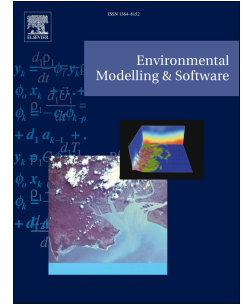


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# A MATLAB software platform for modelling vertically-integrated non-hydrostatic flows with moment equations

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1 **Abstract**

2 This work presents a software platform to compute depth-integrated non-hydrostatic  
3 coastal and open channel flows. The software is based on the **V**ertically-**A**veraged  
4 and **M**oment (VAM) equations model. The VAM model uses the weighted residual  
5 method to account for the non-hydrostaticity and non-uniformity of flow. After  
6 presenting the governing equations, numerical scheme and structure of the graphical  
7 user interface, the software is applied to solve coastal and open channel flow tests  
8 with significant non-hydrostaticity and non-uniformity of flow. The results are  
9 compared with hydrostatic model computations and laboratory measurements. Where  
10 a hydrostatic-based simulation poorly reproduces experimental observations, the  
11 higher-order model equations by this software platform generate excellent  
12 predictions. These findings highlight the potential of this user-friendly software  
13 platform for modeling open channel, river and nearshore flows and processes, where  
14 non-hydrostatic pressure and non-uniform velocity shall be accounted for, which  
15 entails a meaningful advance for the hydraulic community.

16 *Keywords:* non-hydrostatic flows; depth-integrated model; software platform; coastal  
17 applications; open channel tests

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## 18 **Software availability**

19 - Name of software: VAM Model Software

20 - Contact emails: [ag2caoro@uco.es](mailto:ag2caoro@uco.es), [fncaintero@ias.csic.es](mailto:fncaintero@ias.csic.es), [rafael.bergillos@uco.es](mailto:rafael.bergillos@uco.es),

21 [ag2gaojp@uco.es](mailto:ag2gaojp@uco.es)

22 - Requirements: MATLAB Runtime

23 - Availability: The software platform is freely available at

24 [https://github.com/Frncch/VAM\\_Model\\_Software](https://github.com/Frncch/VAM_Model_Software)

## 25 **1. Introduction**

26 Most of the user-friendly models used for the simulation of open channel flows are  
27 based on the solution of the Saint-Venant equations, where it is assumed that the  
28 distribution of fluid pressure is hydrostatic and the vertical distribution of the  
29 velocity component in the direction of motion is uniform (U. S. Corps of Engineers,  
30 2002; Bladé et al., 2014; Teng et al., 2017). In free surface flows with a vertical  
31 length scale negligible as compared to the horizontal one, the Saint-Venant based  
32 models provide accurate approximations. However, in open channel transitions, near  
33 hydraulic structures, in dam break waves over movable beds, in eroded granular  
34 dikes by overtopping, or in river flows with a significant variation of bottom  
35 topography, among others, the variation of the vertical velocity is non-negligible and  
36 the vertical acceleration is, therefore, non-zero. Thus, the pressure distribution is  
37 non-hydrostatic with non-negligible dispersive effects in the equations of motion  
38 (Castro-Orgaz and Hager, 2017). The inclusion of such dispersive effects in a  
39 vertically-averaged framework allows to accurately solve a large portfolio of river  
40 and coastal flow processes (Kim and Lynett, 2011; Castro-Orgaz and Hager, 2017;  
41 Cantero-Chinchilla et al. 2018a; b). Coastal flows are typically simulated by using  
42 Boussinesq-type solvers, such as the short waves module in MIKE21 (Warren and  
43 Bach, 1992) or the Celeris model (Tavakkol and Lynett, 2017). The dam-break flood  
44 waves in rivers and the flow in the surf zone and the swash region are important

45 examples of non-hydrostatic flows, where Boussinesq equations have been  
46 extensively applied (Brocchini and Peregrine, 1996; Yamazaki et al., 2009; Cantero-  
47 Chinchilla et al., 2016). The use of non-hydrostatic models is required to provide  
48 realistic predictions in these flows (Wei and Jia, 2014a; b), albeit with a significant  
49 penalty in the numerical treatment of dispersive terms, due to the presence of higher-  
50 order derivatives of the depth-averaged velocity field (Castro-Orgaz and Hager  
51 2017). However, given the still high computational cost of three-dimensional (non-  
52 hydrostatic) models for large-scale simulations, e.g. in a river basin, the development  
53 and use of depth-integrated simulation models, where the vertical coordinate is  
54 removed from the governing equations, is a common practice for non-hydrostatic  
55 modelling (Zijlema and Stelling, 2005; 2008; Yamazaki et al., 2009; Bai and  
56 Cheung, 2013; Wei and Jia, 2013; Aissiouene et al., 2015; Escalante et al., 2018,  
57 among others).

58 Essentially, three families of vertically-averaged models have been developed so  
59 far to simulate non-hydrostatic flows. The most widely used family is known as  
60 Boussinesq-type equations. This family considers the non-hydrostaticity through the  
61 inclusion of additional dispersive terms in the governing equations, demanding the  
62 numerical discretization of high-order velocity derivatives (Madsen et al., 1991;  
63 Madsen and Sørensen, 1992; Peregrine, 1966; Brocchini, 2013; Castro-Orgaz and  
64 Hager, 2017). The issue of increasing the order of spatial derivatives in a model  
65 appears subtle, but getting a robust solver when discretizing spatial derivatives of  
66 higher order than two is a formidable task when dealing with dam-break like  
67 numerical simulations. Another different (although related) family is the non-  
68 hydrostatic extension of the shallow water equations, which is based on a multilayer  
69 discretization of layer-averaged Reynolds-Averaged Navier-Stokes (RANS)  
70 equations (Stansby and Zhou, 1998; Casulli, 1999; Zijlema and Stelling, 2005;  
71 Fernandez-Nieto et al., 2018; Escalante et al., 2019). Although both Boussinesq  
72 equations and multilayer models are very useful tools, each one has its own  
73 disadvantages: the presence of high-order derivatives in the Boussinesq equations,

74 and the increased computational cost by increasing the number of layers to simulate a  
75 flow in the multilayer models. Finally, there is a third family of vertically-averaged  
76 models for the prediction of non-hydrostatic flows (Steffler and Jin, 1993; Khan and  
77 Steffler, 1996a; b), which is based on the use of the weighted residual method to  
78 derive the vertically-averaged and moment equations from the RANS equations. The  
79 modelling system of equations is known as **V**ertically **A**veraged and **M**oment (VAM)  
80 equations model (Cantero-Chinchilla et al., 2018a). The weighting function,  $\Phi =$   
81  $2(z - \bar{z})/h$ , is used in the VAM model to derive the moment equations while  
82 applying the weighted residual method, where  $\bar{z}$  is the elevation of the centroid of a  
83 section ( $\bar{z} = z_b + h/2$ ). The VAM model is a physical system of equations that is still  
84 unfamiliar to the scientific community to some extent. It was developed by Steffler  
85 and Jin (1993) and then solved by Khan and Steffler (1996a; b) for flows in weirs  
86 and spillways, producing extremely accurate results for non-hydrostatic flows.

87 Apparently, publication of these results in hydraulics journals discouraged the use  
88 of VAM models by numerical modellers, who in this branch of science routinely  
89 apply Saint-Venant equations. The increased numbers of equations to solve in the  
90 VAM equations, and the apparent complexities in their structure when compared to  
91 Saint-Venant equations, unfairly relegated the use of VAM equations to a few  
92 isolated works conducted by their development team. However, complexities of a  
93 model are not in the number of equations to solve, but rather, on the mathematical  
94 structure of the terms to discretize. In this regard, the highest order derivative in the  
95 VAM equations is of order one, which defines a system of first order partial  
96 differential equations. This feature is attractive from the view point of numerical  
97 discretization, since higher order derivatives are absent. Discretization of dispersive  
98 terms is implicit for stability of the elliptic step of the solver, while the hyperbolic  
99 step is explicit and subject to the Courant-Friedrich-Lewy condition ( $CFL < 1$ ).  
100 Despite the VAM model was recently demonstrated to be extremely accurate  
101 predicting a number of river and coastal flows (Cantero-Chinchilla et al., 2018a); we  
102 acknowledge that the apparent complexities of the VAM model are a penalty in

103 terms of its diffusion and use by peers, since the development and application of the  
 104 VAM equations and numerical scheme are time-consuming and not immediate tasks.

105 Therefore, the motivation of this paper is to present a user-friendly software  
 106 platform that easily allows the application of the VAM model to simulate non-  
 107 hydrostatic flows in a number of coastal and open channel flow cases. First, the  
 108 governing equations and numerical scheme of the VAM model are shown in Section  
 109 2. Then, the structure of the graphical user interface (GUI) and the tests included in  
 110 the software platform are described in Section 3. Section 4 shows several  
 111 applications of the software platform. Finally, the conclusions of the work are  
 112 summarized in Section 5. The VAM model software platform is freely available on  
 113 GitHub ([https://github.com/Frnccch/VAM\\_Model\\_Software](https://github.com/Frnccch/VAM_Model_Software)) and the MATLAB  
 114 scripts of the VAM model are provided as supplementary material to this paper.

