Multifractal detrended fluctuation analysis of sheep livestock prices in origin

P. Pavón-Domínguez [a,*], S. Serrano [b], F.J. Jiménez-Hornero [a], J.E. Jiménez-Hornero [c], E. Gutiérrez de Ravé [a], A.B. Ariza-Villaverde [a]

[a] Department of Graphics Engineering and Geomatics, Gregor Mendel Building (3rd floor), Campus Rabanales, University of Córdoba, 14071 – Córdoba, Spain

[b] Department of Food Hygiene and Technology, Darwin Building, Campus Rabanales, University of Córdoba, 14071 – Córdoba, Spain

[c] Department of Computing and Numerical Analysis, Leonardo da Vinci Bulding, Campus Rabanales, University of Córdoba, 14071 – Córdoba, Spain

A B S T R A C T

The multifractal detrended fluctuation analysis (MF-DFA) is used to verify whether or not the returns of time series of prices paid to farmers in original markets can be described by the multifractal approach. By way of example, 5 weekly time series of prices of different breeds, slaughter weight and market differentiation from 2000 to 2012 are analyzed. Results obtained from the multifractal parameters and multifractal spectra show that the price series of livestock products are of a multifractal nature. The Hurst exponent shows that these time series are stationary signals, some of which exhibit long memory (Merino milk-fed in Seville and Segureña paschal in Jaen), short memory (Merino paschal in Cordoba and Segureña milk-fed in Jaen) or even are close to an uncorrelated signals (Merino paschal in Seville). MF-DFA is able to discern the different underlying dynamics that play an important role in different types of sheep livestock markets, such as degree and source of multifractality. In addition, the main source of multifractality of these time series is due to the broadness of the probability function, instead of the long-range correlation properties between small and large fluctuations, which play a clearly secondary role.

1. Introduction

The application of certain physical methods to economic and financial time series has been widely used in order to detect characteristics that cannot be deduced from classical economic theories [1]. Multifractal Detrended Fluctuation Analysis (MF-DFA) [2] has become a widely applied technique for analyzing price fluctuations. This method, which links

* Corresponding author. Tel.: +34 957212126.
E-mail address: g22padop@uco.es (P. Pavón-Domínguez).

the Detrended Fluctuation Analysis (DFA) [3] to the fractal theory [4], provides correct results for time series that are affected by trends and yields a more in-depth knowledge of certain of their features. The DFA has been used to determine of (mono-) fractal scaling behaviors in different non-stationary time series [5–7]. However, it is unable to capture complex dynamics in a time series or to characterize their scaling properties when the processes are governed by more than one (theoretically infinite) scaling exponent [8]. As seen in several studies, MF-DFA allows for a reliable multifractal characterization of time series from different fields, such as seismic events [9,10], wind speed [11], vehicle traffic speed [12], sunspot observations [13], streamflow rivers [14], magnetic fields [15], as well as applications in Medicine [16]. In recent years, several authors have suggested that MF-DFA is also a useful tool for analyzing time series of prices [17], one of its main advantages being its ability to accurately quantify correlation properties masked by polynomial trends. In addition, it has been found that MF-DFA is able to discern the different underlying dynamics in different types of markets [18].

Fluctuations in certain speculative prices of goods and scaling properties through detrended fluctuation analysis methods have been studied in detail. For example, the time scale property on the process of returns in the fluctuations of cotton prices was observed in Ref. [19]. Economic indices [20], stock exchange prices [21] and stock markets [22–26] have also been studied. Concerning agricultural markets, the use of MF-DFA has focused on world prices, commodities and future markets (see e.g. Refs. [27–29]), although few studies concentrate on prices of agricultural products in national markets, as with Korea [30]. Therefore, no attention has been paid to local markets and to prices received by farmers. It should be born in mind that agricultural prices directly affect the income paid to farmers, this being one of the issues most relevant to the European Union's agricultural policy [31]. For this reason, sheep livestock regional markets have been selected, in which price fluctuations are mainly due to the inherent characteristics of markets themselves.

Despite the fact that slaughter prices in livestock markets are determined by consumer preferences, slaughter age and seasonality, farmers cannot usually forecast price developments. This is because the pricing of agricultural products responds to different variables and interests that make prediction and change somewhat uncertain over the months. Hence, farmers face uncertainty about the economic consequences of their actions owing to difficulties in predicting prices [32]. The uncertainty of the agents is attributable to the excessive price fluctuations of agricultural and livestock products, which are undesirable because they could affect inflation and social stability in the most extreme cases [33]. Therefore, the main goal of this paper was to verify the existence of multifractal characteristics in time series of agricultural-livestock prices at a regional level, using livestock sheep markets in Andalusia (southern region of Spain) as an example. Furthermore, because two different types of multifractality in time series can be distinguished [2]: (i) Multifractality due to a broad probability density function for the values of the time series and (ii) Multifractality due to different long-range (time-) correlations of small and large fluctuations, this circumstance is also analyzed in this study. This work has been organized in different sections where price time series, together with the multifractal detrended fluctuation analysis, are described. The relation between MF-DFA and the standard multifractal analysis can also be found herein. The multifractal nature of the time series prices is explored and the results of MF-DFA are provided, together with the calculation of the multifractal spectra.

