevation tracker plant arranged in a rectangular grid with an east-west distance ( $d_{EW}$ ) of 20 m and north-south distance ( $d_{NS}$ ) of 14 m (Figure 5).



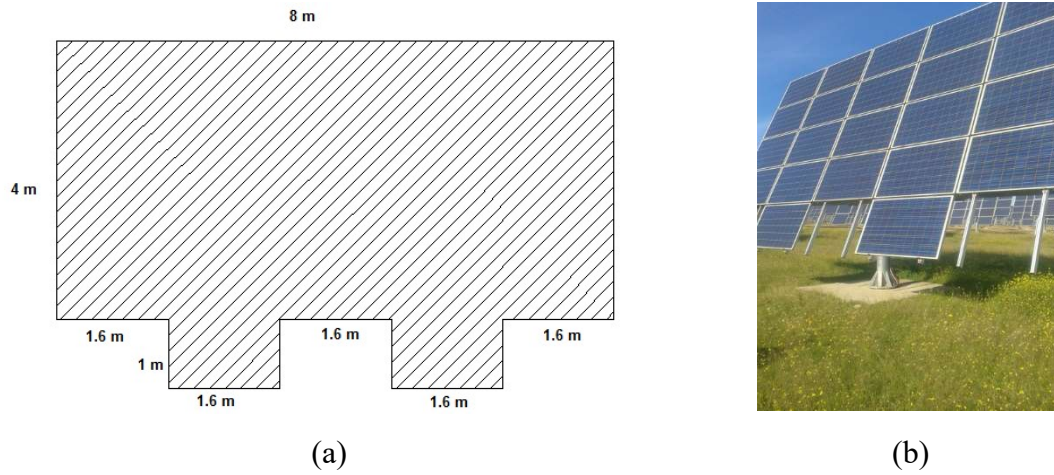
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286

*Figure 5. Application example: Spatial distribution of the collectors of the El Molino PV plant (Cordoba, Spain).*

287

288



(a)

(b)

289

*Figure 6. Application example: Shape and dimensions of the photovoltaic collectors in the El Molino PV plant (Cordoba, Spain).*

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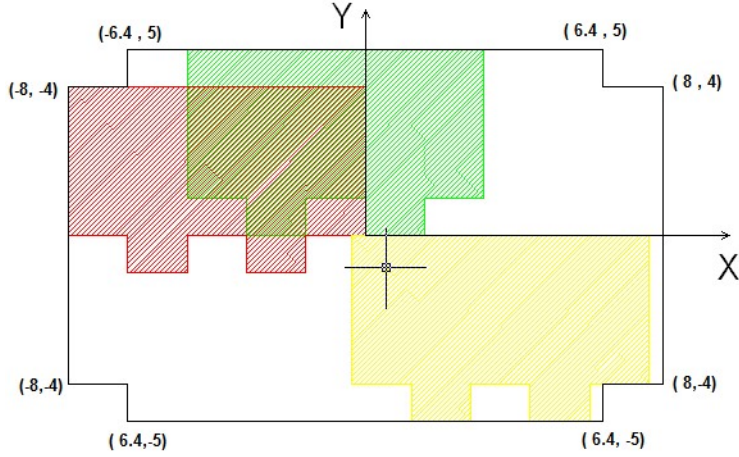
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292 Based on the geometry of the collectors (Figures 6a and 6b), Figure 7 shows the  
 293 envelope  $\Sigma$  for the collectors' surface. In practice, the polygons constituting the  
 294 collectors (Figure 6a) only have right angles. Therefore, the surrounding polygon  $\Sigma$  has  
 295 only right angles (Figure 7), simplifying the test to determine whether the  $\vec{d}_i$   
 296 included in  $\Sigma$ . Therefore, in this example, each  $\vec{d}_i$  is included in the  $\Sigma$  envelope if  
 297 condition (23) or (24) is verified.

298  $|d_{iX}| < 8 \text{ m}$  and  $|d_{iY}| < 4 \text{ m}$  (23)

299  $|d_{iX}| < 6.4 \text{ m}$  and  $|d_{iY}| < 5 \text{ m}$  (24)

300



301

302 *Figure 7. Application example: Shape and dimensions of polygon  $\Sigma$  enveloping the set*  
 303 *of polygons generated by sliding  $\Pi_0$  onto the origin of the coordinates (values in*  
 304 *metres).*

305

306 The analysis method proposed here was applied every five minutes for one astronomical  
 307 year. As this methodology involves a dichotomous test to establish whether or not there  
 308 is shading at a specific collector orientation defined by its azimuth ( $\gamma$ ) and elevation ( $\alpha$ ),  
 309 a binary search for the elevation limit between the shaded and non-shaded areas has  
 310 been programmed for fixed azimuth values. It has been verified that eight iterations are  
 311 sufficient for estimating this limit with an error below 0.3 deg.

