A guided data projection technique for classification of sovereign ratings: the case of European Union 27

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Abstract

Sovereign rating has had an increasing importance since the beginning of the financial crisis. However, credit rating agencies opacity has been criticised by several authors highlighting the suitability of designing more objective alternative methods. This paper tackles the sovereign credit rating classification problem within an ordinal classification perspective by employing a pairwise class distances projection to build a classification model based on standard regression techniques. In this work the ϵ-SVR is selected as the regressor tool. The quality of the projection is validated through the classification results obtained for four performance metrics when applied to Standard & Poors, Moody's and Fitch sovereign rating data of U27 countries during the period 2007-2010. This validated projection is later used for ranking visualization which might be suitable to build a decision support system.

Keywords: ordinal regression, ordinal classification, country risk, sovereign risk, rating agencies, financial crisis

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1. Introduction

The sovereign rating industry is relatively new and has rapidly grown since Standard & Poors (S&P) published the first ranking of sovereign issuers in January 1961, followed by Moody's in 1974 and Fitch in 1994. Rating the creditworthiness of sovereign issuers has drawn growing attention due to the fact that the national governments are by far the largest borrowers in capital markets, outnumbering 60% of debt issued [1]. Sovereign ratings are a condensed assessment of each government's ability and willingness to service its debts in full and on time [2], distilling a multitude of credit risk information into a single letter on a credit quality scale. The main advantage of the sovereign ratings is the providing of a way of comparing investment and their credit quality to international private investors due to "the lack of consistent standards on government accounting across borders" [1]. Furthermore, in the framework of Basel Accords [3], they play a public function in determining the capital requirements for banks, securities firms and insurance companies according their assets and liabilities [3]. In this way, the role of sovereign ratings in structured finances has been accentuated by both market and regulatory practices.

The European debt crisis is a dramatic example of sovereign rating's role in the today financial market functioning and their economic consequences. The rating downgrade was focused on the so-called PIGS countries, i.e., Portugal, Ireland, Greece and Spain, but led significant spillovers across other European countries with solid macroeconomic and fiscal fundamentals [4]. As a result, the Eurozone financial markets have been under the pressure of the widening of sovereign bond and credit default swap spread, threatening the very existence of the European Union [5].

In the face of these developments, many policymakers and commentators have stated that Credit Rating Agencies (CRAs) precipitated the European crisis by the timing and extent of the downgrades [1]. Their critics highlight some of the disadvantages of the credit risk assessment process carried out by CRAs, such as their inherent conflict of interest within their business model or

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their adequacy of performance and lack of transparency [3].

The "issuer-pays" model, which is the most common remuneration practice among CRAs, lends to business more than two thirds of their total revenues [3]. CRAs also publish unsolicited ratings [1], being considered less reliable and less accurate because they are based on publicly available data. Therefore, CRAs face a moral hazard problem, in which they have an incentive to overestimate the creditworthiness of the issuers and a restrain to avoid the loss of their credibility with the investors.

Besides the above issue, several studies pointed out that CRAs provide different rating for the same entity [6] and the markets react differently to rating changes made by each agency [4]. These disagreements are more frequent for sovereign ratings than for corporate ones [7], between one or two notches in the finer risk-scale [6], and may be explained by the use of varying economic and non-economic factors and different weights on these factors, as well as the different methodologies [8]. Even though CRAs publish their rating methodologies, the precise models are not officially disclosed because of their business practice. In addition, the qualitative part of the rating approach makes it harder to identify the relationship between the assessment criteria and the resulting sovereign ratings [2], aggravating the problem of opacity in the rating process.

In order to complement or replace human or institutions decisions, many statistical and machine learning techniques have been applied to financial and business issues [9, 10, 11, 12]. In this sense, the sovereign rating problem is a multiclass classification problem in which the items require to be classified into naturally ordered classes. However, even though the ordered nature of most of the financial classification problems, most of the solutions apply nominal classification techniques. This paper deals with the problem of sovereign rating within an ordinal classification framework.

Ordinal classification (also known ordinal regression) is often addressed by the so called threshold models [13]. These methods assume that ordinal response is a coarsely measured latent continuous variable, and model it as real intervals in one dimension. Some examples are the Proportional Odds Model (POM) [14],

a large margin based algorithm [15] or the Support Vector Ordinal Regression (SVOR) [16]. Other approaches, such as the proposed by Frank and Hall [17], tackle ordinal classification by combining several binary classifiers.

The method employed in the present work can be considered in threshold models category, although instead of trying to learn the latent representations of the patterns, this latent positions are imposed by a guided projection procedure called Pairwise Class Distances projection (PCD). The projection is build by considering the relative positions of the patterns in the input space regarding adjacent classes, so that the ordinal structure of the data is exploited for improving the quality of the dimensionality reduction process.

The proposal is studied and validated based on a real sovereign rating dataset, which includes the rating assigned by the three leaders agencies to the 27 European Union countries during the period 2007-2010. The performance is compared to single-model state-of-the-art nominal and ordinal classifiers. Experimental results demonstrate the robustness of the method by using four ordinal classification performance metrics. In addition, the projected predicted values for unseen data are studied in the experimental section in order to interpret the classification and within class values of several EU countries.

As a summary of our motivation, the main objectives of this paper are the following. First, and more generally, to provide a tool that can complement the rating provided by CRAs. Secondly, to address the sovereign rating modelling with an ordinal perspective, this is, to apply ordinal classification methods and ordinal performance metrics to the task of sovereign rating. Finally, to prove the usefulness of the PCD projection which is not only able to obtain a good classification performance, but also to place a pattern in a relative order within its class, which brings additional information to the predicted class labels that can be helpful for decision making tasks and data analysis.

The rest of the paper is organized as follows. Section 2 briefly presents some related works, specifically highlighting machine learning methods applied to credit rating. Section 3 presents the problem formulation and the goals of ordinal classification, and also describes and analyses the PCD projection

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together with the associated classifier. In addition, the section provides basic background in Support Vector Machines (SVM) for the regression case. In the following section, the PCDOC algorithm is compared to other nominal and ordinal methods, and the model is internally analysed to provide additional information for potential decision makers. Finally, Section 5 ends with some conclusions.

2. Machine learning state-of-the-art for sovereign rating

This section briefly presents the related state-of-the-art works. An in-depth study is out of the scope of the current paper, but there are several review papers regarding statistical and machine learning techniques applied to financial and business issues [9, 10, 11, 12].