## 115 **2. Theoretical and numerical basis of the VAM model**

### 116 *2.1 Governing equations*

117 The governing equations of the one-dimensional (1D) VAM model are derived  
 118 from the vertically-averaged RANS continuity equation,  $x$ - and  $z$ -momentum  
 119 equations, and moment of  $x$ - and  $z$ -momentum equations (Steffler and Jin, 1993;  
 120 Khan and Steffler, 1996a; b). They can be expressed in conservative form as follows  
 121 (Cantero-Chinchilla *et al.*, 2018a; see Hydraulic Reference Manual of the VAM  
 122 model in the supplementary material for a complete derivation of the model  
 123 equations):

$$124 \quad \frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S}_o + \mathbf{S}_\tau, \quad (1)$$

$$\mathbf{U} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix} = \begin{bmatrix} h \\ q \\ u_1 \\ h\bar{w} \\ \frac{h^2 w_*}{12} \end{bmatrix}, \mathbf{F} = \begin{bmatrix} q \\ \frac{q^2}{h} + g \frac{h^2}{2} \\ \frac{qu_1}{h} \\ q\bar{w} \\ \frac{hqw_*}{12} \end{bmatrix} = \begin{bmatrix} U_2 \\ \frac{U_2^2}{U_1} + g \frac{U_1^2}{2} \\ U_2 \frac{U_3}{U_1} \\ U_2 \frac{U_4}{U_1} \\ U_2 \frac{U_5}{U_1} \end{bmatrix}, \quad (2)$$

$$\mathbf{S}_o = \begin{bmatrix} 0 \\ -\frac{\partial}{\partial x} \left( \frac{u_1^2 h}{3} + \frac{hp_1}{2\rho} + \frac{2hp_2}{3\rho} \right) - gh \frac{\partial z_b}{\partial x} - \frac{p_1}{\rho} \frac{\partial z_b}{\partial x} \\ \frac{1}{2\rho} \left( \frac{\partial p_1}{\partial x} - \frac{p_1}{h} \frac{\partial h}{\partial x} \right) - \frac{4p_2}{h\rho} \frac{\partial \bar{z}}{\partial x} \\ \frac{1}{6} \frac{\partial (hu_1 w_*)}{\partial x} + \frac{p_1}{\rho} \\ \left[ -\frac{h\bar{w}}{2} \frac{\partial q}{\partial x} + \left[ q\bar{w} - \frac{hu_1 w_*}{6} \right] \frac{\partial \bar{z}}{\partial x} \right] \\ + \frac{\partial}{\partial x} \left[ \frac{h^2 u_1}{10} \left( \bar{w} + \frac{w_b}{3} + \frac{w_s}{3} \right) \right] - h\bar{w}^2 - \frac{2hp_2}{3\rho} \end{bmatrix}, \mathbf{S}_\tau = \begin{bmatrix} 0 \\ -\frac{\tau_b}{\rho} \\ 0 \\ -\frac{\tau_b}{\rho} \frac{\partial z_b}{\partial x} + \frac{1}{2\rho} \frac{\partial h\tau_b}{\partial x} \\ -\frac{h\tau_b}{2\rho} \left( \frac{\partial \bar{z}}{\partial x} + \frac{\partial z_b}{\partial x} \right) \end{bmatrix}, \quad (3)$$

127 where  $\mathbf{U}$ ,  $\mathbf{F}$  and  $\mathbf{S}$  are, respectively, the vector of unknowns, fluxes and source terms,  
 128  $(U_1, \dots, U_5)$  are the conservative flow variables, related to the conservation laws as  
 129 follows:  $U_1$  (continuity equation),  $U_2$  ( $x$ -momentum equation),  $U_3$  (moment of  $x$ -  
 130 momentum equation),  $U_4$  ( $z$ -momentum equation) and  $U_5$  (moment of  $z$ -momentum  
 131 equation). Subscripts  $o$  and  $\tau$  refer to the inviscid and the turbulent stress source  
 132 terms, respectively,  $x$  is the longitudinal coordinate,  $t$  is the time,  $\rho$  is the fluid  
 133 density,  $g$  is the gravitational acceleration,  $q$  is the discharge per unit width,  $h$  is the  
 134 flow depth,  $u_1$  is the  $x$ -velocity at the free surface in excess of the mean  $q/h$ ,  $z_b$  is the  
 135 river bed elevation,  $\bar{w}$  is the depth-averaged  $z$ -velocity [ $\bar{w} = (w_s + w_b)/2 + 2w_2/3$ ],  
 136 with  $z$ -as the Cartesian vertical coordinate,  $w_s$  is the  $z$ -velocity at the free surface,  $w_b$



137 is the  $z$ -velocity at the bed,  $w_2$  is the mid-depth  $z$ -velocity in excess of the average  
 138 between vertical velocities at the bed and free surface,  $w_*$  is the vertical velocity  
 139 difference between bed and free surface ( $w_* = w_b - w_s$ ),  $p_1$  is the bed pressure in  
 140 excess of the hydrostatic,  $p_2$  is the mid-depth deviation from the linear non-  
 141 hydrostatic law,  $\tau_b$  is the bed shear stress, and  
 142  $\overline{w^2} = (\overline{w})^2 + \frac{w_b^2}{12} + \frac{w_s^2}{12} - \frac{w_b w_s}{6} + \frac{1}{20} (2\overline{w} - w_b - w_s)^2$ . Equations (1)–(3) are derived  
 143 from the general vertically-integrated mass, momentum and moment of momentum  
 144 equations, modelling the  $(x, z)$  velocity components and fluid pressure with  
 145 distributions containing perturbation parameters  $u_1(x, t)$ ,  $w_2(x, t)$ ,  $p_1(x, t)$  and  $p_2(x, t)$   
 146 to deviate the flow from hydrostatic conditions with horizontal and depth-  
 147 independent velocity. The particular choice used in Eqs. (1)–(3) is (Steffler and Jin  
 148 1993; Khan and Steffler 1996a; b; see Hydraulic Reference Manual of the VAM  
 149 model in the supplementary material for a complete derivation of the model  
 150 equations):

$$151 \quad u(x, z, t) = u_0 + u_1 \left( 2 \frac{z - z_b}{h} - 1 \right), \quad (4)$$

$$152 \quad w(x, z, t) = \left( w_b + w_2 4 \frac{z - z_b}{h} \right) \left( 1 - \frac{z - z_b}{h} \right) + w_s \frac{z - z_b}{h}, \quad (5)$$

$$153 \quad p(x, z, t) = \left( \rho g h + p_1 + p_2 4 \frac{z - z_b}{h} \right) \left( 1 - \frac{z - z_b}{h} \right). \quad (6)$$

154 Note that the perturbation parameters are functions of  $x$  and  $t$ , but not of the  
 155 vertical coordinate  $z$ . To mathematically close the model, additional physical  
 156 statements are required. The kinematic boundary conditions at the bed and free  
 157 surface levels are, respectively,

$$158 \quad w_b = u_b \frac{\partial z_b}{\partial x} = \left( \frac{q}{h} - u_1 \right) \frac{\partial z_b}{\partial x}, \quad (7)$$

$$159 \quad w_s = \frac{\partial h}{\partial t} + u_s \frac{\partial z_s}{\partial x} = -\frac{\partial q}{\partial x} + \left( \frac{q}{h} + u_1 \right) \frac{\partial(z_b + h)}{\partial x}, \quad (8)$$

160 where  $u_b$  and  $u_s$  are the  $u$ -velocity components at the bed and free surface,  
 161 respectively. Two reactive equations, i.e. where temporal derivatives of conserved  
 162 variable are absent, are prescribed to provide closure to the system of Eqs. (1)–(3).  
 163 The first is obtained combining Eqs. (7)–(8), resulting

$$164 \quad \frac{h^2 w_*}{12} = \frac{h^2}{12} \left( \frac{q}{h} - u_1 \right) \frac{\partial z_b}{\partial x} + \frac{h^2}{12} \frac{\partial q}{\partial x} - \frac{h^2}{12} \left( \frac{q}{h} + u_1 \right) \frac{\partial(z_b + h)}{\partial x}. \quad (9)$$

165 The second is the moment of continuity equation, which reads (see Hydraulic  
 166 Reference Manual of the VAM model in the supplementary material):

$$167 \quad h\bar{w} = -\frac{h}{2} \frac{\partial q}{\partial x} + q \frac{\partial \bar{z}}{\partial x} + \frac{1}{6} \frac{\partial(h^2 u_1)}{\partial x}. \quad (10)$$

169 The VAM model system (Eqs. 1–3 and Eqs. 9–10) is based on a simple  
 170 turbulence closure where the normal stresses are neglected and the shear stress is  
 171 considered to vary linearly between  $\tau_b$  at bed and zero at the surface. The latter is  
 172 modelled using the Manning's equation parametrized with the near-bed non-  
 173 hydrostatic velocity field (Cantero-Chinchilla *et al.* 2018a; see also Hydraulic  
 174 Reference Manual of the VAM model in the supplementary material). If  
 175  $u_1 = w_2 = p_1 = p_2 = 0$  the VAM Eqs. (1)–(3) reduce to the Saint-Venant equations. In  
 176 this case the moment equations and the vertical momentum equation are not needed,  
 177 given the lack of perturbation parameters, e.g. only  $h$  and  $q$  are computed.