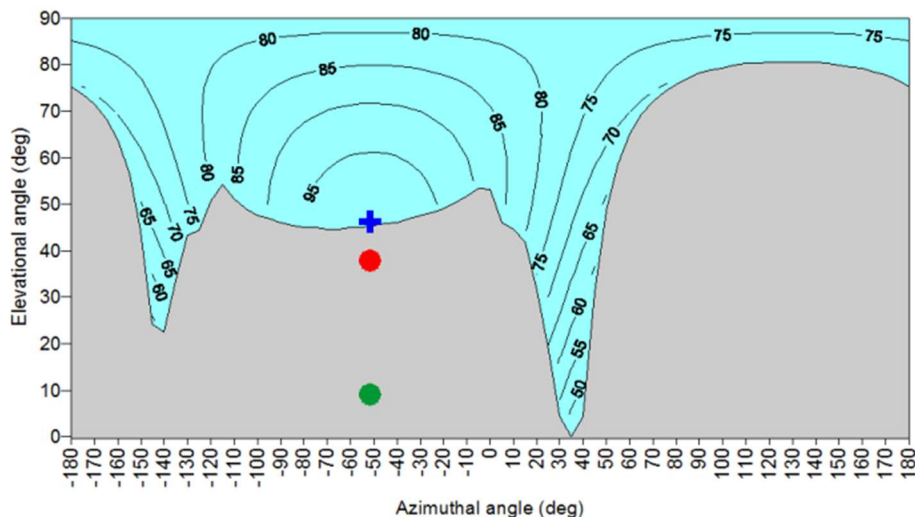
312 For Julian Day 349 at 8:20 (true solar time; TST), Figure 8 shows a cylindrical chart  
 313 representing the boundary between the shaded (grey region, corresponding to the cases  
 314 for which at least one  $\vec{a}_i$  is included in  $\Sigma$ ) and not-shaded (blue region, corresponding to  
 315 the cases for which all vectors  $\vec{a}_i$  are not included in  $\Sigma$ ) areas. Moreover, the irradiance  
 316 isolines were included in the non-shaded area. As the proposed methodology only  
 317 considers the collector positions at which there would be no shading, a single irradiance  
 318 model is assumed. Therefore, Liu-Jordan's equation [34] (Equation 25) was considered  
 319 as it was used by Fernandez-Ahumada et al. [15], where  $I_B$  and  $I_D$  are direct and  
 320 diffuse irradiances, respectively, and  $\rho$  is the albedo. In this study,  $\rho=0.2$  is considered  
 321 following [34]. Therefore, it is possible to determine the solar irradiance captured by the  
 322 collectors for each orientation without shading using equation (25).

323 
$$I = \frac{\vec{s} \cdot \vec{n}}{s^2 k} I_B + \frac{1 + k \vec{n} \cdot \vec{s}}{2} I_D + \rho \frac{1 - k \vec{n} \cdot \vec{s}}{2} (I_B + I_D) \quad (25)$$

324 Similarly, Figure 8 presents the collector orientation for three different tracking  
325 strategies:

- 326 a. Astronomical tracking with no shading (ATNS, represented by a green circle):  
327 tracking governed by an astronomic equation for an ideal PV plant where the  
328 distances between the collectors are sufficiently large to avoid shading.
- 329 b. Maximum irradiance tracking with no shading (MITNS, represented by a red  
330 circle): the optimal tracking strategy proposed by Fernandez-Ahumada et al.  
331 [15], which seeks maximum irradiance levels on an ideal isolated collector that  
332 is not affected by shadows from adjoining collectors.
- 333 c. Maximum irradiance backtracking (MIBT, represented by a blue cross): tracking  
334 strategy proposed in this study, which seeks maximum irradiance levels while  
335 avoiding shading between the collectors by backtracking when necessary.