2. Methods

2.1. Price data time series

Sheep farming is widespread around the world, and a wide range of products is obtained from it (wool, meat and milk). Because of the adaptability of these animals to marginal sites and the possibility of making better use of natural resources, sheep livestock have traditionally been located in less-favored areas [34]. In Andalusia, sheep farming is related to extensive systems and is more meat production-oriented [35]. Data obtained from the database of annual reports of the Consejería de Agricultura y Pesca (regional government of Andalusia, Spain) have been used. We analyzed 5 weekly price data time series for nearly thirteen years, from January 2000 to April 2012. The sheep time series of prices paid on the farm were: Merino milk-fed (Seville), Merino paschal (Cordoba), Merino paschal (Seville), Segureña milk-fed (Jaen) and Segureña paschal (Jaen). The market where the price was recorded is noted in brackets.

Merino is the most widespread breed around the world, which can be attributed to its traditional specialization in wool production, although the selection of this breed is currently focused on meat production. Segureña is a rustic-type breed from the Mediterranean region, quite hardy and well adapted to the specific conditions of low rainfall and scant pastures. Segureña meat shows evident market differentiation, as it is more valued by consumers owing to its quality, while the Merino breed lacks clear market differentiation and its meat can be considered as being more generic. In addition, Segureña lamb possesses an official quality label linked to its geographical origin [36]. Moreover, the milk-fed lamb is breast-fed and slaughtered at a very early age, between 20 and 40 days, with a weight of around 12 kg. Its meat is white to pink in color and has a low fat content, while the paschal lamb is usually slaughtered at around 120 days of age, having reached a slaughter weight of over 23 kg. The paschal meat is dark pink in color and is greasier than the milk-fed meat. Fig. 1 depicts these five time series which, as with other commodities, show an oscillatory behavior, but this is not enough to establish the existence of long-term correlations and the presence of scaling behavior. Thus, an in-depth study with a suitable tool, such as the MF-DFA, can clarify and properly characterize the underlying features in these series. In Spain, lamb has traditionally been a food with a strong familiar and festive character, this being the reason that meat consumption as well as its prices increases in December [37], as can be seen in Fig. 2, in which weekly prices for the analyzed year are averaged. Sheep prices exhibit a seasonal pattern, which is noted for every breed and slaughter weight.

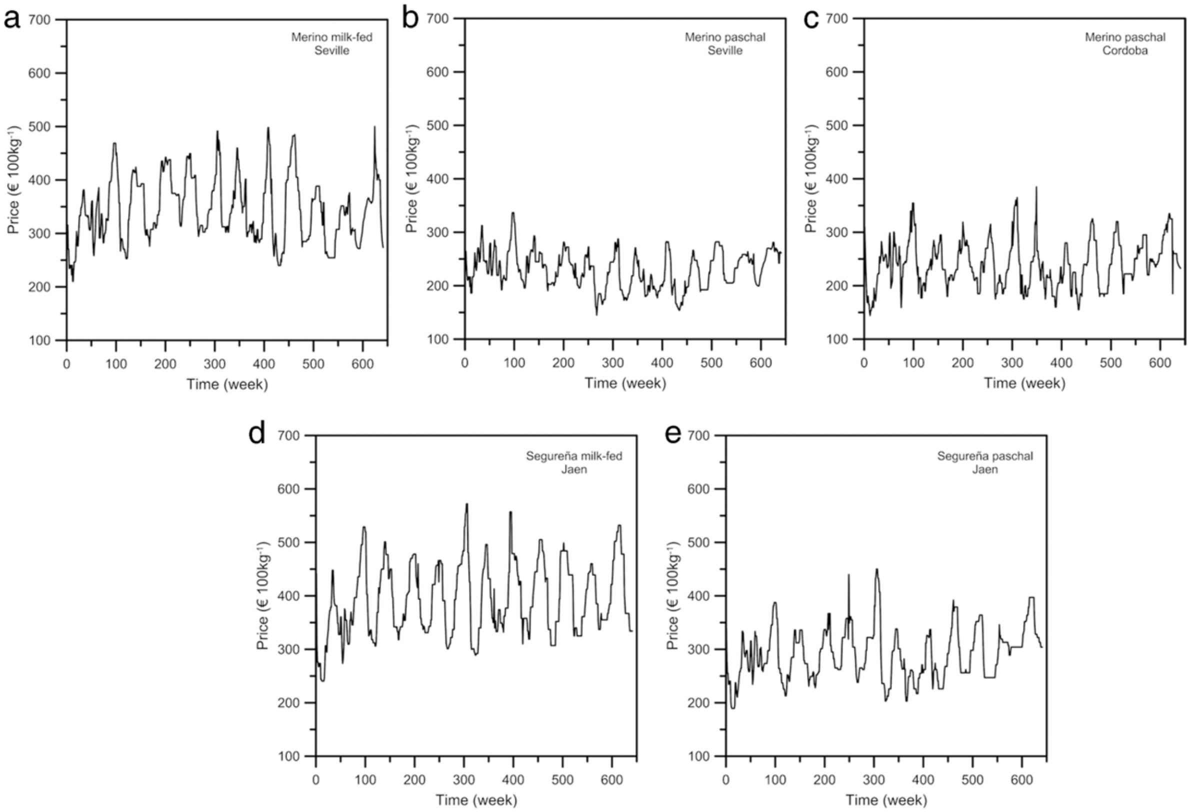

Fig. 1. Time series of (a) Merino milk-fed, Seville; (b) Merino paschal, Seville; (c) Merino paschal, Cordoba; (d) Segureña milk-fed, Jaen; and (e) Segureña paschal, Jaen; from 2000 to 2012.