178 A significant feature of the VAM equations as compared to Boussinesq equations  
 179 is that a wave breaking criteria is not needed, given that the vertical profile of  
 180 horizontal velocity  $u$  is modeled, thereby introducing dissipation into the solution.  
 181 The VAM model was demonstrated to produce the transition from solitary wave to

182 broken waves during run-up on a beach without any empirical formulation for wave  
183 breaking (Cantero-Chinchilla *et al.* 2018a). If  $u_1 = w_2 = p_2 = 0$ , the only perturbation  
184 parameter retained is  $p_1$ , and the VAM Eqs. (1)–(3) reduce to the Vertically-  
185 Averaged equations (VA) by removing the moment equations (Khan and Steffler  
186 1996a). The VA system of equations is basically equivalent to a Boussinesq-type  
187 system, albeit with a factor affecting the dispersive term (Cantero-Chinchilla *et al.*  
188 2016). The VAM equations can model with great accuracy dispersive effects up to  
189  $kH = 8$ , where  $k$  is the wave number and  $H$  is the still water depth. However, the  
190 accuracy of the VA model is acceptable only to  $kH = 2$  (Steffler and Jin, 1993; see  
191 Hydraulic Reference Manual of the VAM model in the supplementary material for a  
192 detailed comparison of VAM and VA models). Therefore, the advantage of the VAM  
193 model over other types of models, such as those based on Boussinesq equations, is  
194 that dispersive effects are modelled with great accuracy up to large  $kH$  values, and  
195 that wave breaking is automatically accounted for by the model equations. This  
196 comparison is not aimed at discouraging the use of Boussinesq-type equations, which  
197 are valuable numerical models by improving their linear dispersion relation and  
198 implementing a wave breaking criterion. The comparison focuses on presenting the  
199 VAM equations as a viable alternative to model the same flows, showing some of its  
200 important features.

## 201 2.2 Numerical scheme

202 The system of Eqs. (1)–(7) is solved using a semi-implicit hybrid finite volume-  
203 finite difference numerical scheme (Cantero-Chinchilla *et al.*, 2018a; see  
204 supplementary material, where the MATLAB scripts are freely available.  
205 Fundamentals of the numerical method implemented are explained in detail the  
206 Hydraulic Reference Manual accompanying the VAM code). The solution strategy  
207 uses a splitting approach. The homogeneous part of Eq. (1), i.e. with no source terms,  
208 is solved first in an advection step using a Godunov-type finite-volume scheme. The

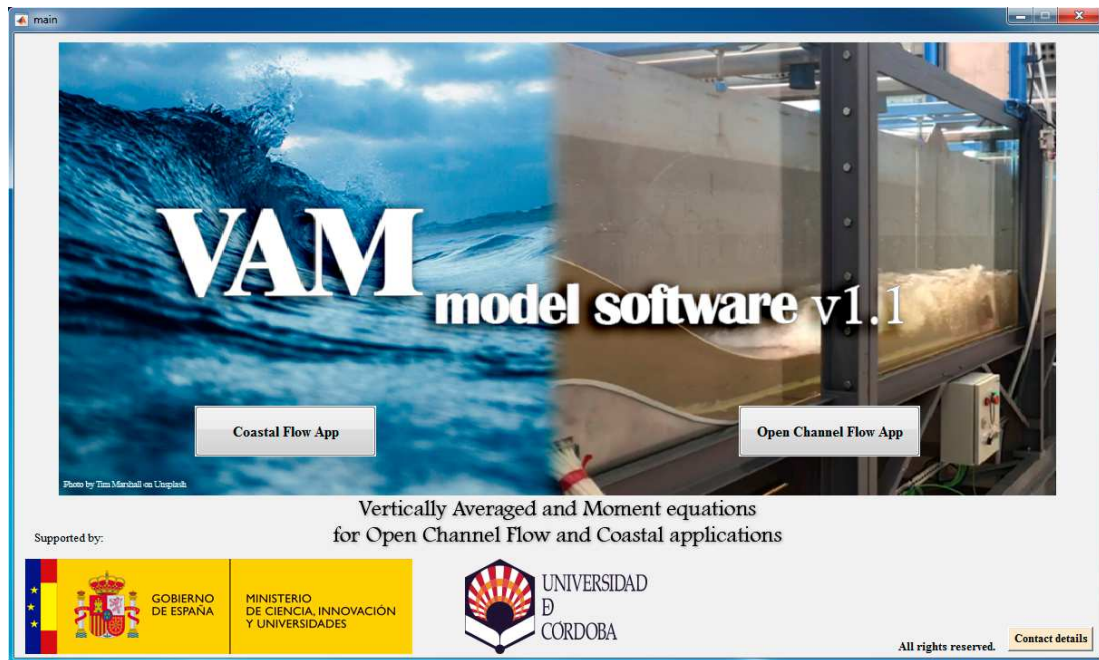
209 fourth-order total variation diminishing monotone upstream centered scheme for  
210 conservation laws (MUSCL-TVD-4<sup>th</sup>) and the *minmod* limiter are used to reconstruct  
211 the flow variables at the cell-interfaces (Yamamoto and Daiguji, 1993; Kim et al.,  
212 2009; Kim and Lynett, 2011; Cantero-Chinchilla et al., 2016).

213 The weighted surface-depth gradient method (WSDGM) (Aureli et al., 2008) and a  
214 discretization of the gravity term in the  $x$ -momentum equation to grant the C-  
215 property of the numerical scheme are implemented. The numerical flux at the cell-  
216 interface is determined using the HLLC approximate Riemann solver (Toro, 2001,  
217 2009). In this step, the computed flow depth is the final value for the actual time. The  
218 homogeneous solution is then updated by including the effects of the inviscid terms  
219 using a centred implicit finite-difference scheme resulting from the application of the  
220 backward Euler formula. The system of algebraic equations is iteratively solved  
221 using the Newton-Raphson method. The equations resulting from this step are non-  
222 linear and implicit, and, for this reason, an iterative solver is needed. The discretized  
223 system of equations and Jacobian matrix implemented are described in the Hydraulic  
224 Reference Manual accompanying the VAM MATLAB scripts as supplementary  
225 material. Note that, in this step, the two reactive equations [Eqs. (9) and (10)] are  
226 used for model closure. In a final step, the inviscid non-hydrostatic solution is  
227 updated by including the effect of the turbulent source terms using a centred finite-  
228 difference scheme and the forward Euler formula. The time step is subjected to the  
229 CFL condition to ensure numerical stability, which was found by numerical  
230 experimentation to be restricted to  $CFL < 0.5$ . A detailed description of the numerical  
231 scheme is available in Cantero-Chinchilla et al. (2018a), and the MATLAB scripts  
232 can be downloaded as supplementary material.

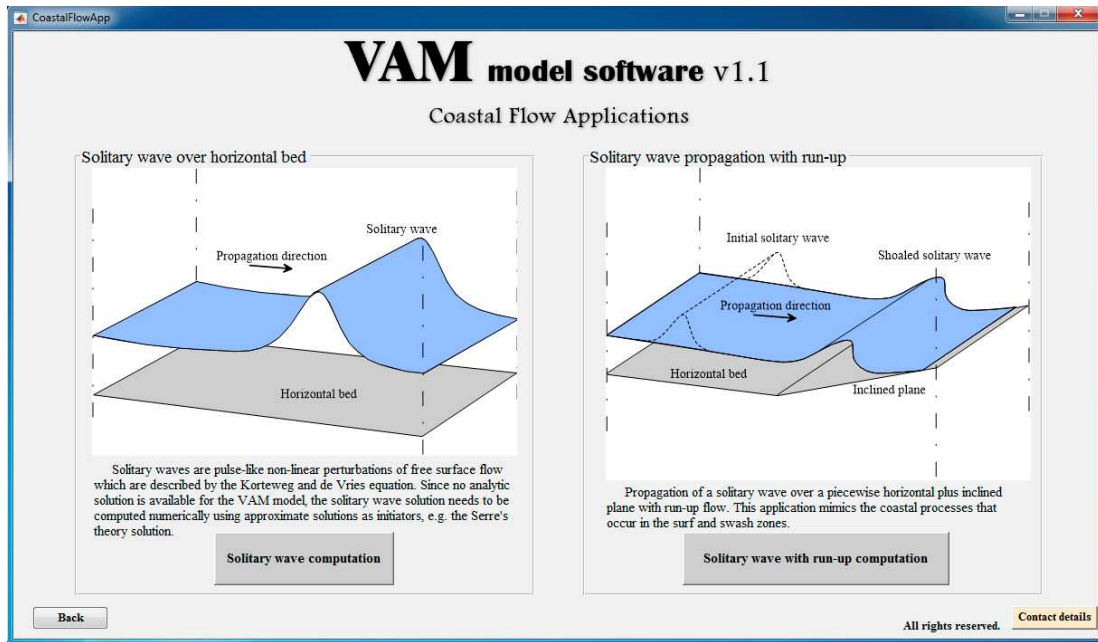
### 233 **3. Description and structure of the software platform**

234 The initial window of the software platform is shown in Fig. 1. It allows selecting  
235 coastal or open channel flow applications. Within the first option, two kinds of

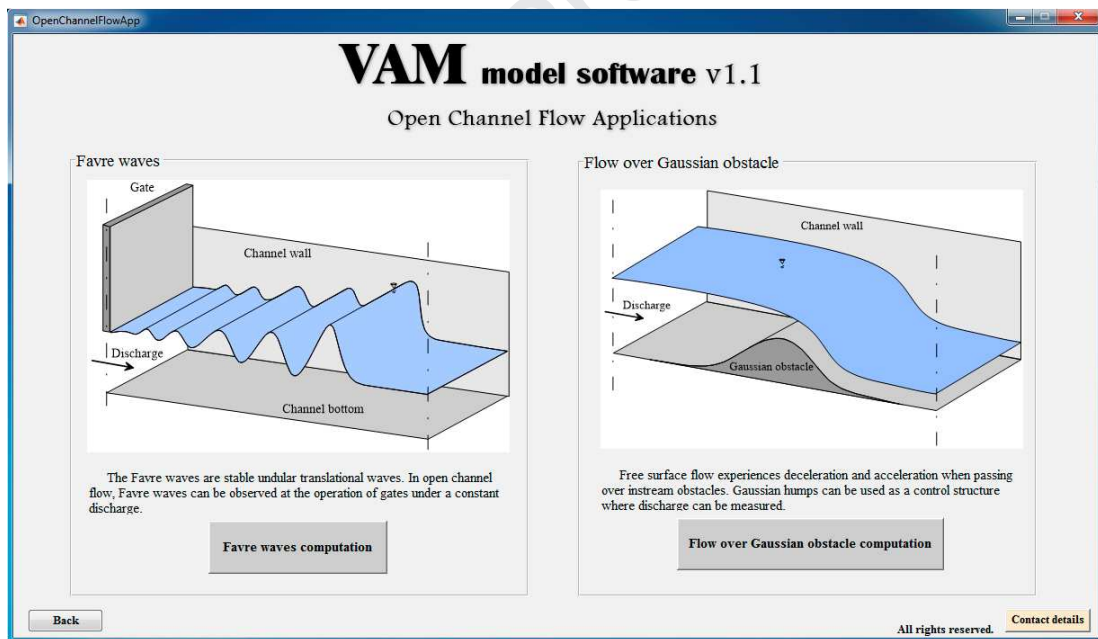
236 problems can be modelled: (i) solitary wave propagation and (ii) solitary wave  
237 propagation and run-up on inclined planes (Fig. 2). On the other hand, the open  
238 channel flow applications consist of: (i) Favre waves and (ii) flow over Gaussian  
239 obstacles (Fig. 3). A flow chat with the inputs and outputs of each application of the  
240 software platform is shown in Fig. 4. These applications are further detailed in  
241 Sections 3.1 and 3.2.