In the accounting and finance domain, bankruptcy prediction and credit scoring are the two major research problems [11]. More related to the current work, sovereign debt rating issue is growing in attention of the machine learning scientific community, although most of the methodologies have been focussing on corporate bonds rather than on sovereign risk [18]. In the second case, to assess the ability of a sovereign to honour its debt, some works applied statistical techniques such as Discriminant Analysis (DA) [19, 20, 21]. In spite of the ease to explain behaviour of the statistical models, the issue with applying these methods to the bond-rating prediction problem is that the multivariate normality assumptions for independent variables are frequently violated in financial data sets [22], which makes these methods theoretically invalid for finite samples [23]. This justify the use of alternative methods such as machine learning ones. The literature recognizes the unsuitability of these approaches to deal with sovereign rating problem because they ignore the ordered nature of ratings and ordered response models have been later introduced to overcome this limitation [2, 24, 25, 26, 27].

Machine learning methods have been applied to model sovereign ratings, e.g. Artificial Neural Networks (ANNs), which do not rely on parametric assump-

tions of normality of data, independence of the explanatory variables, stationary or sample-path continuity. The better performance of ANNs compared to previous statistical methods have been highlighted by Cosset and Roy [28], Cooper [29], Yim and Mitchell [30] and Benell [18], among others. Although this approach is not without its problems, such as the risk of over-fitting, the difficulty entailed in defining the physical structure of the network, and the tendency to fall into local optima [31]. Support Vector Machines (SVM) [32, 33] have being widely used for financial problems in the recent years [11], for instance the standard SVM classifier has been applied to financial time series forecasting [34] or to corporate credit rating prediction [35]. Later, new SVM models have been evaluated for credit scoring, for example, weighted SVM models such as the Least Squares SVM (LSSVM), where the hyper-parameters selection and training are based on the Area Under receiver operating characteristics Curve (AUC) maximization [36]. Yu et. al present a modified LSSVM to consider the prior knowledge that different classes may have different importance for model building so that more weight should be given to important classes [37]. Finally, soft computing techniques have been considered for financial problems. For instance, bank performance prediction were tackled with fuzzy SVM models by Chaudhuri and De [38] or with ensemble systems by Ravi et. al [39]. In addition, hybrid machine learning approaches have been applied to credit scoring [40] and bank rating [41].

However, most of the machine learning works deals with the problem as a binary classification problem, because several classifiers, such as SVM, are naturally designed for binary classification tasks. This would limit the applicability to evaluate credit as 'risk' or 'non-risk'. More recent approaches not only use a multi-class focus, but a limited number of them also consider an ordinal perspective of the problem. Van Gestel et al. [42] propose a whole process model to develop rating systems. In this work the classifier side is implemented by adding SVM terms to the linear model of the ordinal logistic regression so that the final model is both accurate and readable.

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3. Ordinal classification

This section presents the formulation of the ordinal classification problem, highlighting how threshold models work and presenting some state-of-the-art ordinal classifiers. Then, the PCD projection and the PCD based classifier are presented. Next, for clarifying purposes, the PCD projection is explained by using two of the input variables of one of the datasets considered in this paper. The last subsection is devoted to present the Support Vector Machine for regression, since this is the regressor selected for the experiments.

3.1. Problem formulation

When ordinal regression problems are tackled, the main objective is to map an input space, $\mathcal{X}$, to a finite set, $\mathcal{C} = \{C_1, C_2, \ldots, C_Q\}$, by using a mapping function $\phi : \mathcal{X} \to \mathcal{C}$. The important point is that the label set has an order relation $C_1 \prec C_2 \prec \ldots \prec C_Q$, where the symbol $\prec$ denotes the given order between different ranks. For each label, the rank is defined as $\mathcal{O}(\mathcal{C}_q) = q$, i.e. the position of the label in the ordinal scale. Patterns are represented by a set where each element contains a K-dimensional feature vector $\mathbf{x} \in \mathcal{X} \subseteq \mathbb{R}^K$ and a class label $y \in \mathcal{C}$. The training dataset T is composed of N patterns $T = \{(\mathbf{x}_i, y_i) \mid \mathbf{x}_i \in \mathcal{X}, \ y_i \in \mathcal{C}, i = 1, \ldots, N\}$, with $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{iK})$.

Considering the above definitions, ordinal classification is different from nominal classification in the evaluation of the classifier performance and also in the fact that the classifier should exploit the ordinal data disposition. For the former, as an example, although accuracy (*Acc*) has been widely used in classification tasks, it is not suitable for some type of problems, such as imbalanced datasets [43] (very different number of patterns for each class), and ordinal datasets [44]. Then, the performance metrics must consider the order of the classes so that errors between adjacent classes should be considered less important than the ones between separated classes in the ordinal scale. In our sovereign rating example, a misclassification predicting class C_3 (*Strong payment capacity*) when the real class is C_1 (*Highest quality*) should be more penalized

than the case when the prediction is C_2 (*High quality*). For this issue, some specific ordinal performance measures are needed [45, 46] (see Subsection 4.2 for measures definition). On the other hand, according to Hühn and Hüllermeier [47], the nature of the problem implies that the class order is somehow related to the distribution of patterns in the space of attributes $\mathcal{X}$, and therefore the classifier must exploit this a priori knowledge about the input space.

Different approaches have been proposed for addressing ordinal classification. For example, Kramer et. al [48] map the ordinal scale by assigning numerical values. Other alternative is to transform the ordinal classification problem into a nested binary classification problem [17, 49], and then classification predictions are obtained by combining the results of the binary classifications. These methods can be regarded as general frameworks that can adapt any generic classifier to the ordinal classification methods. Frank et. al [17] propose *A Simple Approach to Ordinal Regression (ASAOR)*, a general method that enables standard classification algorithms to make the use of order information in attributes. For the training process, the method transforms the Q-class ordinal problem into $Q - 1$ binary class problems. More sophisticated schemes are applied in other works, for instance, Li and Lin [50, 51] derive the Extended Binary Classification (EBC) framework and apply it to SVM, naming the method to as *EBC(SVM)*.

Other approaches are the commonly called threshold models. The current work relies in this kind of models. These methods assume that ordinal response is a coarsely measured latent continuous variable, and model it as real intervals in one dimension. Based on this assumption, the algorithms seek a direction in which the samples are projected and a set of thresholds that divide the direction into consecutive intervals representing ordinal categories. This approach is the most widely used for ordinal classification [13]. From a statistical background, one of the firstly proposed threshold methods is the Proportional Odds Model (POM), specifically designed for ordinal regression [14].