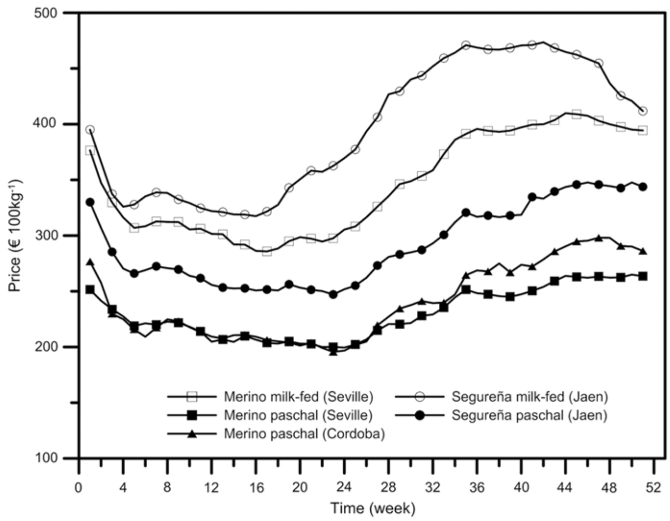

Fig. 2. Weekly average time series for the five studied sheep prices, from 2010 to 2012.

In all five time series, the existence of a clear seasonal behavior is evinced, which is convenient to remove before the multifractal procedure is applied. In order to confirm this periodicity, autocorrelation functions have been calculated for each time series. Autocorrelation functions (Fig. 3a–e) show a sinusoidal trend which slowly declines. The existence of a seasonal pattern is confirmed though the cyclic behavior of the latter, which is repeated at every time lag of around 50 data (weeks) that correspond to 1 year. This seasonal effect has been removed by subtracting a seasonal coefficient from each price data. The seasonal coefficients represent the average deviation from the overall mean of all of the values for a specific week of the year. When weekly prices are below the overall mean, a negative coefficient is obtained, while those

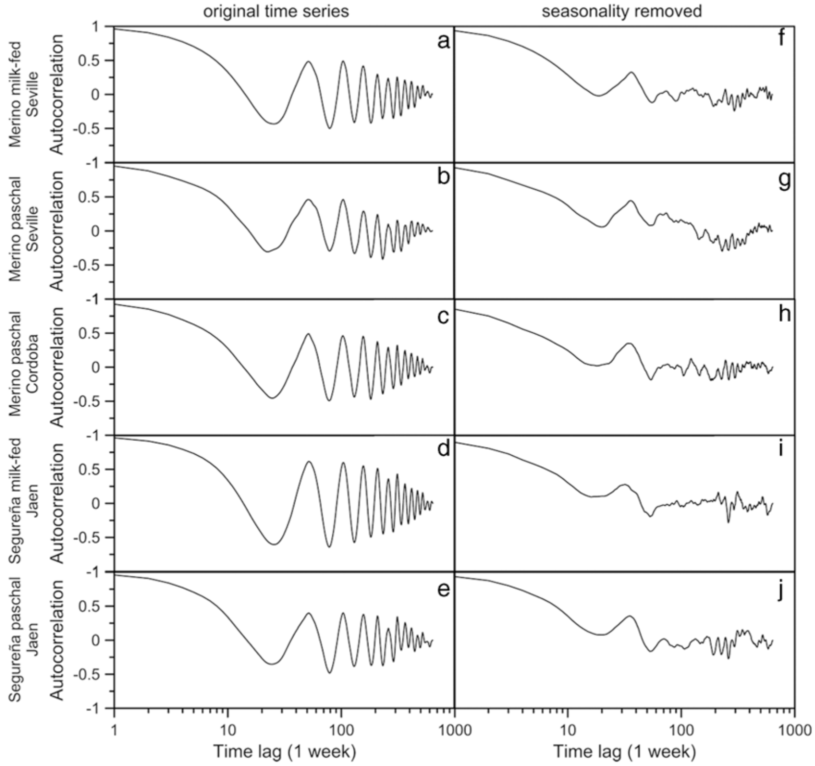

Fig. 3. Autocorrelation functions for the original and the seasonally adjusted time series.

months with higher prices have positive seasonal coefficients. This procedure was performed for each price time series. As a consequence of the removal of the seasonal component in the five price time series, the autocorrelation functions do not exhibit the typical shape of time series governed by a periodic component (see Fig. 3f–j). Thus, the seasonality was removed in the residual time series defined above. Finally, the return time series, $r(t)$, were calculated from the residual time series as follows:

$$r(t) = \ln P(t+1) - \ln P(t) \tag{1}$$

$P(t)$ is defined as the price at the week t.

2.2. Description of the MF-DFA

We studied the MFDFA to characterize the MF properties of the returns for sheep price time series. The MF-DFA procedure was proposed by Kantelhardt et al. [2] for the study of non-stationary time series which are affected by trends or cannot be normalized. This method, which aims to identify the scaling behavior of the fluctuations of the time series for different qth order moments, is based on the detrended fluctuation analysis [3]. In fact, the three first steps are essentially identical to the traditional DFA. The five steps run as follows [2]:

Step 1: Determine the "profile"

Let us suppose that x_k is a time series of length N, and that this series is of compact support ($x_k = 0$ for an insignificant fraction of the values only), it is feasible to determine the "trajectory" or "profile" $Y(i)$, which is given by the accumulation of the signal, as follows:

$$Y(i) = \sum_{k=1}^{i} [x_k - \bar{x}], \quad i = 1, \ldots, N. \tag{2}$$

The profile is determined by subtracting for each record x_k the average value $\bar{x}$ of the time series. Subtraction of the mean is not compulsory, since it would be eliminated by the later detrending in the third step.

Step 2: Divide the profile $Y(i)$ into $N_s = \text{int}(N/s)$ non-overlapping segments of equal length s. Since the length N of the series is often not a multiple of the considered time scale s, a short part at the end of the profile may remain. In order to retain this part of the series, the same procedure is carried out again starting from the opposite end. Thereby, $2N_s$ segments are obtained for each s value.