243 **Figure 1.** Initial window of the software platform.

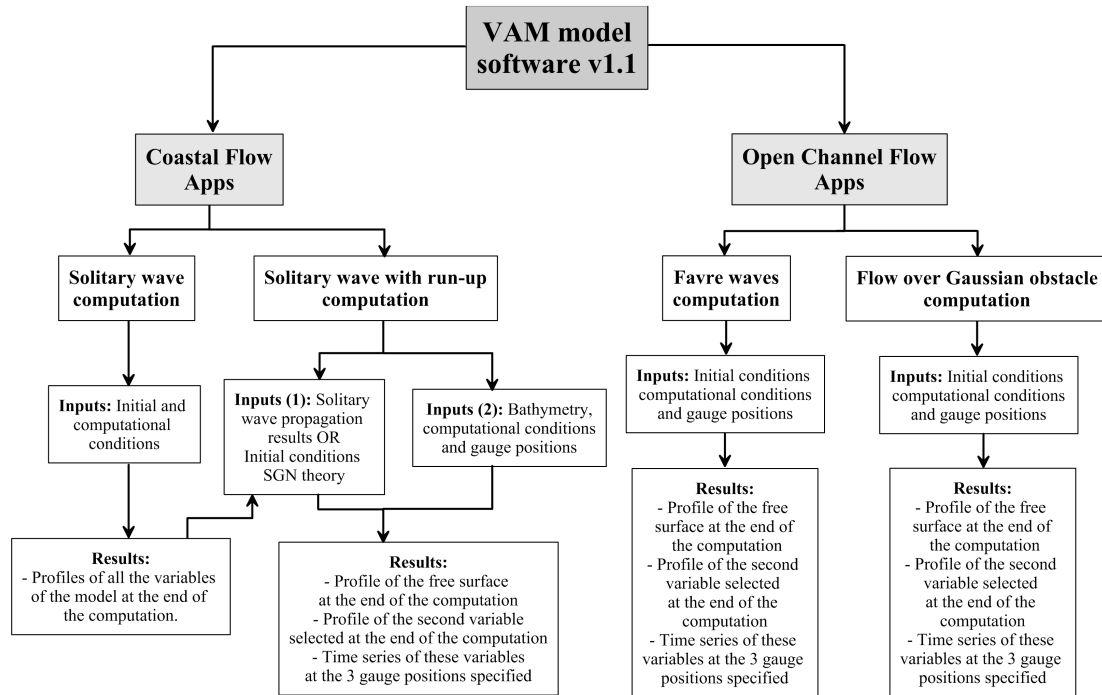


244

245 **Figure 2.** Coastal flow applications window.

246

247 **Figure 3.** Open channel flow applications window.



248

249 **Figure 4.** Flow chart with the input conditions and the results of each application.250 

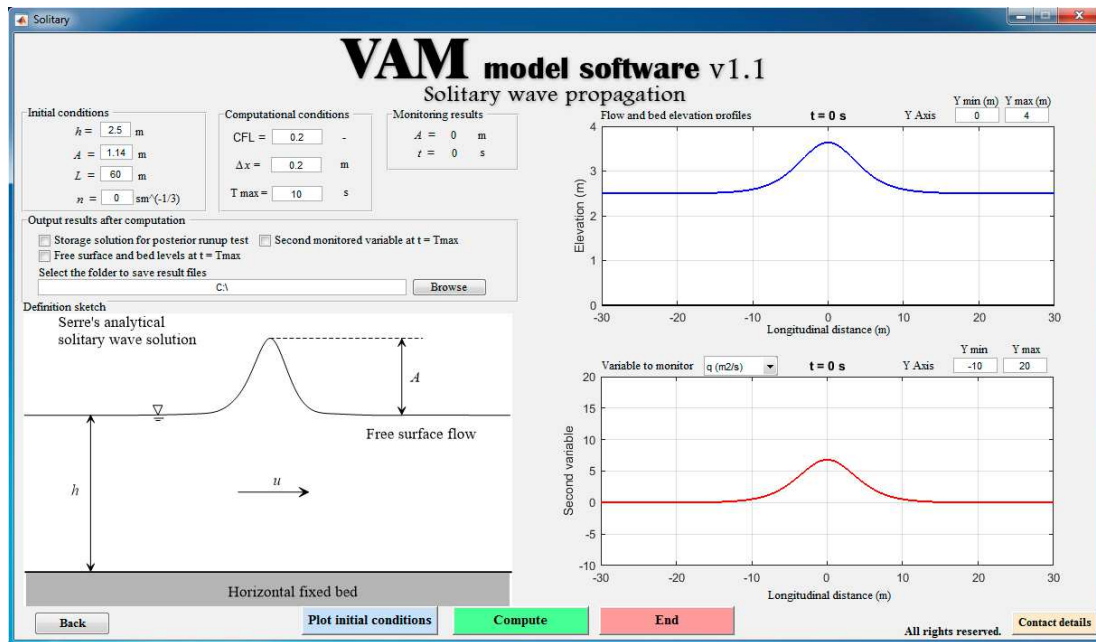
### 3.1 Coastal flows

251 

#### 3.1.1 Solitary wave propagation

252 The generation and propagation of a solitary wave is a challenging test for  
 253 numerical models, since a balance is required between non-linearity and dispersion  
 254 to produce a wave propagating with a permanent form. Since there is not known  
 255 analytical solution for the solitary wave solutions embedded into the VAM model, it  
 256 is determined numerically as follows. The analytical solution of the Serre-Green-  
 257 Naghdi equations (Castro-Orgaz and Hager, 2017) is considered as initial condition  
 258 for the application of the VAM model. When the model is run with this initial  
 259 condition, an unsteady motion involving a re-shaping of the wave profile occurs over  
 260 a few time steps. After routing a few seconds a wave of stable form results, which is  
 261 the solitary wave solution of the VAM model (Cantero-Chinchilla *et al.*, 2018a).

262 The sketch of the problem is shown in Fig. 5. The following input variables are  
 263 required: water depth ( $h$ ), wave amplitude ( $A$ ), length of the simulation reach ( $L$ ),  
 264 Courant-Friedrich-Lewy number (CFL), cell width ( $\Delta x$ ) and simulation time ( $T_{\max}$ ).  
 265 The user can select the second variable to monitor ( $q$ ,  $u_0$ ,  $u_1$ ,  $p_1$ ,  $p_2$  or  $w_2$ ) and the Y-  
 266 limits of the graphs, as well as plot the initial conditions, stop the simulation and save  
 267 the solitary wave solution to a file (Fig. 5).



268

269 **Figure 5.** Window for computation of solitary wave propagation.

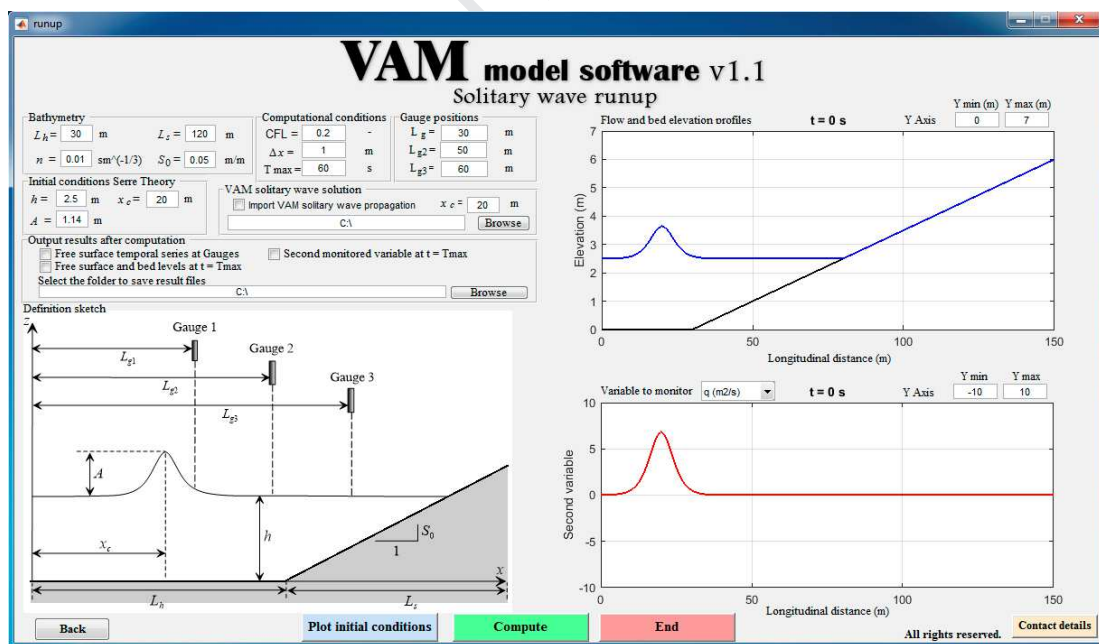
### 270 3.1.2 Solitary wave propagation and run-up on a plane beach