Gaussian Processes for Ordinal Regression (GPOR) [52] presents a probabilistic approach to ordinal regression based on Gaussian processes where a

threshold model that generalizes the *probit* function is used as the likelihood function for ordinal variables. SVM have also been adapted to fit into this generic form of threshold models. Chu et. al. [16], proposes two new support vector approaches for ordinal regression, Support Vector Ordinal Regression (SVOR) where multiple thresholds are optimized in order to define parallel discriminant hyperplanes for the ordinal scales. Two approaches are presented: *SVOR-EX* when the inequality constraints are explicitly added to the optimization problem, and SVOR with implicit constraints or *SVOR-IM* where the samples from different categories are allowed to contribute errors for each threshold.

In this article, rather than learning the latent variable, direct hints are provided to the threshold model via a dimensionality reduction process that exploits the a priori knowledge of an ordinal space distribution of patterns in the input space. For each class, the distances of the patterns belonging to that class and to the adjacent classes are used. This projection is named to as Pairwise Class Distances (PCD), and the associated classifier using PCD is called PCD Ordinal Classifier (PCDOC). Here, PCDOC is implemented by using ϵ-SVR [33, 53, 54] as the base regressor.

3.2. Threshold models

Threshold models consider the ordinal scale as the result of coarse measurements of a continuous variable, which is assumed to be difficult or impossible to be measured [13]. A threshold model is based on the following equation:

$$
f(\mathbf{x}, \mathbf{b}) = \begin{cases}
C_1, & \text{if } g(\mathbf{x}) \leq b_1, \\
C_2, & \text{if } b_1 < g(\mathbf{x}) \leq b_2, \\
\vdots \\
C_Q, & \text{if } g(\mathbf{x}) > b_{Q-1},
\end{cases}
\tag{1}
$$

where $g : \mathcal{X} \to \mathbb{R}$ is the function that projects data space into the 1-dimensional latent space $\mathcal{L} \subseteq \mathbb{R}$ and $b_1 < \ldots < b_{Q-1}$ are the thresholds that divide the space into ordered intervals corresponding to the classes.

In the PCDOC proposal it is assumed that a model $\phi : \mathcal{X} \to \mathcal{L}$ can be found that links data items $\mathbf{x} \in \mathcal{X}$ with their latent space representation $\phi(\mathbf{x}) \in \mathcal{L}$. For simplicity, $\mathbf{b}$ are fixed in opposition to other models such as POM. Instead of paying attention to the thressholds, the keys of the method are the projection procedure, which exploits the ordinal structure of the space $\mathcal{X}$, and the explicit compression that the projection does on the margins between classes in $\mathcal{L}$ (see Section 3.5).

3.3. Pairwise Class Distances projection

This section explains the proposed guided projection and how it is used to train a generic regressor for performing ordinal classification.

To describe the Pairwise Class Distance (PCD) projection, first, we define a measure $w_{\mathbf{x}^{(q)}}$ of "*how well*" a pattern $\mathbf{x}^{(q)1}$ is placed within all the instances of class C_q. The goodness of its position is given by a function of the Euclidean distance between the evaluated pattern and the patterns in adjacent classes. On the assumption of an ordinal input space, patterns of adjacent classes may be closer than patterns of non-adjacent classes. The minimum distance from a pattern $\mathbf{x}_i^{(q)}$ to patterns in the previous class (δ_{p}) is:

$$\delta_{\mathrm{p}}\left(\mathbf{x}_i^{(q)}\right) = \min_{\mathbf{x}_j^{(q-1)}} \left\{ ||\mathbf{x}_i^{(q)} - \mathbf{x}_j^{(q-1)}|| \right\}, \quad 1 \leq j \leq n_{q-1}, \tag{2}$$

where $||\mathbf{x} - \mathbf{x}'||$ is the Euclidean distance between $\mathbf{x}$ and $\mathbf{x}'$ and n_{q-1} represents the number of patterns of class C_{q-1}. And similarly, the minimum distance from a pattern $\mathbf{x}_i^{(q)}$ to patterns in the next class (δ_{n}) is:

$$\delta_{\mathrm{n}}\left(\mathbf{x}_i^{(q)}\right) = \min_{\mathbf{x}_j^{(q+1)}} \left\{ ||\mathbf{x}_i^{(q)} - \mathbf{x}_j^{(q+1)}|| \right\}, \quad 1 \leq j \leq n_{q+1}. \tag{3}$$

Then, the measure of "*goodness*" of a pattern $\mathbf{x}_i^{(q)}$ is defined as:

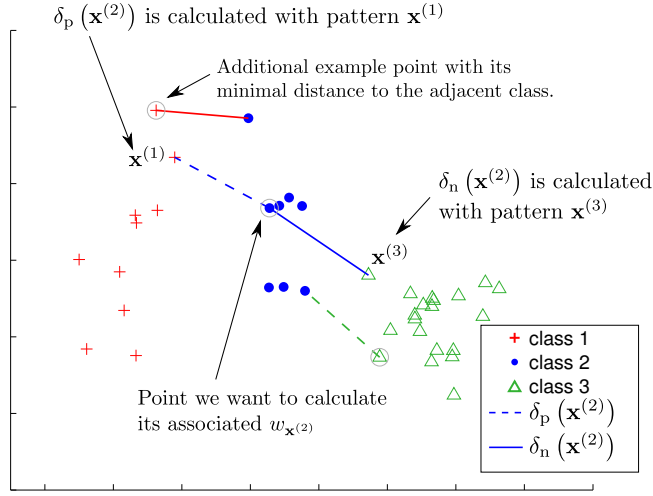[1]The notation $\mathbf{x}^{(q)}$ will be used to indicate that pattern $\mathbf{x}$ belongs to class C_q

Figure 1: General idea about how the "*goodness*" measure is obtained. For each pattern of class C_2, δ_{p} and δ_{n} are represented using lines. As an example, value $w_{\mathbf{x}^{(2)}}$ of $\mathbf{x}^{(2)}$ is calculated by using $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(3)}$.

$$w_{\mathbf{x}_i^{(q)}} = \frac{\delta_{\mathrm{p}}\left(\mathbf{x}_i^{(q)}\right) + \delta_{\mathrm{n}}\left(\mathbf{x}_i^{(q)}\right)}{\max_{\mathbf{x}_j^{(q)}}\left\{\delta_{\mathrm{p}}\left(\mathbf{x}_j^{(q)}\right) + \delta_{\mathrm{n}}\left(\mathbf{x}_j^{(q)}\right)\right\}}, \quad 1 \leq j \leq n_q, \tag{4}$$

where n_q represents the number of patterns of class C_q and we set $\delta_{\mathrm{p}}\left(\mathbf{x}_i^{(1)}\right) = 0$ and $\delta_{\mathrm{n}}\left(\mathbf{x}_i^{(Q)}\right) = 0$ to keep into account the extreme classes. In this way, the sum of the minimum distances of a pattern with respect to adjacent classes is normalized across all patterns of the class, so that $w_{\mathbf{x}_i^{(q)}}$ has a maximum value of 1 for the best positioned pattern.