Step 3: For each segment v the estimated values $y_v(i)$ are calculated by a polynomial fit by using the least-squares method. For estimating $y_v(i)$, several fits may be used such as linear, quadratic, cubic, or higher order polynomials (conventionally called DFA1, DFA2, DFA3, etc.). Since the detrending of the time series is done by the subtraction of the polynomial fits from the profile, different order DFAs differs in their capacity to eliminate trends in the series. In MF-DFA*m* (*m*th order (MF-) DFA) trends of order m in the original series are eliminated. Later, considering the previous fits, the possible trends are removed by subtracting $y_v(i)$ from $y(i)$ for each segment v. Then the variance for each s value is calculated as the fluctuations of the time series into each v segment as follows:

$$F^2(s, v) = \frac{1}{s}\sum_{i=1}^{s}\{Y[(v-1)s+i] - y_v(i)\}^2 \tag{3}$$

for each segment v, $v = 1, \ldots, 2N_s$ and for $v = N_s + 1, \ldots, 2N_s$ in equation:

$$F^2(s, v) = \frac{1}{s}\sum_{i=1}^{s}\{Y[N-(v-N_s)s+i] - y_v(i)\}^2 . \tag{4}$$

Step 4: Generally, we are interested in how the generalized q dependent fluctuation functions $F_q(s)$ depends on the time scale s for different values of q. The fluctuation function of qth order is computed by averaging over all the fluctuations of the segments, defined as:

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}\left[F^2(s, v)\right]^{q/2}\right\}^{1/q} . \tag{5}$$

In general, the index variable q can take any real value except zero due to the diverging exponent (see Eq. (7)). For $q = 2$, the standard DFA procedure is retrieved. Hence, repeating steps 2–4 for several time scales s, $F_q(s)$ will increase with increasing s. For $q = 0$ the fluctuation function needs to be estimated by applying a logarithmic averaging procedure:

$$F_{q=0}(s) = e^{\left[\frac{1}{4N_s}\sum_{v=1}^{2N_s}\ln[F^2(v,s)]\right]}. \tag{6}$$

Step 5: Finally, the scaling behavior of the fluctuation functions is determined by analyzing log–log plots of $F_q(s)$ versus s for each value of q. If the series x_k is long-range power-law correlated, $F_q(s)$ increases for large values of s as a power-law, as follows:

$$F_q(s) \sim s^{h(q)}. \tag{7}$$

As a consequence, the exponents $h(q)$ can be estimated by determining the slopes of log–log plots $F_q(s)$ versus s for each q value. Only if small and large fluctuations scale differently, will there be a significant dependence of $h(q)$ on q. When the time series exhibits a multifractal nature, the exponent $h(q)$ and q are related in a decreasing sense due to small and large fluctuations scaling differently. Negative q values describe the scaling behavior of intervals with small fluctuations, whereas those of positive q values characterize the scaling behaviors of intervals with large fluctuations. When the contribution of the small fluctuations are comparable to the contribution of the large fluctuations, the time series is considered as a monofractal signal, and thus $h(q)$ is independent of q. This fact is due to the scaling behavior of the variance $F^2(s, v)$ being identical for all segments v, and the averaging procedure in Eq. (5) will just give this identical scaling behavior for all values of q. To determine the range of s values that will be considered for calculating the fluctuation function it is necessary to take account that systematic deviations from the scaling behavior in Eq. (7) occur for very small scales, usually for s values smaller than 10. In addition, for very large scales, the fluctuation function becomes statistically unreliable because of the amount of intervals considered is very scarce. In agreement with this fact, s values larger than $N/4$ have usually been ruled out in previous works.

2.3. Relation to standard multifractal analysis

As was also demonstrated by Kantelhardt et al. [2], for stationary, normalized series defining a measure with compact support, the multifractal scaling exponents $h(q)$ defined in Eq. (7) are directly related, as shown below, to the scaling exponents $\tau(q)$ defined by the standard partition function-based multifractal formalism.

Suppose that the series X_k of length N is a stationary, positive and normalized sequence. Then the detrending procedure in step 3 of the MF-DFA method is not required, since no trend has to be eliminated. Thus, the DFA can be replaced by the standard fluctuation analysis (FA), which is identical to the DFA except for a simplified definition of the variance for each segment v, $v = 1, \ldots, N_s$, in step 3 (see Eq. (3)).

$$F^2_{\mathrm{FA}}(v, s) = [Y(vs) - Y((v-1)s)]^2. \tag{8}$$

Table 1
Results obtained from the statistical analysis of sheep price time series, from 2000 to 2012.

Sheep	Market	Mean (€ 100 kg^{-1})	Minimum (€ 100 kg^{-1})	Maximum (€ 100 kg^{-1})	Standard deviation (€ 100 kg^{-1})	Skewness	Kurtosis
Merino milk-fed	Seville	345,02	210,35	500,00	60,684	0.330	−0.603
Merino paschal	Seville	229,88	145,00	336,57	34,551	0.128	−0.386
Merino paschal	Cordoba	240,41	144,24	385,00	43,666	0.370	−0.223
Segureña milk-fed	Jaen	394,22	240,40	572,00	68,571	0.239	−0.725
Segureña paschal	Jaen	291,20	189,32	450,00	49,492	0.497	0.032

Inserting this simplified definition into Eq. (5) and using Eq. (7), we obtain

$$\left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}|Y(vs)-Y((v-1)s)|^q\right\}^{\frac{1}{q}} \sim s^{h(q)}. \tag{9}$$