271 The second application of the software platform is the solitary wave propagation  
 272 and run-up on a constant slope beach. This case is relevant since wave breaking and  
 273 run-up flows are responsible of coastal erosion and flooding issues. The VAM model  
 274 reproduces the wave breaking without using any empirical breaking criteria to switch  
 275 off dispersive terms, as typically done in Boussinesq-type models (Brocchini 2013;  
 276 Castro-Orgaz and Hager 2017). The wave breaking mimicking ability of the VAM  
 277 model is due to the modeled velocity profile, which introduces differential advection



278 of momentum  $[\frac{1}{3}\partial(u_1^2h)/\partial x]$  and can, thus, counterbalance the effects of the  
 279 dynamic component of fluid pressure, accounted for in the VAM model by the  
 280 perturbation parameters  $p_1$  and  $p_2$ . The geometrical configuration of the problem is  
 281 shown in the lower-left part of Fig. 6. The initial conditions for this test can be  
 282 imported from the solution of the solitary wave propagation problem detailed in  
 283 Section 3.1.1. Alternatively, the analytical solution of the Serre-Green-Naghdi  
 284 equations can be considered as initial condition through the definition of water depth  
 285 ( $h$ ), wave amplitude ( $A$ ) and wave position ( $x_c$ ), as can be observed in Fig. 6.

286 Apart from these inputs, the following variables should be specified in the  
 287 interface: length of the horizontal stretch of the profile ( $L_h$ ), length of the sloped  
 288 stretch ( $L_s$ ), Manning roughness coefficient ( $n$ ), beach slope ( $S_0$ ), Courant-Friedrich-  
 289 Lewy number (CFL), cell width ( $\Delta x$ ), simulation time ( $T_{\max}$ ) and positions of the  
 290 three gauges indicated in the definition sketch ( $L_{g1}$ ,  $L_{g2}$  and  $L_{g3}$ ). The user can also  
 291 plot the initial conditions, start and finish the simulation, plot the evolution of the  
 292 free surface and the second variable previously selected, and save the results (Fig. 6).



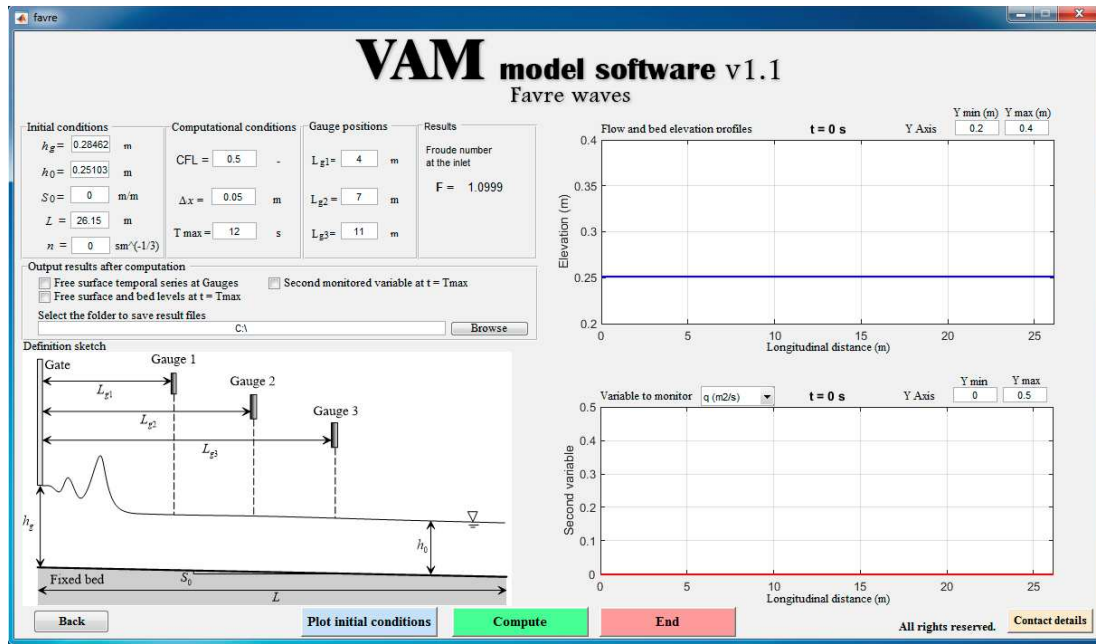
293

294 **Figure 6.** Window for computation of solitary wave run-up.

295 3.2 *Open channel flows*296 3.2.1 *Favre waves*

297 Favre waves are dispersive trains of free-surface undulations generated at the head  
298 of a bore or after a rapid opening/closing of a gate (Favre, 1935; Soares-Frazaio and  
299 Zech, 2002; Soares-Frazaio and Guinot, 2008). Hydrostatic models are dispersionless  
300 and, thus, unable to predict Favre waves. Therefore, Favre waves can only be  
301 modelled in a depth-integrated framework by shallow-water models accounting for  
302 non-hydrostatic pressures and, thus, dispersive effects (Soares-Frazaio and Zech,  
303 2002).

304 The first open channel flow application of the software platform allows simulating  
305 Favre waves generated by the opening of a sluice gate. As can be observed in Fig. 7,  
306 the input variables required to run this application are the distance from the bed to  
307 the gate ( $h_g$ ), downstream water depth ( $h_0$ ), bed slope ( $S_0$ ), length of the canal ( $L$ ),  
308 Manning roughness coefficient ( $n$ ), Courant-Friedrich-Lewy number (CFL), cell  
309 width ( $\Delta x$ ), maximum time of the simulation ( $T_{max}$ ) and positions of the gauges ( $Lg_1$ ,  
310  $Lg_2$ , and  $Lg_3$ ). The definition sketch of the lower-left part in Fig. 7 indicates the  
311 geometrical configuration of the test, including some of the aforementioned  
312 variables.



313

314 **Figure 7.** Window for computation of Favre waves.

315 The interface allows plotting the initial conditions as well as the evolution of the  
 316 free surface elevation and the second variable selected by the user during the  
 317 simulation period. The simulation is run by the user through the *Compute* button and  
 318 can be ended at any time by pressing the *End* button. The free surface temporal series  
 319 at the gauge positions as well as the spatial distribution of the free surface and the  
 320 second selected variable can be saved through the selection of the corresponding  
 321 items and the specification of the path to save the result files (Fig. 7).

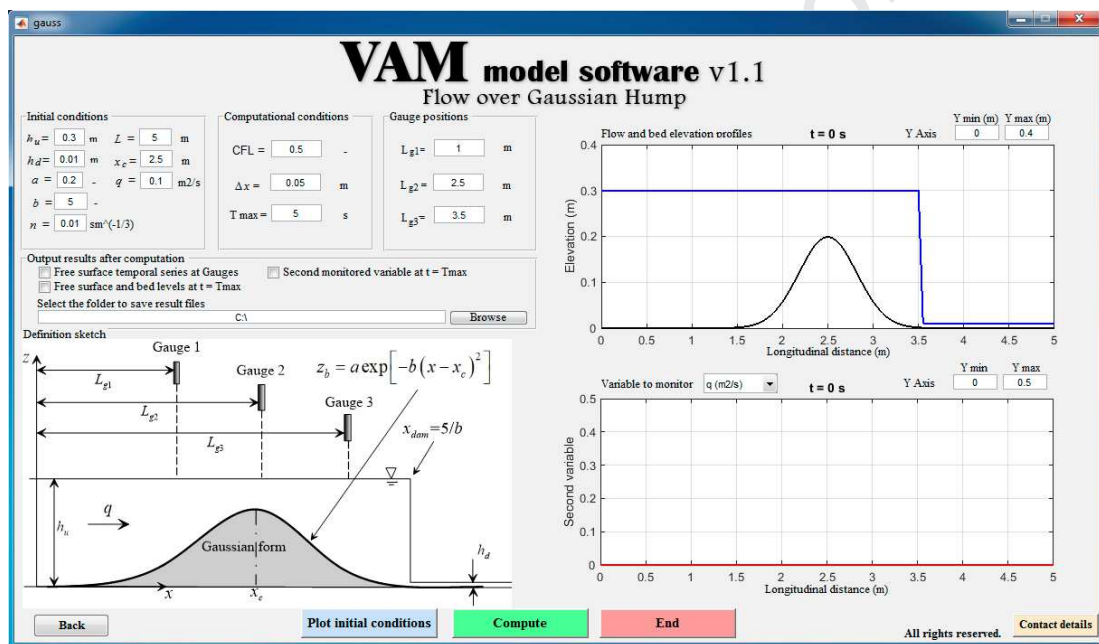
322 

### 3.2.2 Flow over a Gaussian obstacle

323 The last application allows reproducing the water waves evolving over a Gaussian  
 324 obstacle. The initial conditions are of dam-break type, with a significant variation in  
 325 the river depth downstream, as shown in the definition sketch of Fig. 8. This test  
 326 aims at mimicking the wave propagation over an obstacle, like those occurring over a  
 327 river bed mound when an undular bore propagates in a river. This type of water wave  
 328 problem was previously used by Nadiga et al. (1996) to test Boussinesq-type models,

329 given that it is challenging. The particular case of flow over obstacle with an initially  
 330 uniform distribution of water depths can be also simulated with the software  
 331 platform. For that, the two initial values of water depth specified by the user (denoted  
 332 by  $h_u$  and  $h_d$ , see Fig. 8) should be equal.