Figure 1 shows the idea of minimum distances for each pattern with respect to the patterns of the adjacent classes, considering patterns of the second class. As an example, the $w_{\mathbf{x}^{(2)}}$ value is obtained for the pattern $\mathbf{x}^{(2)}$ (marked with a circle).

These values, $w_{\mathbf{x}_i^{(q)}}$, are used to derive a latent variable $l_i \in \mathcal{L}$. But, first of all, thresholds need to be fixed so the intervals on $\mathcal{L}$ which correspond to each class are delimited, and the calculated values for l_i may be correctly positioned. Moreover, in the test phase, predicted values $\hat{l}_i$ of unseen data would be classified

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in different classes according to these thresholds (see Subsection 3.4) (which is indeed the way all threshold models work). For the sake of simplicity, $\mathcal{L}$ is limited to the interval $[0, 1]$, and the thresholds are uniformly positioned in this interval:

$$\mathbf{b} = \{b_1, b_2, \ldots, b_Q\} = \{1/Q, 2/Q, \ldots, Q/Q = 1\}. \tag{5}$$

Consequently, the centres c_q for $\mathcal{L}$ values belonging to class C_q are defined as:

$$\mathbf{c} = \{c_1, c_2, \ldots, c_Q\}; \quad c_1 = 0, \ c_Q = 1, \ c_q = \frac{\frac{q-1}{Q} + \frac{q}{Q}}{2}, 2 \leq q \leq Q - 1. \tag{6}$$

A very intuitive idea is now considered to construct l_i values for training inputs $\mathbf{x}_i^{(q)}$. If $\mathbf{x}_i^{(q)}$ has a high value of $w_{\mathbf{x}_i^{(q)}}$ (i.e. minimum distances δ_{p} and δ_{n} are quite similar), the l_i value should be closer to c_i (because the pattern is clearly *well located* within its class). If a low value of $w_{\mathbf{x}_i^{(q)}}$ is obtained (i.e. δ_{p} and δ_{n} are very different), this means that the pattern is closer to one of these classes, and the resulting l_i value should be closer to the closest adjacent class, $q - 1$ or $q + 1$. The following expression is reflecting this idea:

$$l_i = \phi\left(\mathbf{x}_i^{(q)}\right) = \begin{cases} c_1 + (1 - w_{\mathbf{x}_i^{(1)}}) \cdot \frac{1}{Q}, & \text{if } q = 1, \\\\ c_q - (1 - w_{\mathbf{x}_i^{(q)}}) \cdot \frac{1}{2Q}, & \text{if } q \in \{2, \ldots, Q-1\} \text{ and} \\ & \qquad \delta_{\mathrm{p}}(\mathbf{x}_i^{(q)}) \leq \delta_{\mathrm{n}}(\mathbf{x}_i^{(q)}), \\\\ c_q + (1 - w_{\mathbf{x}_i^{(q)}}) \cdot \frac{1}{2Q}, & \text{if } q \in \{2, \ldots, Q-1\} \text{ and} \\ & \qquad \delta_{\mathrm{p}}(\mathbf{x}_i^{(q)}) > \delta_{\mathrm{n}}(\mathbf{x}_i^{(q)}), \\\\ c_Q - (1 - w_{\mathbf{x}_i^{(Q)}}) \cdot \frac{1}{Q}, & \text{if } q = Q, \end{cases} \tag{7}$$

where $w_{\mathbf{x}_i^{(q)}}$ is defined in Eq. (4), and c_q is the centre of class interval corresponding to C_q (see Eq. (6)). This expression guarantees that all l values lie in the correct class interval (the threshold vector $\mathbf{b}$ delimiting class intervals is

defined in Eq. (5)). This methodology for data projection is called *Pairwise Class Distances* (PCD).

3.4. PCD based ordinal classifier

Once the PCD projections have been obtained for all training inputs, this projection can be used to construct a classifier based on any regression tool. The regressor, g, will approximate the mapping from the input space to the PCD projections, $g : \mathcal{X} \rightarrow \mathcal{L}$. In other words, a new training set is derived changing the target variable, $T' = \{(\mathbf{x}_i, \phi(\mathbf{x}_i) = l_i) \mid (\mathbf{x}_i, y_i) \in T\}$, where l_i is obtained from Eq. (7). This dataset is used to construct the regressor g. For the test phase, this regressor will be applied to obtain an estimated latent variable value, $\hat{l} = g(\mathbf{x})$, and using the thresholds $\mathbf{b}$ in Eq. (5) and the general expression of the threshold model in Eq. (1), the predicted class label $\hat{y}$ will be obtained.

It is expected that formulating the problem as a regression problem would help the model to capture the ordinal structure of the input and output spaces, and their relationship. In addition, due to the nature of the regression problem, it is expected that the performance of the classification task will be improved regarding metrics that consider the difference between the predicted and actual ranks or the correlation between the target and predicted ranks. Experimental results confirm this hypothesis in Section 4.4.

3.5. Analysis of the PCD projection with sovereign rating datasets

For better understanding how the PCD projection takes advantage of the ordinal disposition of the data, two different input variables have been selected from the sovereign rating datasets of Section 4.1. Our objective is to show how the proposed projection works in a two dimensional representation. The variables selected are "GDP per capita" (PC_GDP) and "Government effectiveness" (GV_EFFECT), for years 2007-2009, their values being standardised. This selection is done because these variables are suitable for showing the ordinal structure of the data in the input space, not because of feature selection criteria, which is out of the scope of this paper. Fitch agency countries patterns
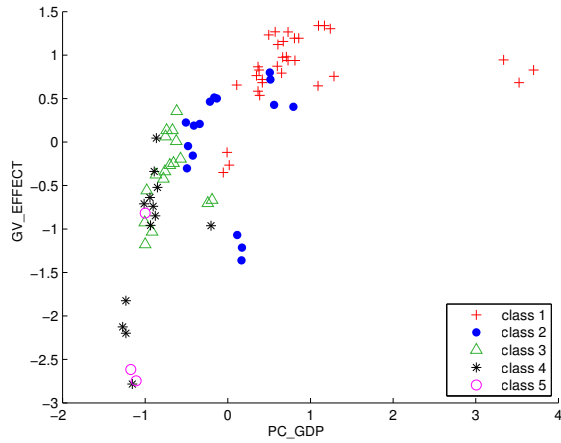
(a) Patterns of Fitch's dataset considering PC_GDP and GV_EFFECT variables.
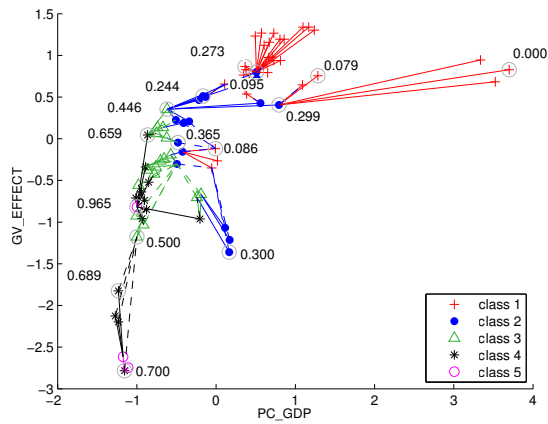
(b) Example of the generated l_i values on the Fitch's dataset.