For simplicity, it can be assumed that the length N of the series is an integer multiple of the scale s, obtaining $N_s = N/s$ and therefore

$$\sum_{v=1}^{N/s}|Y(vs)-Y((v-1)s)|^q \sim s^{qh(q)-1}. \tag{10}$$

This already corresponds to the multifractal formalism. In fact, a hierarchy of exponents H_q similar to our $h(q)$ has been introduced based on Eq. (10) in Ref. [38]. In order to also relate the MF-DFA to the standard textbook box counting formalism [38], we have employed the definition of the profile in Eq. (2). It is obvious that the term $Y(vs) - Y((v-1)s)$ in Eq. (10) is identical to the sum of the numbers X_k within each segment v of size s. This sum is known as the box probability $p_s(v)$ in the standard multifractal formalism for normalized series x_k,

$$p_s(v) = \sum_{k=(v-1)s+1}^{vs} x_k = Y(vs) - Y((v-1)s). \tag{11}$$

The scaling exponent $\tau(q)$ is usually defined via the partition function $Z_q(s)$,

$$z_q(s) = \sum_{v=1}^{N/s}|p_s(v)|^q \sim s^{\tau(q)} \tag{12}$$

where q is a real parameter as in the MF-DFA above. Sometimes, $\tau(q)$ is defined with an opposite sign (see, e.g. Ref. [38]). Using Eq. (11) it is seen that Eq. (12) is identical to Eq. (10), and, analytically, the relationship between the two sets of multifractal scaling exponents is obtained,

$$\tau(q) = qh(q) - 1. \tag{13}$$

Thus, it is shown that $h(q)$ defined in Eq. (7) for the MF-DFA is directly related to the classical multifractal scaling exponent $\tau(q)$. Another way to characterize a multifractal series is the singularity spectrum $f(\alpha)$, that is related to $\tau(q)$ via a Legendre transform [38].

$$\alpha = \frac{d\tau(q)}{dq} \quad \text{and} \quad f(\alpha) = q\alpha - \tau(q). \tag{14}$$

3. Results

Table 1 illustrates some of the main descriptive statistics. It is apparent from this table that regarding slaughter age, the highest prices occur for milk-fed lambs, while those of Segureña-type breed reach higher prices than those of Merino. These facts are due to the fact that Spanish consumers have traditionally preferred milk-fed lambs over paschal lambs [39]. Furthermore, the Segureña breed has a differentiated quality feature. Moreover, price oscillations are more marked for milk-feds, taking into account the above-mentioned increased demand. In general, frequency distributions are slightly displaced to the left (Skewness), whereas the Kurtosis coefficient exhibits a shape that is similar to a slightly flattened normal distribution.

MF-DFA is carried out over the returns derived from the five time series of sheep prices. The log–log plots of the fluctuation functions $F_q(s)$ versus s for the time series of price records were obtained by employing linear fits to remove trends in the series. For all the studied cases, the corresponding log–log plots of the fluctuation functions are plotted in Fig. 4 using q moments ranging from −5.5 to 5.5 with 0.5 intervals and the scale s ranging from 10 to $N/4$, N being the length of the time series. In general, for smaller time scales, negative q moments add chaotic features in the fluctuation functions and, consequently, these values might be removed. Moreover, several works have confirmed that the correlations of the