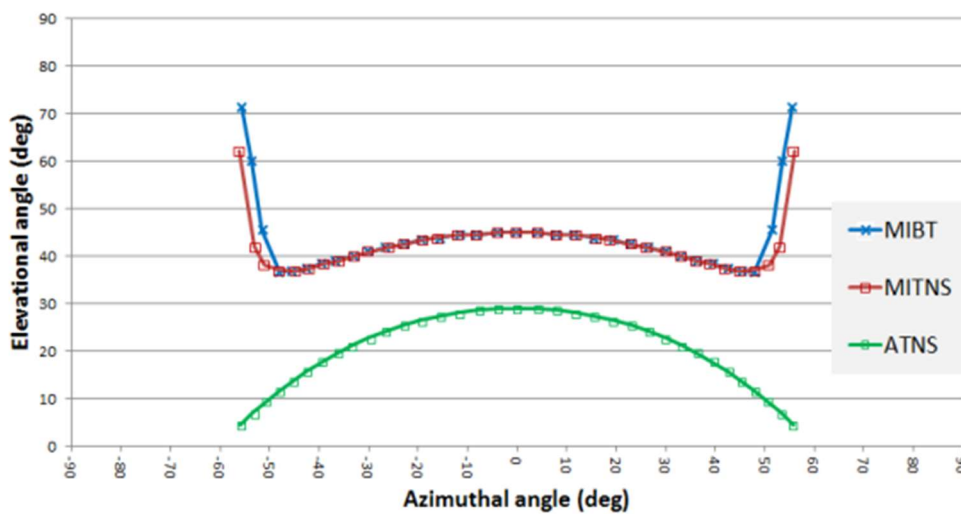
336 Therefore, for this day and time, this novel backtracking approach proposes that the  
337 tracker should point towards the maximum irradiance direction within the non-shaded  
338 region (blue cross in Figure 8). Figure 8 also shows that the orientations corresponding  
339 to ATNS and MITNS are within the region where there are shadows between the  
340 collectors and, consequently, the irradiance captured by the PV modules is reduced.  
341 However, it should be noted that, in this case, the minimum and maximum limits of the  
342 azimuth or elevation are not considered. Consequently, if these constructive limits exist,  
343 they should also be represented as additional restrictions in the cylindrical charts.



344  
345 *Figure 8. Application example: Splitting of the spatial directions and selection of the*  
346 *angles ( $\gamma$ ,  $\alpha$ ) that optimise irradiance ( $W/m^2$ ) for the reference collector in the El*  
347 *Molino PV plant (Cordoba, Spain) on Julian day 349 at 8:20 TST.*

348

349 Moreover, based on the method outlined above, the path to be tracked by the collector  
 350 for the day of study can be proposed. Therefore, Figure 9 shows the trajectories  
 351 corresponding to the three different analysed tracking strategies: ATNS (green line),  
 352 MITNS (red line), and MIBT (blue line). As shown, the proposed MIBT trajectory (blue  
 353 curve) exhibits sections where it does not coincide with the MITNS trajectory (red  
 354 curve) corresponding to the maximum solar irradiance collection under an ideal  
 355 situation with no shading. For these periods, backtracking is proposed as the movement  
 356 that optimises energy collection by the plant, as it considers the real shadows between  
 357 the collectors, which reduce the levels of irradiance from their optimal values  
 358 considered by MITNS.



359  
 360 *Figure 9. Application example: Potential collector pointing trajectories of the PV plant*  
 361 *"El Molino" (Cordoba, Spain) on the Julian day=349*

362  
 363 Finally, the daily radiation was determined for each approach to compare energy  
 364 production under the three potential strategies (ATNS, MITNS, and MIBT). The values  
 365 for the three cases were obtained by integrating equation (25) on representative days.  
 366 Therefore, although the three tracking strategies imply no shading between collectors, in  
 367 contrast to MIBT, ATNS and MITNS are only valid for isolated trackers and not for  
 368 plants with many PV modules. Accordingly, the simulated energy production of these  
 369 two ideal tracking strategies can be considered as maximum potential values and should  
 370 be used as a reference to evaluate the improvements made by the proposed tracking  
 371 method.

372

373 Table 1. Comparative analysis of the energy production levels of PV solar plants with  
 374 different tracking strategies.

Month	MIBT (kWh/kWp)	MITNS (kWh/kWp)	ATNS (kWh/kWp)	Decrease in MIBT vs. MITNS (%)	Increase in MIBT vs. ATNS (%)
January	82.4	84.2	83.7	2.16	-1.51
February	114.7	116.0	114.0	1.09	0.59
March	144.3	146.3	144.4	1.36	-0.11
April	160.8	163.0	161.4	1.35	-0.39
May	177.1	179.6	178.4	1.41	-0.76
June	250.5	251.1	244.5	0.24	2.44
July	291.0	291.3	280.7	0.08	3.68
August	269.3	269.6	259.0	0.09	4.01
September	197.7	198.4	192.1	0.34	2.93
October	125.4	127.3	125.9	1.49	-0.35
November	86.6	88.5	88.0	2.11	-1.52
December	65.0	67.4	67.4	3.54	-3.49
<b>Year</b>	<b>1965.0</b>	<b>1982.7</b>	<b>1939.5</b>	<b>0.89</b>	<b>1.31</b>