Figure 2: Illustration of the PCD projections with PC_GDP and GV_EFFECT variables of Fitch's dataset corresponding to patterns of years 2007-2009. Points of different classes are plotted with different symbols and colours.

labelling is used for this illustration example, as can be seen in Figure 2a. The figure shows that the data have a clear ordinal distribution through the input space, and how a separation in classes is difficult, some of them being clearly overlapped. It can be noticed the majority of patterns of each class are situated in regions of the space having adjacent classes patterns in neighbour regions.

Figure 2b presents the PCD concepts applied to the patterns in Figure 2a.

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The minimum distances are illustrated with lines of the same colour than the class. The minimum distance of a point to the next class patterns are marked with solid lines, while the minimum distances to the previous class are marked with dashed lines. For some example points (surrounded by a grey circle), the value of the PCD projection using Eq. (7) is shown near the point. It can be easily seen that the l value increases for patterns of the higher classes, and this value varies depending on the position of the pattern $\mathbf{x}^{(q)}$ in the space with respect to the patterns $\mathbf{x}^{(q-1)}$ and $\mathbf{x}^{(q+1)}$ of adjacent classes.

3.6. Support Vector Machines for regression

The Support Vector Machines (SVM) [32] are perhaps the most common kernel learning method for statistical pattern recognition. Initially formulated for binary classification problems, they have been extended for multi-class environments [55], ordinal classification problems [16] and for standard regression [54]. The later is explained in this section since it is the base regressor used in our proposal to model the PCD projection in the experiments.

SVM are linear models with an optimization process that implicitly selects a subset of patterns for building the model. These patterns are named as *support vectors*. The first formulation of SVM is known as the hard-margin approach, but it presents the overfitting problem, which severally downgrades the generalization performance. In contrast, the soft-margin approach is achieved with the inclusion of slack-variables ξ_i in the optimization process [32], and it improves the performance of the model.

The ϵ-SVR model is defined by the hyperplane given by:

$$f(\mathbf{x}) = \hat{y}(\mathbf{x}) = \mathbf{w} \cdot \phi(\mathbf{x}) + b, \tag{8}$$

where ϕ is a mapping function which transforms the patterns representation in the attributes or input space $\mathcal{X}$ to a high dimensional Reproducing Kernel Hilbert Space (RKHS). The reproducing kernel function is used, defined as $k(\mathbf{x}, \mathbf{x}') = \langle \phi(\mathbf{x}) \cdot \phi(\mathbf{x}') \rangle$, where $\langle \cdot \rangle$ denotes inner product in the RKHS. This

transformation is known as the *kernel trick* and it allows to overcome the limitations of linear models. The most common mapping function is the Gaussian kernel, defined as:

$$k(\mathbf{x}, \mathbf{x}') = \exp^{-\gamma \|\mathbf{x} - \mathbf{x}'\|^2}.$$

where γ is a parameter associated with the width of the Gaussian kernel.

The model training consist on finding the weight vector $\mathbf{w}$ that minimizes the following regularized error function:

$$C \sum_{i=1}^{N} E_\epsilon(\hat{y}(\mathbf{x}_i) - y_i) - \frac{1}{2} \|\mathbf{w}\|^2.$$

where $\hat{y}(\mathbf{x}_i)$ is given by Eq. 8 and E_ϵ is the *ϵ-insensitive error function* proposed by [56]:

$$E_\epsilon(\hat{y}(\mathbf{x}) - y) = \begin{cases} 0, & \text{if } |\hat{y}(\mathbf{x}) - y| < \epsilon \\ |\hat{y}(\mathbf{x}) - y| - \epsilon, & \text{otherwise.} \end{cases} \tag{9}$$

The E_ϵ function gives zero error if the absolute difference between the prediction $\hat{y}(\mathbf{x})$ and the actual target y is less than ϵ (being $\epsilon > 0$) and it produces more sparse solutions that other functions such as the quadratic error function which is widely used in regression problems [57].

The optimization problem can be reformulated by introducing slack variables. For each point $\mathbf{x}_i$ two slack variables are needed ($\xi_i \geq 0$ and $\xi_i^* \geq 0$, where $\xi_i > 0$ corresponds to a point for which target value $y_i > \hat{y}(\mathbf{x}_i) + \epsilon$, and $\xi_i^* > 0$ corresponds to a point for which $y_i < \hat{y}(\mathbf{x}_i) - \epsilon$).

A target point lies inside the ϵ-tube if $\hat{y}_i(\mathbf{x}_i) - \epsilon \leq y_i \leq y_i + \epsilon$. The inclusion of the slack variables allows points to lie outside the tube provided that the slack variables are non-zero. The corresponding conditions are:

$$y_i \leq \hat{y}_i(\mathbf{x}_i) + \epsilon + \xi_i,$$
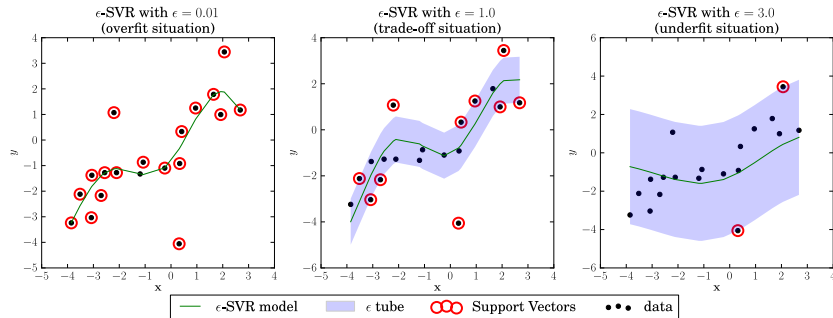$$y_i \geq \hat{y}_i(\mathbf{x}_i) - \epsilon - \xi_i^*.$$

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Figure 3: Example of the influence of the ϵ parameter in the ϵ-SVR model (overfit, trade-off and underfit situations are presented).