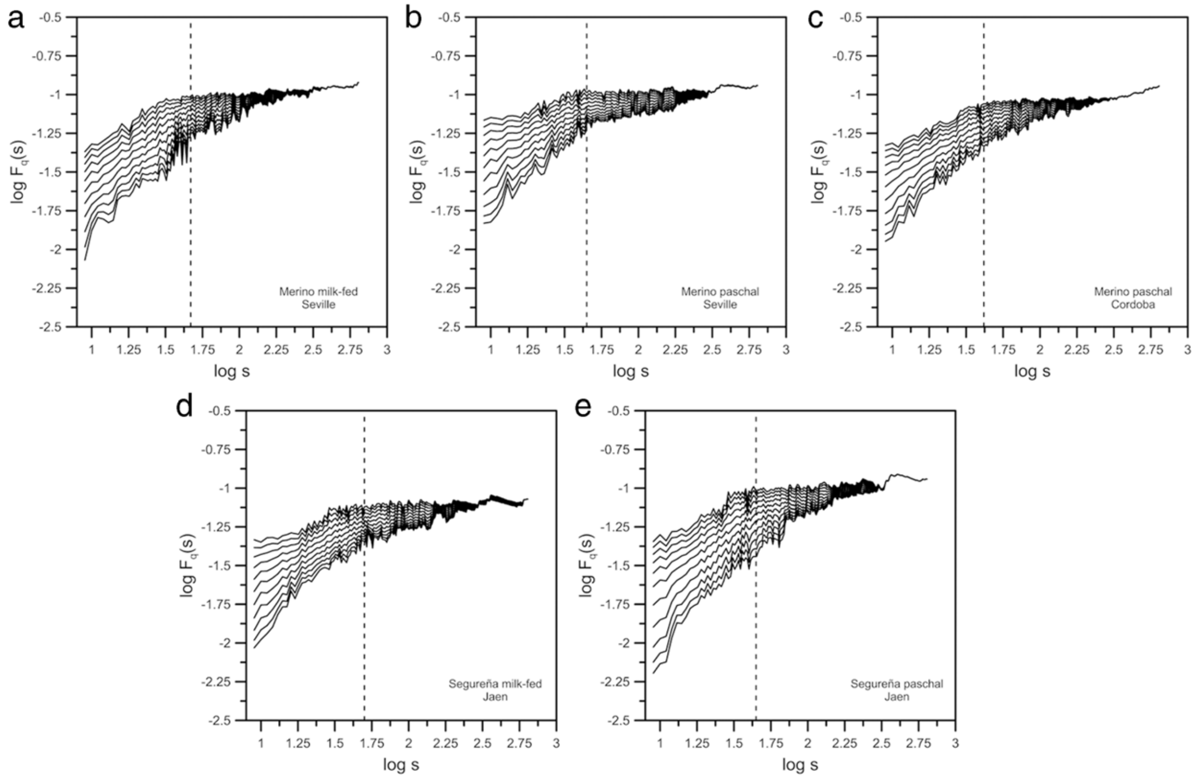

Fig. 4. Multifractal fluctuation function $F_q(s)$ obtained from MF-DFA1 for the time series (a) Merino milk-fed, Seville; (b) Merino paschal, Seville; (c) Merino paschal, Cordoba; (d) Segureña milk-fed, Jaen; and (e) Segureña paschal, Jaen. Dashed line represents the s value for the crossover.

time series do not often follow the same scaling law for all time scales s (see e.g. Refs. [12,13]). In these cases, one or more crossovers (s_x) between different scaling regimes are observed in the fluctuation functions $F_q(s)$. As can be seen from Fig. 4, there is at least one crossover time scale in each of the time series analyzed, which is represented with a dashed line. Thus, in this study, the maximum values ($s_{\max}$) for the fits are determined by the crossover time scale. These crossovers do not depend on the special values of q and are different for each time series considered. Despite the fact that seasonality had been removed before applying the multifractal procedure, $s_{\max}$ values range between 42 and 51 weeks, being close to the annual periodicity. For each time series, $h(q)$ is estimated by the slope of the linear fits between $s_{\min}$ and $s_{\max} = s_x$ for each log–log plot of $F_q(s)$ versus s for each q moment. Hence, $s_{\min}$ and $s_{\max}$ values have been established for each case for the log–log plots of $F_q(s)$ versus s, being fitted to an adequate straight line for each q moment. Coefficients of determination are higher than 0.90 for each studied case. These time scale intervals represent the range in which multifractal properties are maintained.

As can be noted from Fig. 5, $h(q)$ are decreasing functions which exhibit a strong dependence on q, which suggest that these time series of sheep prices are characterized by a multifractal behavior. The exponent $h(q)$ describes the scaling behavior of the qth order fluctuation function. For positive values of q, $h(q)$ describes the scaling behavior of intervals with large fluctuations, while for negative values of q, $h(q)$ describes the scaling behavior of segments with small fluctuations. In order to evaluate and compare the degree of multifractality of the studied time series, the range of the $h(q)$ is calculated. The larger the range, the greater the variability in the distribution of high and low fluctuations is. A greater degree of multifractality is related to more violent price fluctuations [40]. Table 2 shows that the lowest variability in price fluctuations is found for milk-fed in Seville (0.353). Perhaps this coincides with better distribution channels existing in a large market as Seville. By contrast, highest price fluctuations occur in paschal lamb markets in Cordoba, in which the highest $\Delta h(q)$ values are reached (0.579), Cordoba being a minor local market with poor marketing possibilities. $\Delta h(q)$ shows similar values for the other markets (ranging from 0.436 to 0.465).

Based on the known relationship between the Hurst exponent and $h(q)$ for $q = 2$ for small scales, it is possible to directly compute the Hurst exponents for the sheep prices time series. While for stationary time series, $h(2)$ is identical to the well-known Hurst exponent H [2], for non-stationary signals the relation between the exponent $h(2)$ for small scales and the Hurst exponent H is $H = h(2) - 1$ [14]. Thus, $h(2)$ allows one to determine whether a time series is stationary or non-stationary and detect its long-range correlations. We have found that the Hurst exponent for Merino milk-fed (Seville), Merino paschal (Seville), Merino paschal (Cordoba), Segureña milk-fed (Jaen) and Segureña paschal (Jaen) are $0.587+/-0.012$, $0.513+/-0.009$, $0.404+/-0.011$, $0.461+/-0.011$, $0.604+/-0.014$, respectively. Owing to the fact that $h(2)$ are all lower than 1, all of these five time series are stationary signals. Moreover, where the Hurst exponent ranges

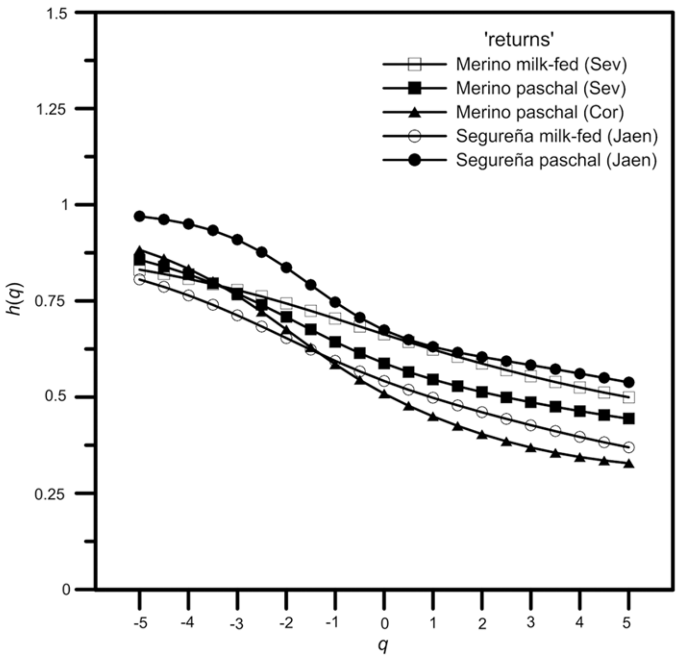

Fig. 5. Generalized Hurst exponent $h(q)$ as a function of q for the original time series of sheep prices.