333 Apart from these two values, the following input variables should be specified:  
 334 constants for the mathematical definition of the Gaussian bed profile ( $a$ ,  $b$  and  $x_c$ ),  
 335 Manning roughness coefficient ( $n$ ), length of the canal ( $L$ ), river flow per unit width  
 336 ( $q$ ), Courant-Friedrich-Lewy number (CFL), cell width ( $\Delta x$ ), simulation time ( $T_{\max}$ )  
 337 and gauge positions ( $L_{g1}$ ,  $L_{g2}$  and  $L_{g3}$ ).



338

339 **Figure 8.** Window for computation of flows over Gaussian humps.

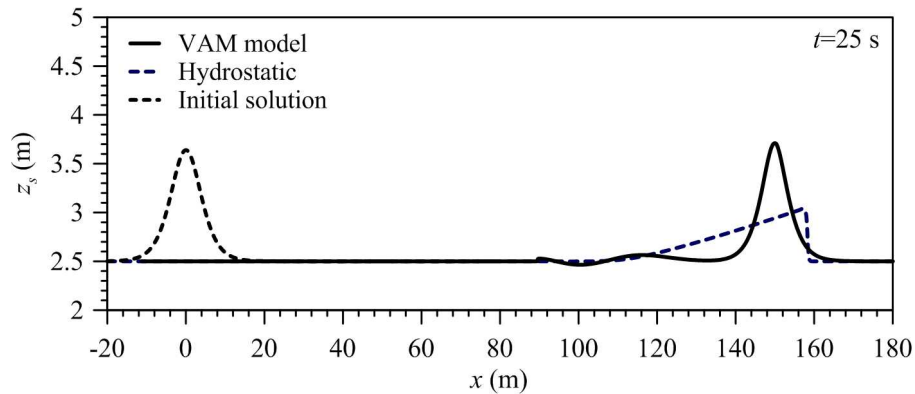
340 As in the rest of applications, the user can also depict the initial conditions through  
 341 the *Plot initial conditions* button, start the simulation by means of the *Compute*  
 342 button and end the computation by pressing the *End* button at any time. The  
 343 evolution of the free surface elevation and the second variable selected by the user  
 344 are depicted in the right part of the window, and the results can be also stored  
 345 through the selection of the items and the specification of the path (Fig. 8).

346 The computational time of the software platform depends on the computational  
347 conditions specified for each case, that is, on the Courant-Friedrich-Lewy number  
348 (CFL), the number of cells used, and the maximum simulation time ( $T_{max}$ ). The  
349 computational time is also dependent on the characteristics of the computer  
350 employed for the simulations.

#### 351 **4. Application examples of the software platform**

##### 352 *4.1 Solitary wave propagation*

353 This section reports an application of the *solitary wave propagation* module and  
354 compares the results with the hydrostatic version of the VAM model. The initial  
355 conditions settled in the software platform window (Fig. 5) were  $h = 2.5$  m,  $A = 1.14$   
356 m,  $L = 80$  m,  $n = 0 \text{ sm}^{-1/3}$ ,  $\text{CFL} = 0.1$  and  $T_{\max} = 10$  s. Fig. 9 shows the initial wave  
357 and the solitary wave solutions with the hydrostatic and non-hydrostatic versions of  
358 the VAM model. It is observed that, after 25 seconds, the wave propagated over the  
359 horizontal fixed bed is transformed into a shock by the hydrostatic computation,  
360 whereas the non-hydrostatic wave propagation solution obtained with the VAM  
361 software platform produces a solitary wave. The train of small ripples generated at  
362 the tail of the solitary wave is produced during the transformation of the initial  
363 solitary wave (analytical solution of the Serre-Green-Naghdi equations) into the  
364 solitary wave solution of the VAM equations. If a large computational time is  
365 prescribed, the ripples are splitted from the solitary wave tail, and the solitary wave  
366 continues its propagation isolated. This non-hydrostatic solution obtained by using  
367 the *solitary wave propagation* module can be imported to *the solitary wave with run-*  
368 *up* module, as it is detailed in Section 4.2.



369

370 **Figure 9.** Solitary wave propagation obtained with the hydrostatic (grey solid line)  
 371 and non-hydrostatic (black solid line) versions of the VAM model after 25 s of  
 372 routing. The initial solution is also shown (black dashed line).

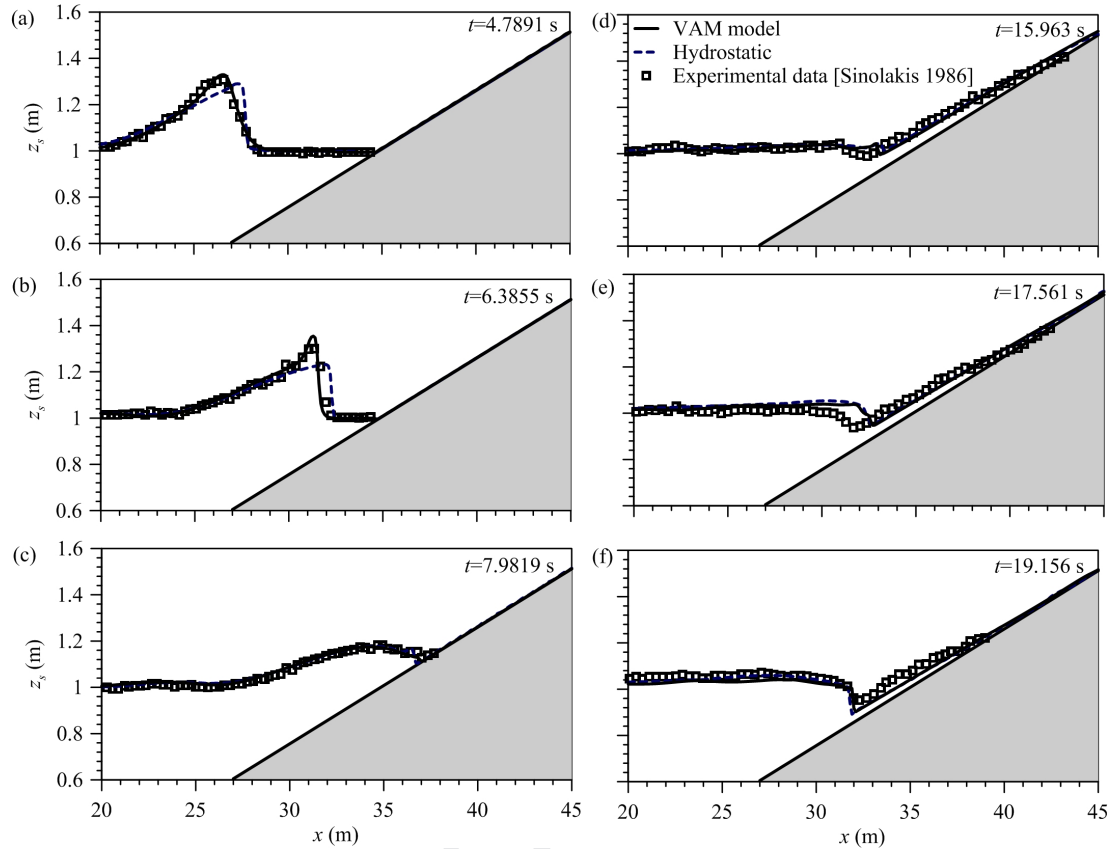
#### 373 4.2 Solitary wave run-up on plane beach

374 This section deals with the application of the present software platform to address  
 375 solitary wave run-up on a plane beach. The experimental data of solitary wave run-  
 376 up by Synolakis (1986) are selected to evaluate the accuracy of the corresponding  
 377 module of the software platform. The experiments were conducted in a wave tank of  
 378 the W. M. Keck Laboratories at the California Institute of Technology. The  
 379 dimensions of the wave tank were 37.73 m (length) x 0.61 m (height) x 0.39 m  
 380 (width). At a distance of 14.68 m from the wave generator, an inclined plane beach  
 381 of slope 1:19.85 was installed, see Fig. 10.

382 From the experimental dataset collected by Synolakis (1986), a solitary wave of  
 383  $A/h = 0.3$  with  $h = 1$  m was selected here. This wave amplitude will introduce  
 384 significant non-linear effects on the solitary wave propagation. First, the solitary  
 385 wave is propagated over a horizontal frictionless bed using the *solitary wave*  
 386 *propagation* module of the software platform. This module uses a moving-grid  
 387 subroutine that allows the propagation of the solitary wave in a short length, which is  
 388 usually defined to produce an unaltered non-hydrostatic variable  $p_1$  at the moving

389 boundaries. As the solitary wave amplitude is continuously monitored in the screen,  
390 the program can be stopped when the user identifies the desired amplitude and, then,  
391 the output result file is automatically generated. Later, the output of the *solitary wave*  
392 *propagation* module is used as initial condition in the *solitary wave propagation with*  
393 *run-up* module. This module was run six times to compare the results with the data  
394 by Synolakis (1986), setting the maximum computational time at  $t = 4.7891$  s (Fig.  
395 10a),  $t = 6.3855$  s (Fig. 10b),  $t = 7.9819$  s (Fig. 10c),  $t = 15.963$  s (Fig. 10d),  $t =$   
396  $17.561$  s (Fig. 10e), and  $t = 19.156$  s (Fig. 10f).