At this point, the error function for support vector regression can be written as:

$$C \sum_{i=1}^{N} (\xi_i + \xi_i^*) + \frac{1}{2} \|\mathbf{w}\|^2,$$

which must be minimized subject to the constraints $\xi_i \geq 0$ and $\xi_i^* \geq 0$. The corresponding dual problem for minimizing this function implies using two kinds of Lagrange multipliers (α_i and α_i^*). More details can be found in [57].

Finally, predictions of new inputs can be expressed in terms of the kernel function:

$$y(\mathbf{x}) = \sum_{i=1}^{N} (\alpha_i - \alpha_i^*) k(\mathbf{x}, \mathbf{x}_i) + b,$$

where α_i and α_i^* are the Lagrange multipliers.

To conclude this section, Figure 3 shows different ϵ-SVR models for several ϵ values on an artificial regression dataset. The figure shows the support vectors of the model. It can be observed how the ϵ simultaneously determines the suitable fitting of the model to the data, as well as the sparseness of the model.

4. Experiments

This section presents the description of the sovereign rating dataset, the ordinal classification performance metrics, the dataset experimental design and related methods experiments configuration, and the comparison of these methods

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Table 1: A comparison of the rating labels from CRAs. The observations have been grouped into broader categories in the second column.

	Broader rating categories		S&P	Moody's	Fitch
	Highest quality	C_1	AAA	Aaa	AAA
	High quality	C_2	AA+	Aa1	AA+
			AA	Aa2	AA
			AA-	Aa3	AA-
Investment grade	Strong payment capacity	C_3	A+	A1	A+
			A	A2	A
			A-	A3	A-
	Adequate payment capacity	C_4	BBB+	Bbb1	BBB+
			BBB	Bbb2	BBB
			BBB-	Bbb3	BBB-
	Likely to fulfill obligations, ongoing uncertainty	C_5	BB+	Bb1	BB+
			BB	Bb2	BB
			BB-	Bb3	BB-
Speculative grade	High credit risk	C_6	B+	B1	B+
			B	B2	B
			B-	B3	B-
	Very high credit risk	C_7	CCC+	Caa1	CCC+
			CCC	Caa2	CCC
			CCC-	Caa3	CCC-
	Near default with possibility of recovery	C_8	CC	Ca	CC
			C	C	C
	Default	C_9	SD	-	SD
			D	-	D

to the proposed one. The section concludes with the analysis of the predicted projection of the patterns for the generalization set, i.e. the 2010 year. The datasets used in this section are available in a public website[2].

4.1. Data

A set of economic descriptors of countries have been collected. The dataset contains 108 annual observations of long-term foreign-currency sovereign credit ratings of 27 EU sovereign borrowers during the period from 2007 to 2010. The credit rating data is from the publicly available historical information provided by the three leader agencies, Standard and Poor's, Moody's and Fitch, as of

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[2]http://www.uco.es/grupos/ayrna/asoc-sovereign-ratings

Table 2: Description of the input variables. Note the business cycle approach has been considered through the inclusion of Gross Domestic Product (GDP) growth, fiscal and current account balance, inflation and unemployment as a three years average.

Variable name	Unit of measurement	Rating Influence
Real GDP growth	Rate	Positive
GDP per capita	Euros per inhabitant	Positive
Government debt	Percent of GDP	Negative
Fiscal balance	Percent of GDP	Positive
External debt	Percent of exports	Negative
Foreign reserves	Percent of imports	Positive
Current account balance	Percent of GDP	Negative
Inflation	Index (2005=100)	Negative
Unemployment	Rate	Negative
Unit labour cost	Index (2005=100)	Negative
Government effectiveness	Percentile	Positive

December 31st of each year. The three CRAs use similar rating scales with 23-risk points (Table 1), in which the triple-A notation means the best quality issue. The observations have been grouped into broader categories according to Hill et al. [6], which are shown in the second column.

By using the broader rating categories of Table 1 the problem would be a 9 ordinal label classification problem. However, during the time span considered, all the European issuers were rated into the five first categories. Consequently, only five classes are considered for the classification problem (C_1, C_2, C_3, C_4, C_5).

Many empirical studies, such as those from Cantor and Packer [7] and Bissoyondal-Beheninck [2], among others, have investigated the determinants of sovereign ratings, showing that they are mainly driven by economic variables. Based on them, eleven variables have been selected (see Table 2), ten economic indicators as well as one non-economic [58]. The data were taken from Eurostat, except Government Effectiveness indicator provided by the World Bank as well as the External Debt figures, which were completed with information provided by Central Banks of Cyprus, Malta, Sweden and Romania.

Notice that the methodology proposed attempts to estimate the rating provided by CRAs based on a reduced set of publicly available indicators in comparison with the large number criteria taken into account by CRAs. All the

information about those criteria is accessible online into the CRA web pages, being comparable across the rating process, but they differ in the way in which they are classified and weighted by the firms' analysts [59, chap. III].

Three datasets are generated, one for each of the CRAs considered (Fitch, Moody's and S&P). The input variables are the same for all of them, but the rating is different depending on the CRA taken into account. The dataset for each CRA has been split in two subsequent time period sets used as training and generalization sets. The first set includes 81 observations, described by the correspondent variables, from the 27 EU sovereign borrowers during the period 2007-2009, whereas, in the second one, data are from 2010.

4.2. Ordinal classification performance metrics

In this work, four evaluation metrics have been considered which quantify the classifier's performance for N predicted ordinal labels in a given dataset $\{\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_Q\}$, with respect to the true targets $\{y_1, y_2, \ldots, y_Q\}$.

The accuracy (Acc) and the Mean Absolute Error (MAE) are included since they are typically used in the literature, and nominal and ordinal classification methods usually optimize these metrics. The accuracy, also known as Correct Classification Rate, or as Mean Zero-One Error when expressed as an error, is the rate of correctly classified patterns:

$$Acc = \frac{1}{N} \sum_{i=1}^{N} [\![\hat{y}_i = y_i]\!],$$

where $[\![c]\!]$ is the indicator function, being equal to 1 if c is true, and to 0 otherwise. Acc values ranging from 0 to 1 represent a global performance on the classification task.