Table 2
Parameters obtained from the log–log plots of the fluctuation functions $F_q(s)$.

Sheep	Market	$s_{\max}$ (weeks)	$\Delta h(q)$	$h(2)$
Merino milk-fed	Seville	47	0.353	0.587
Merino paschal	Seville	45	0.436	0.513
Merino paschal	Cordoba	42	0.579	0.404
Segureña milk-fed	Jaen	51	0.465	0.461
Segureña paschal	Jaen	45	0.448	0.604

between 0.5 and 1, the time series exhibits a long memory or persistence, whereas, if the Hurst exponent ranges between 0 and 0.5, the time series shows a short memory or anti-persistence. Where H is equal to 0.5, the time series is uncorrelated. Merino milk-fed (Seville), Merino paschal (Seville) and Segureña paschal (Jaen) time series are governed by long memory (persistence), which indicates that, generally, high prices fluctuations are followed by high fluctuations, and low prices fluctuations are followed by low fluctuations. Since Merino paschal (Seville) time series has a H value amounting to around 0.5, this might be considered a quasi-uncorrelated signal. Merino paschal (Cordoba) and Segureña milk-fed (Jaen) time series show short memory (anti-persistence), thus high price fluctuations are followed by low fluctuations, and vice versa.

Kantelhardt et al. [2] mentioned that there are two sources of multifractality in time series. It is due to a broad probability density function, on the other hand, and on the other, to different long-range correlations of small and large fluctuations, bearing in mind that both can coexist. Analyzing the corresponding shuffled time series is an easy way to clarifying the kind of multifractality. Multifractality due to the broadness of the probability density function cannot be removed with a shuffling procedure, whereas multifractality due to different long-range correlations of small and large fluctuations are destroyed through a random shuffling process. Hence, if the multifractality only corresponds to the long range correlation, we may find $h(q)_{shuffled} = 0.5$, and where the source of multifractality is exclusively due to the width of the probability density function, both $h(q)$ and $h(q)_{shuffled}$ are the same value. If both kinds of multifractality are present, the shuffling process will only be characterized by a weaker multifractality than the original one. Since we were interested in the possible source of multifractality, a shuffling procedure of the returns time series was performed. Shuffled time series were obtained by following the procedure proposed by Matia et al. [22], which consists of: First, generating pairs of random integer numbers $(m, n \leq N)$, N being the length of the time series; second, interchanging entries m and n; and finally, repeating these steps $20N$ times, for the purpose of ensuring adequate random shuffling.

Fig. 6 evinces that there are notable differences concerning the source of multifractality among time series. In general, the main source of multifractality in these time series is the broadness of the probability density function, due to the fact that shuffled returns also exhibit a multifractal nature. $h(q)$ and $h(q)_{shuffled}$ curves overlap for Merino paschal (Seville) and Segureña milk-fed (Jaen), which indicates that the source of multifractality is solely responsible for the multifractal nature