397 As observed by Synolakis (1986), during the propagation of the solitary wave  
398 along the plane beach, the wave deformed and the skewness increased, leading to an  
399 instability that induced the wave breaking. The software results are in good  
400 agreement with laboratory measurements, resulting, however, a minor phase shift at  
401 the hydraulic jump position formed during the drawdown phase at  $t = 17.561$  s (Fig.  
402 10e). Note that when a hydraulic jump is formed, the turbulence is significant and,  
403 thus, the simple turbulence module implemented in the VAM model may not be  
404 accurate enough. Further, the present vertically-averaged computational results  
405 cannot reproduce the inherent three-dimensional flow effects in hydraulics jumps.  
406 Therefore, despite the limitations, the computational results obtained with the VAM  
407 model are considered satisfactory in a depth-integrated sense. As in the case of the  
408 solitary wave propagation (Section 4.1), the agreements between the software  
409 platform results and the experimental data are significantly more satisfactory than  
410 those obtained with the hydrostatic version of the VAM model, especially before the  
411 inception of wave breaking (Fig. 10).



412

413 **Figure 10.** Solitary wave run-up obtained with the hydrostatic (grey dashed line) and  
 414 non-hydrostatic (black solid line) versions of the VAM model at different times.  
 415 Comparisons with experimental measurements by Synolakis (1986) (black squares).

#### 416 4.3 Favre waves

417 In this section, the software platform is tested to estimate three different data sets  
 418 of Favre waves generated by using different Froude numbers for the inlet conditions:  
 419  $F = 1.081$  (Fig. 11a),  $F = 1.104$  (Fig. 11b), and  $F = 1.192$  (Fig. 11c). Once the flow  
 420 depth at rest in the channel is set ( $h_0$ ), the inlet tailgate depth ( $h_g$ ) is calculated using  
 421 the expression by Soares-Frazão and Zech (2002), i.e.  $F = [h_g (h_g + h_0) / (2h_0^2)]^{1/2}$ .  
 422 The input variables for the software platform in the *Favre waves* module to generate  
 423 Fig. 11 are shown in Table 1.



424 The non-hydrostatic free-surface predictions of the VAM model obtained with the  
 425 software platform are compared with the experimental data by Soares-Frazão and  
 426 Zech (2002). These data were collected in an experimental flume of the laboratory of  
 427 the Civil Engineering Department at the University of Louvain. A gate located at a  
 428 10 m distance from the beginning of the flume separates two regions: the upstream  
 429 reservoir and the channel of length 26.15 m. The gate was suddenly opened and the  
 430 free surface measured at different downstream locations. Measurements at distances  
 431 from the gate of 13.15 m, 15.65 m and 13.5 m were used for the tests with  $F = 1.081$ ,  
 432  $F = 1.104$  and  $F = 1.192$ , respectively (Table 1).

433 To extend the comparison, the hydrostatic computation accomplished using the  
 434 main code of the VAM model, i.e. neglecting  $p_1$ ,  $p_2$ ,  $u_1$  and  $w_2$ , and thus equivalent to  
 435 solving Saint-Venant equations, is also depicted in Fig. 11. In addition, the results  
 436 obtained by applying the widely-used (hydrostatic) IBER model software for river  
 437 flows (Bladé et al., 2014; Cea and Bladé, 2015; Cea et al., 2016) are included to  
 438 allow the readers to compare the results between both hydraulics software platforms.  
 439 The results by IBER were generated using the input parameters indicated in Table 1.  
 440 The 2D geometry in the IBER model was set to keep invariant the transverse  
 441 direction (i.e. resembling 1D conditions). The comparisons between the time series  
 442 of the free surface obtained for the three Froude numbers are shown in Fig. 11.

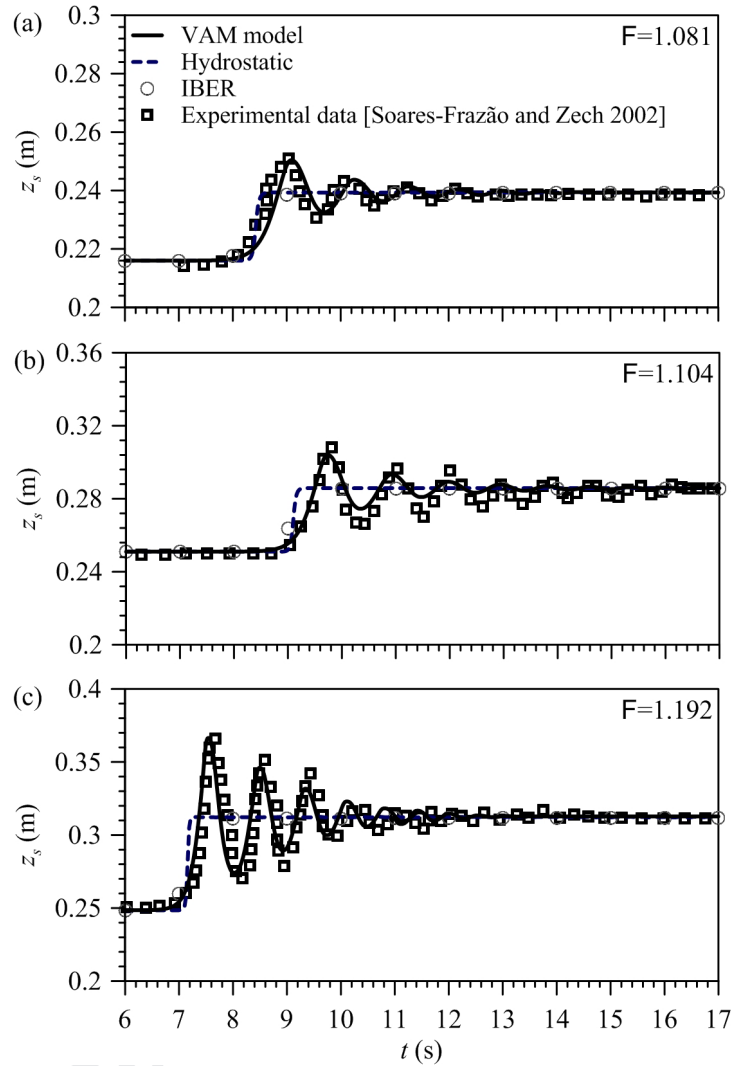
443 Table 1. Input variables for the *Favre* waves module of the software platform to  
 444 generate Fig. 11.

<b>F</b>	<b>1.081</b>	<b>1.104</b>	<b>1.192</b>
$h_g$ (m)	0.2394	0.286	0.3127
$h_0$ (m)	0.216	0.251	0.2485
$S_0$	0	0	0
$L$ (m)	40	40	40

$n$ (s/m <sup>1/3</sup> )	0.01	0.01	0.01
CFL	0.2	0.2	0.2
$\Delta x$ (m)	0.05	0.05	0.05
$T_{\max}$ (s)	17	17	17
$L_{g1}$ (m)	13.15	15.65	13.15

445

446 In Fig. 11 it is observed that the results of the VAM model software platform  
447 produce excellent approximations to the laboratory measurements by Soares-Frazão  
448 and Zech (2002) in the three cases for the whole time series. On the contrary, the  
449 agreements between experimental data and results obtained with both the hydrostatic  
450 version of the VAM model and the IBER model are significantly worse, due to the  
451 lack of non-hydrostatic modelling. Thus, Favre waves are not captured by hydrostatic  
452 simulations. This remarks the importance and necessity of using non-hydrostatic  
453 models, as the VAM model software platform presented in this work, to predict  
454 Favre wave during gate manoeuvres in open channel flows.



455

456 **Figure 11.** Depth-hydrographs of Favre waves obtained with the hydrostatic (grey  
 457 dashed line) and non-hydrostatic (black solid line) versions of the VAM model and  
 458 with the IBER model (grey dots). Comparisons with experimental measurements by  
 459 Soares-Frazão and Zech (2002) (black squares).

#### 460 4.4 Flows over a Gaussian obstacle

461 This section addresses the flow over Gaussian obstacle for three different values of  
 462 the discharge per unit width ( $q = 0.0561 \text{ m}^2/\text{s}$ ,  $q = 0.07703 \text{ m}^2/\text{s}$  and  $q=0.10144 \text{ m}^2/\text{s}$ )

463 experimentally tested by Sivakumaran (1981). The experiments were conducted in a  
 464 laboratory flume with dimensions of 9.15 m (length) x 0.75 m (height) x 0.44 m  
 465 (width). A curved bed with a maximum height of 0.2 m was built in the flume.  
 466 Several values of unit discharge were tested, and the free surface position and bed  
 467 pressure head were measured for each run at different positions along the flume.

468 The input variables for the software platform in the Flow over Gaussian obstacle  
 469 module are shown in Table 2. Fig. 12 depicts the comparison between the  
 470 experimental data of the free surface and bed pressure measured by Sivakumaran  
 471 (1981) and the results obtained with the three models mentioned in the previous  
 472 section: the non-hydrostatic version of the VAM model obtained with the GUI, the  
 473 hydrostatic version of the VAM model and the IBER model.