The Mean Absolute Error (MAE) is the average deviation in absolute value of the predicted ranks from the true ranks [44]:

$$MAE = \frac{1}{N} \sum_{i=1}^{N} e(\mathbf{x}_i),$$

where $e(\mathbf{x}_i) = |\mathcal{O}(y_i) - \mathcal{O}(\hat{y}_i)|$ is the distance between the true and the predicted ranks, and, then, MAE values range from 0 to $Q - 1$. Acc is typically used in the literature together with MAE since the former does not reflect the category order in the errors consideration [16, 45, 51, 52, 60].

However, more robust performance metrics have been proposed for ordinal classification. For instance, Baccianella et al. proposed the *average MAE* ($AMAE$) [44] as a more robust alternative to MAE for imbalanced datasets, which is very common in ordinal classification (in general, extreme classes are usually associated to rare situations). The $AMAE$ is defined as the mean performance of the MAE across classes:

$$AMAE = \frac{1}{Q} \sum_{j=1}^{Q} MAE_j = \frac{1}{Q} \sum_{j=1}^{Q} \frac{1}{n_j} \sum_{i=1}^{n_j} e(\mathbf{x}_i),$$

where $AMAE$ values range from 0 to $Q - 1$ and n_j is the number of patterns of class j.

Finally, the Kendall's τ_b is a statistic used to measure the association between two measured quantities. τ_b has been recently advised as a suitable measure for ordinal classification since it is independent of the values used to represent classes [61]. Its robustness is achieved by working directly on the set of pairs corresponding to different observations. Specifically, τ_b is a measure of rank correlation [62]:

$$\tau_b = \frac{\sum \hat{c}_{ij} c_{ij}}{\sqrt{\sum c_{ij}^{*2} \sum c_{ij}^2}}, i = 1, ..., N, j = 1, ..., N,$$

where $\hat{c}_{ij}$ is $+1$ if $\hat{y}_i$ is greater than (in the ordinal scale) $\hat{y}_j$, 0 if $\hat{y}_i$ and $\hat{y}_j$ are the same, and -1 if $\hat{y}_i$ is lower than $\hat{y}_j$, and the same for c_{ij}. τ_b values range from -1 (maximum disagreement between the prediction and the true label), to 0 (no correlation between them) and to 1 (maximum agreement).

4.3. Experimental design and comparison methods

The experiments have being carried out by using a hold-out experimental design of the three datasets described at Section 4.1. The training dataset consist on patterns belonging to years 2007-2009. The generalization or test dataset consist on the 2010 year patterns. It must be paid attention to the Moody's dataset since during the period 2007-2009 (training period) there were not ratings of class C_5. However, in 2010 (test period), Greece was ranked as C_5 by this CRA. Given that all classifiers were trained for a four class dataset, none of them was able to correctly classify Greece during the test phase.

Due to the deterministic nature of all the compared methods, only one run of each method has been performed for each dataset and the generalization performance for several classification metrics is reported.

The ordinal regression methods used for comparison purposes have been selected according to their similarities to the proposal, and also because of the implementation availability. The ordinal methods considered are: ASAOR with a C4.5 base classifier (as suggested by Frank and Hall [17], ASAOR(C4.5)), EBC with SVM as base classifier (as suggested by Li and Lin [50, 51], EBC(SVM)), GPOR [52], SVOR-EX and SVOR-IM [16]. The reader can consult Section 3.1 for more details. In our approach, the Support Vector Regression (SVR) algorithm is used as the base regressor, so the method is called SVR-PCDOC. The ϵ-SVR implementation available in the `libsvm` package [63] is used. The authors of GPOR, SVOR-EX, SVOR-IM and EBC(SVM) provide publicly available software implementations of their methods[3]. With respect to the ASAOR method, the C4.5 method available in Weka [64] is used as the underlying classification algorithm since this is the one initially employed by the authors of ASAOR. In this way, the algorithm is identified as ASAOR(C4.5).

In addition, some well known nominal algorithms have been selected in order to compare the ordinal approaches and to check if the ordinal classifiers are tak-

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[3]GPOR (`http://www.gatsby.ucl.ac.uk/~chuwei/ordinalregression.html`), SVOR-EX and SVOR-IM (`http://www.gatsby.ucl.ac.uk/~chuwei/svor.htm`) and EBC(SVM) (`http://home.caltech.edu/~htlin/program/libsvm/`)

ing benefit from the order information. These are C4.5, a standard multinomial logistic regression (Mlogistic), a logistic regression based on simple regression and variable selection (Slogistic), and the Multi-Layer Perceptron (MLP) neural network. A detailed description of these methods can be found in the works of Landwehr et al. [65] and Witten and Frank [66].

Model selection is an important issue and involves selecting the best hyperparameter combination for all the methods compared. All the kernel methods were configured to use the Gaussian kernel. For the support vector algorithms, i.e. EBC(SVM), SVOR-EX, SVOR-IM and ϵ-SVR, the corresponding hyperparameters (regularization parameter, C, and width of the Gaussian functions, γ), were adjusted using a grid search over each of the training set by a 5-fold cross-validation with the following ranges: $C \in \{10^3, 10^2, \ldots, 10^{-3}\}$ and $\gamma \in \{10^3, 10^2, \ldots, 10^{-3}\}$. Regarding ϵ-SVR, the additional ϵ parameter has to be adjusted. The range consider was $\epsilon \in \{10^3, 10^1, \ldots, 10^0\}$. GPOR has no hyper-parameters to fix, since the method optimizes the associated parameters itself. Finally, ASAOR(C4.5), C4.5, Mlogistic and Slogistic have no hyperparameters. For the MLP method, the number of hidden neurons was also adjusted by cross-validation in the training set.

4.4. Experimental results

Table 3 presents the performance results of the different algorithms with the three datasets. Nominal and ordinal classifiers are separated with a horizontal line. In general, it should be pointed out that the performance ranking changes for each metric. However, SVR-PCDOC is very robust when considering all the datasets and all the metrics. It achieves the best performance for all the metrics in the Fitch and S&P datasets. In addition, the second best method for these datasets varies with respect to the metric considered. For example, Slogistic is the second best one in S&P when considering *Acc*. In this dataset, SVOR-IM was in the seventh position for *Acc*, but for metrics which consider the order (MAE, AMAE and τ_b), this method is the second best one. Regarding Moody's dataset, SVR-PCDOC has the best results for MAE and τ_b, but not for *Acc*

Table 3: Comparison of the proposed method to other nominal and ordinal classification methods. The value of different metrics test results (year 2010 prediction) are reported for each dataset. The best result is in bold face and the second best result in italics.