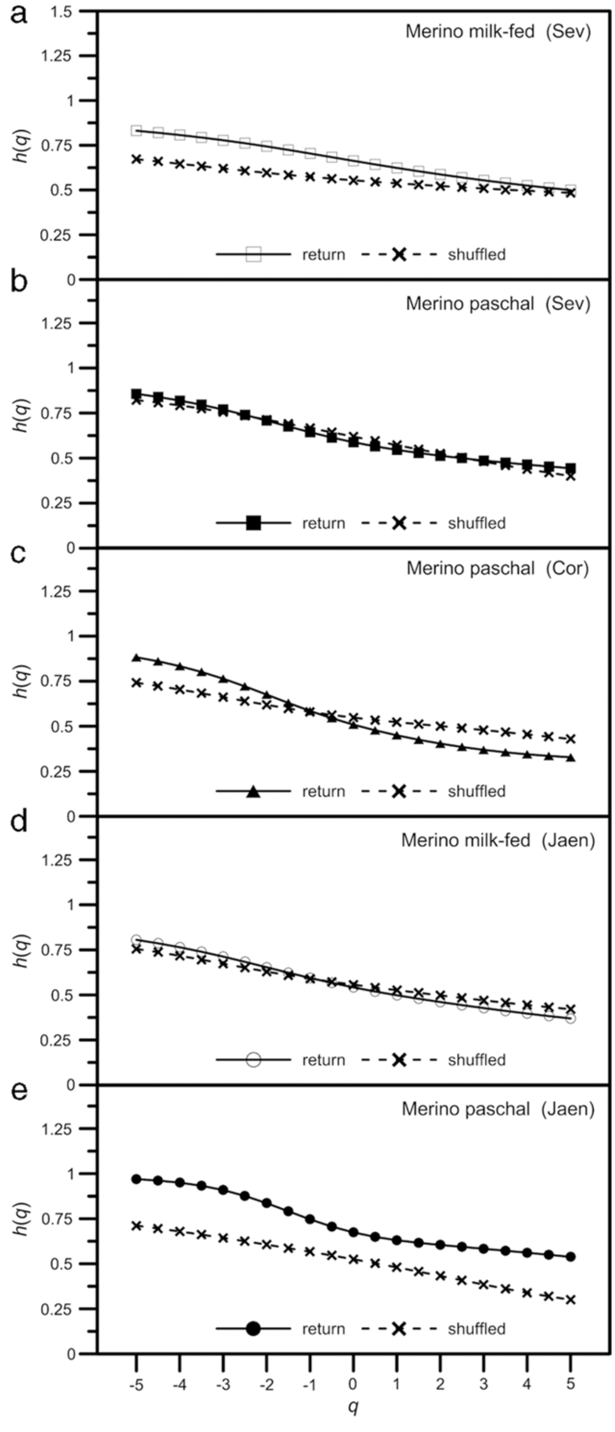

Fig. 6. Comparison of the generalized Hurst exponent $h(q)$ as a function of q for original and shuffled $h(q)_{shuffled}$ time series of sheep prices of series (a) Merino milk-fed, Seville; (b) Merino paschal, Seville; (c) Merino paschal, Cordoba; (d) Segureña milk-fed, Jaen; and (e) Segureña paschal, Jaen.

of both time series (see Fig. 6b and d). By contrast, in Fig. 6a and e a loss of multifractality is noted. For both cases – Merino milk-fed (Seville) and Segureña paschal (Jaen) –, the source of multifractality due to the different long-range correlations of the small and large fluctuations has been destroyed through the shuffling procedure, but the shuffled time series also show a multifractal nature. Therefore, both sources of multifractality coexist in these time series. Merino paschal (Cordoba) (Fig. 6c) is an intermediate case, in which both sources of multifractality also coexist. However, the relevance of the broadness of the probability density function is greater than the long-range correlations.

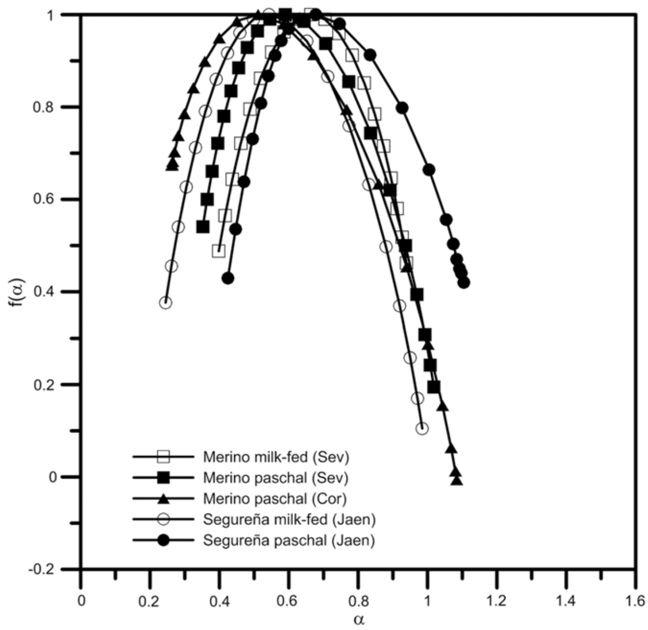

Fig. 7. Multifractal spectra of time series.

Meanwhile, the multifractal spectrum provides an adequate characterization of the multifractal nature of price fluctuations and enables one to describe in detail the price fluctuations among the previously defined time scales. In all cases, multifractal spectra (see Fig. 7) are convex parabolas, which confirm the multifractal behavior of these time series. All of them reach the maximum $f(\alpha)$ value for 1, according to the one dimension of the studied variables. Furthermore, when comparing the spectra, we can observe that their shapes are completely different and consequently, all of the studied markets and breeds are subject to different high and low price fluctuations. Segureña milk-fed spectrum exhibits longer tails than Merino milk-fed, which indicates the existence of greater heterogeneity in the distribution of both low and high price fluctuations. Merino paschal (Seville) spectrum has a longer right tail than the Merino milk-fed (Seville), which evinces greater heterogeneity in the distribution of low price fluctuations. This heterogeneity is even more marked for Merino milk-fed (Cordoba) time series as consequence of the longer length of the right tail of its spectrum.

4. Conclusions

According to the results, time series of livestock prices are a multifractal process, as indicated by the strong q dependence of $h(q)$ and $\tau(q)$ in time series of prices. This q-dependence has different behaviors for $q < 0$ and $q > 0$ in all the studied time series. Thus, it can be stated that the time series of sheep prices are of a multifractal nature. Hence, this study reveals that time series of prices paid to sheep livestock farmers exhibit a multifractal behavior, which can be characterized by using the MF-DFA. This finding allows one to use MFDFA as a suitable tool for the description and characterization of price fluctuations of livestock products in original markets. Multifractal properties are kept from 10 weeks to 42–51 weeks (around one year), which corresponds to the crossover time scale s_x, which is in agreement with the annual seasonality of these markets.

Multifractal results demonstrate that similar time series recorded in nearby markets might show different underlying fluctuations, sources and degrees of multifractality, persistency, anti-persistency and even poor correlation. By analyzing the Hurst exponent values H, it is possible to assert that the time series of sheep prices are stationary signals exhibiting both long and short memory. Moreover, comparing the generalized Hurst exponent of the original time series with the results of the corresponding shuffled series, we found that multifractality of these time series of sheep prices is mainly due to the broadness of the probability function, instead of the long-range correlation properties between small and large fluctuations, which play a clearly secondary role. Therefore, uncertainty in prices depends on the market, the breed and also the slaughter weight. Further studies may deepen the understanding of price fluctuations in different original markets at a regional scale, in light of the consequences of these disturbances, among others, on farmer incomes.

Acknowledgements

The authors gratefully acknowledge the support from the Consejería de Economía, Innovación y Ciencia (Regional Government of Andalusia). Project RNM-3989. The "sequence-determines-credit" approach has been applied for the authors' order.

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