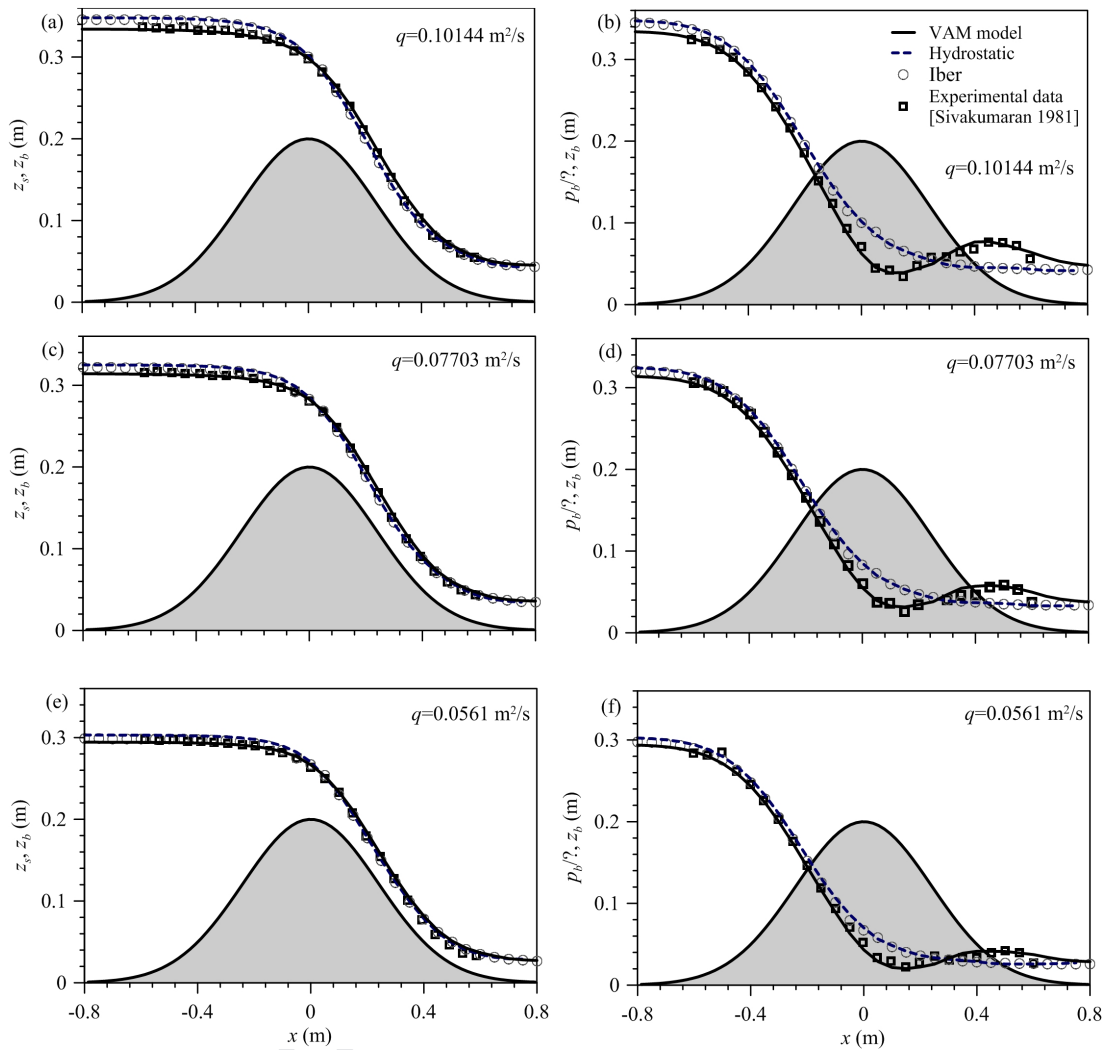
474 Table 2. Input variables for the *flow over Gaussian obstacle* module of the software  
 475 platform to generate Fig. 12.

$q$ (m <sup>2</sup> /s)	<b>0.0561</b>	<b>0.07703</b>	<b>0.10144</b>
$h_u$ (m)	0.35	0.35	0.35
$h_d$ (m)	0.02	0.02	0.02
$a$ (m)	0.2	0.2	0.2
$b$ (m)	8.6806	8.6806	8.6806
$L$ (m)	3.2	3.2	3.2
$x_c$ (m)	1.6	1.6	1.6
$n$ (s/m <sup>1/3</sup> )	0.01	0.01	0.01
CFL	0.5	0.5	0.5
$\Delta x$ (m)	0.05	0.05	0.05
$T_{\max}$ (s)	5	5	5

476

477 As depicted in Fig. 12, the non-hydrostatic results obtained with the VAM model  
478 software platform are in excellent agreement with the experimental free surface and  
479 bed pressure distributions, since the VAM model accounts for vertical flow effects.  
480 The improvements with respect to the simulations carried out with the hydrostatic  
481 VAM and IBER models are more evident on inspecting the bed pressure profiles  
482 results, which are markedly non-hydrostatic. The hydrostatic-based models consider  
483 that the bed pressure equals the water depth and, thus, are unable to reproduce the  
484 bed pressure profiles over the obstacle. Further, the higher free surface levels  
485 upstream of the obstacle obtained with the hydrostatic models result in an unrealistic  
486 prediction of the discharge coefficient of the structure (Castro-Orgaz and Hager,  
487 2017).

488 Curved-shaped obstacles are instream structures used worldwide for water  
489 discharge measurement in open channels and rivers. The measurement of the  
490 discharge is conducted reading the upstream depth and applying a weir flow equation  
491 with an accurate estimation of the discharge coefficient of the structure (Castro-  
492 Orgaz and Hager, 2017). It is clearly depicted in Fig. 12 that the VAM model  
493 produces such an accurate estimation of the discharge capacity of the obstacle. These  
494 results confirm the usefulness of the developed software platform to reproduce non-  
495 hydrostatic conditions in open channel applications.



496

497 **Figure 12.** Free surface and bed pressure distributions for the flow over a Gaussian  
 498 obstacle obtained with the hydrostatic (grey dashed line) and non-hydrostatic (black  
 499 solid line) versions of the VAM model and with the IBER model (grey dots) at  
 500 different times. Comparisons with experimental measurements by Sivakumaran  
 501 (1981) (black squares).

## 502 5. Conclusions

503 In this study, a user-friendly software platform is presented to model depth-  
 504 integrated non-hydrostatic flows through the Vertically-Averaged and Moment

505 (VAM) equations model. The platform comprises four applications: solitary wave  
506 propagation, solitary wave propagation with run-up, Favre waves and flow over  
507 Gaussian obstacles. After the description of the governing equations, numerical  
508 scheme and structure of the software platform, it is applied to solve several coastal  
509 and open channel flow problems.

510 First, the platform is used to reproduce solitary wave propagation over horizontal  
511 fixed bed (with  $A/h = 0.456$  and  $h = 2.5$  m), as well as to model the propagation and  
512 run-up of a solitary wave with  $A/h = 0.3$  and  $h = 1$  m over a plane beach. The results  
513 are compared with the hydrostatic version of the VAM model, thereby resulting that  
514 the non-hydrostatic solutions obtained with the software platform are significantly  
515 more realistic in both cases. In the case of solitary wave propagation and run-up, the  
516 software platform results are also compared to laboratory data collected by Synolakis  
517 (1986). The software outputs reproduce the experimental cross-shore distributions of  
518 the free surface at different times; including the solitary wave deformation before  
519 breaking, wave breaking and run-up.

520 Then, the software is used to model Favre waves for three different Froude  
521 numbers ( $F = 1.081$ ,  $F = 1.104$  and  $F = 1.192$ ) and the results are compared to the  
522 hydrostatic version of the VAM model, the user-friendly hydrostatic model IBER  
523 and experimental data collected by Soares-Frazão and Zech (2002). The agreements  
524 between free-surface time series obtained with the developed software and measured  
525 free surface values are excellent; whereas the results obtained with the hydrostatic  
526 version of the VAM model and the IBER model are significantly worse.

527 Finally, the flow over a Gaussian obstacle is simulated with the three  
528 aforementioned models for three unit discharge runs:  $q = 0.0561$  m<sup>2</sup>/s,  $q = 0.07703$   
529 m<sup>2</sup>/s and  $q = 0.10144$  m<sup>2</sup>/s. The computational results are compared with free surface  
530 and bed pressure distributions measured in a laboratory facility by Sivakumaran  
531 (1981). Again, the fits to the experimental measurements provided by the developed

532 VAM model software platform are better than those obtained with hydrostatic  
533 modelling approaches.

534 The implementation of a numerical solver for the VAM equations is time-  
535 consuming and not evident, and made most of the scientific community working in  
536 vertically-integrated non-hydrostatic flows to tend away from solving them.  
537 However, when the VAM model is properly structured and discretized, a useful  
538 MATLAB application was produced. It permits the VAM platform user to solve this  
539 complex system of equations (as compared to Saint Venant equations) in an easy and  
540 accurate way, thereby producing computational results which may be used to test  
541 other models, like those based on Boussinesq equations.

542 The results of this work remark the importance of the VAM model not only for  
543 modelling non-hydrostatic flows in open channel tests, but also for reproducing  
544 nearshore flows and coastal processes. This model can be used for a wide range of  
545 applications, such as the prediction of free surface profiles at the vicinity of bed  
546 forms and obstacles; the water discharge measurement in open channels and rivers,  
547 the estimation of wave run-up in coastal areas, which is the key variable to delimit  
548 the public domain; or the design of laboratory experiments in wave and hydraulic  
549 flumes, among others. The GUI presented in this paper can be helpful for students,  
550 professors and water managers, whereas the MATLAB routines of the VAM model  
551 provided as supplementary material are intended for engineers and scientists doing  
552 research in this area.

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## **HIGHLIGHTS**

- A software platform to compute depth-integrated non-hydrostatic flows is presented.
- The software is based on the Vertically-Averaged and Moment equations model.
- The software is applied to solve several open channel and coastal flow tests.
- Excellent agreements are obtained between the software results and experimental data.

**Declaration of interests**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

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