375  
 376 Table 1 shows the simulated energy production (kWh) values for each month against  
 377 the peak power (kWp) of the collectors. In line with Fernandez-Ahumada's results [15],  
 378 energy production under MITNS is higher than that under ATNS. Similarly, it has been  
 379 verified that, for several months, energy production by solar plants under MIBT reaches  
 380 values between the optimal values of MITNS and ATNS. Production by MIBT solar  
 381 plants is 0.89% lower than that by MINTS plants, but 1.31% higher than that by ATNS  
 382 plants.

383 The proposed method improves the results obtained by Navarte and Lorenzo [27] in  
 384 their characterisation of the energy losses due to shading in plants with different  
 385 astronomical tracking typologies (one and two-axis). They demonstrated that, in all  
 386 cases, energy production losses increase with GCR. Therefore, in comparison to the  
 387 ideal astronomical tracking, they estimated that the uncertainty of energy production is  
 388 within 2% for GCR=0.09. These results are similar to those published by Panico [28],  
 389 even though this study is restricted to one-axis trackers. Specifically, Panico found that  
 390 the losses due to shading in installations with GCR=0.09 compared to astronomical  
 391 tracking are 2.5% [28]. These values are also within the intervals proposed by Gordon  
 392 and Wenger [26], who demonstrated that energy losses by shading in plants with  
 393 GCR=0.09 depend on the collectors' geometry and spatial layout.

394 Consequently, all published studies indicate that shading causes energy losses in  
395 comparison to energy generation under ideal astronomical tracking. Therefore, this  
396 study shows that solar energy collection by plants with the proposed tracking strategy,  
397 MIBT, is better than that by plants with astronomical tracking and only 0.98% lower  
398 than that by plants with the ideal MITNS tracking. However, owing to the scarcity of  
399 publications in this area, the authors of this paper consider that it is necessary to  
400 continue studying the influence of design parameters on energy collection by plants  
401 with MIBT, as well as to implement this novel tracking strategy in actual PV  
402 installations to evaluate its development.

403

#### 404 **4. CONCLUSIONS**

405 In this study, a new methodology for defining the optimal tracking strategy without  
406 shading of sets of two-axis motion PV tracker collectors is proposed. In contrast to  
407 astronomical tracking, the proposed method indicates that collectors do not have to be  
408 constantly perpendicular to the direct solar rays, as it considers the diffuse and reflected  
409 irradiance, as well as the direct irradiance, reaching PV collectors. Therefore, when  
410 collectors are not shaded, a tracking trajectory seeking maximum irradiance on the  
411 collectors is suggested. However, when the collectors are shaded, backtracking is  
412 proposed. Therefore, based on the concepts of solar vectors and vector algebra, this  
413 method analyses shading between the collectors. However, the proposed technique is  
414 not based on the calculation of the area of polygon intersections; rather, it is based on  
415 whether or not such intersections are present. Consequently, in contrast with other  
416 tracking strategies found in previous studies, this novel method is based on algorithms  
417 that are significantly more simple and fast. Thus, owing to its novelties and advantages,  
418 this method is easier to be used to simulate energy production with different radiative  
419 models and is applicable to situations for which no published generic methods can be  
420 found, such as PV plants:

- 421 i. with non-rectangular surface collectors
- 422 ii. with collectors that are not located on the regular nodes of a geometric mesh
- 423 iii. with different tracking modes
- 424 iv. with trackers located on real topographical surfaces

425 The energy production by PV plants with this new tracking strategy, called MIBT, has  
426 been analysed and compared to two ideal tracking strategies:



427 1) ATNS: Astronomical tracking in an ideal PV plant where the distances between  
428 the collectors are large enough to avoid shading.

429 2) MITNS: optimal tracking that seeks the maximum irradiance levels on an ideal  
430 isolated collector not affected by potential shadows from adjoining collectors  
431 [15].

432 The results show that MIBT improves the energy collection by 1.31% in comparison to  
433 ATNS, and the energy collection is only 0.89% lower than that by MITNS plants.  
434 Therefore, considering these results and the advantages of this method, the authors  
435 consider that this method will not only be useful for designing new facilities, but could  
436 also help to improve the productivity and management of many PV plants by redefining  
437 tracking strategies.

438

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444

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