Method/DataSet	Accuracy			MAE		
	Fitch	Moody's	S&P	Fitch	Moody's	S&P
C4.5	0.6296	0.6667	0.5926	0.4074	0.4074	0.4815
Mlogistic	0.4815	0.7778	0.3704	0.8889	0.3333	0.8889
MLP	0.6667	**0.8519**	0.6667	0.4074	0.2593	0.4444
Slogistic	*0.7407*	0.7778	*0.7037*	*0.2593*	0.2963	0.4074
ASAOR(C4.5)	0.5926	0.6296	*0.7037*	0.4815	0.4815	0.4074
EBC(SVM)	0.6667	*0.8148*	0.6667	0.3333	**0.2222**	0.4074
GPOR	*0.7407*	0.7037	0.6667	0.3704	0.4444	0.4444
SVOR-EX	0.7037	0.7778	0.5926	0.2963	0.2593	0.4444
SVOR-IM	0.6667	*0.8148*	0.6296	0.3333	**0.2222**	*0.3704*
SVR-PCDOC	**0.7778**	*0.8148*	**0.7407**	**0.2222**	**0.2222**	**0.2593**

Method/DataSet	AMAE			$\tau_{\rm b}$		
	Fitch	Moody's	S&P	Fitch	Moody's	S&P
C4.5	0.4400	0.6800	0.5111	0.7621	0.7367	0.7655
Mlogistic	1.1600	0.6467	0.9333	0.5255	0.7719	0.5121
MLP	0.5267	**0.4067**	0.4000	0.7972	0.8097	0.7492
Slogistic	0.2667	0.6200	0.5111	*0.8951*	0.8151	0.8060
ASAOR(C4.5)	0.4533	0.7533	0.4222	0.6989	0.6655	0.7570
EBC(SVM)	0.2822	*0.5356*	0.4222	0.8835	0.8590	0.8052
GPOR	0.5133	0.9200	0.6222	0.7738	0.6869	0.7807
SVOR-EX	*0.2422*	0.5622	0.4444	0.8886	**0.8610**	0.7873
SVOR-IM	0.2756	*0.5356*	*0.3556*	0.8799	0.8525	*0.8370*
SVR-PCDOC	**0.2089**	0.5467	**0.2889**	**0.9224**	**0.8610**	**0.8849**

and AMAE. This is due to the error that the classifier has for Greece pattern in 2010, since it misclassifies Greece as C_3 when it a C_5 pattern, and this is more more penalized by AMAE than by MAE.

Table 4 shows the credit rating granted by the three leader CRAs and the credit rating predicted by the SVR-PCDOC models for the 27 EU countries in the test set (year 2010). Errors have been highlighted in bold face. The data included in this table should be analysed together with the contingency matrices in Figures 4, 5 and 6.

If we take into account the test set (corresponding to year 2010), the number of patterns of each class are the following: $\{9, 5, 6, 5, 2\}$ for Fitch, $\{9, 5, 6, 6, 1\}$ for Moody's and $\{9, 3, 9, 3, 3\}$ for S&P. Considering the three datasets, the total distribution is $\{27, 13, 23, 14, 6\}$. Taking into account this distribution, a com-

Table 4: Ratings for EU countries in the test set (2010): real ratings versus SVR-PCDOC predicted rating.

		Fitch		Moody's		S&P	
Pattern	Acronym	Rating	Pred.	Rating	Pred.	Rating	Pred.
Austria	AUT	1	1	1	1	1	1
Belgium	BEL	2	2	2	2	2	2
Bulgaria	BUL	**4**	**5**	4	4	4	4
Cyprus	CHP	2	2	2	2	**3**	**2**
Czech Republic	CZE	3	3	3	3	3	3
Germany	DEU	1	1	1	1	1	1
Denmark	DNK	1	1	1	1	1	1
Estonia	EST	3	3	3	3	3	3
Finland	FIN	1	1	1	1	1	1
France	FRA	**1**	**2**	1	1	**1**	**2**
Great Britain	GBR	1	1	1	1	**1**	**2**
Greece	GRC	4	4	**5**	**3**	**5**	**4**
Hungary	HUN	4	4	4	4	4	4
Ireland	IRL	**4**	**5**	4	4	3	3
Italy	ITA	2	2	2	2	3	3
Latvia	LAT	5	5	4	4	**5**	**4**
Lithuania	LIT	4	4	4	4	4	4
Luxembourg	LUX	1	1	1	1	1	1
Malta	MAL	3	3	3	3	3	3
Netherlands	NLD	1	1	1	1	1	1
Poland	POL	**3**	**4**	3	3	**3**	**4**
Portugal	PRT	3	3	**3**	**2**	3	3
Romania	ROM	5	5	4	4	5	5
Spain	SPA	2	2	**2**	**1**	**2**	**3**
Slovakia	SVK	**3**	**4**	**3**	**4**	3	3
Slovenia	SVN	**2**	**3**	**2**	**3**	2	2
Sweden	SWE	1	1	1	1	1	1

Errors have been highlighted in bold face.

parative analysis of per class classification error is now presented. The number of patterns in class C_1 is significantly higher than in other classes, particularly with respect to the class C_5. While the number of patterns misclassified is three of 27 in class C_1, it raised to three of six in class C_5. In the case of the intermediate classes, the misclassification errors are lower for class C_4, only two errors, compared to classes C_2 and C_3, with four and six errors, respectively.

The models could classify patterns in an upper or a lower class than that rated by the CRA. In our analysis, we will use the terms "positive error" to mean that model classifies the pattern in upper classes and "negative error" for lower classes compared to the real ones (the higher the class the worse the rating is for the country). The results show that the model for CRA Fitch committed 6 positive errors across the fourth first classes, while the models for the other two

CRA did both type of errors. The errors committed by Moody's model were less biased because it misclassified positively two patterns and negatively other three, being located into the the classes C_2, C_3 and C_5. Finally, the predicted label in the S&P model is very close to real rating. This model committed 7 errors, 4 positive and 3 negative, across all the classes unless C_4. In all cases, the model misclassified patterns in adjacent classes to the real one.

We can distinguish several groups among the misclassified patterns. The first group encompassed the countries that have recently joined to the European Union (Bulgaria, Slovakia, Slovenia and Poland). For these countries, their EU membership represents a qualitative feature that is positively valued throughout the credit risk assessment [67]. The non-inclusion of this aspect in the set of the explanatory variables could partly explain the negative errors committed by the models with respect to these countries.

The second group encloses two great European powers, France and Great Britain. The economic downturn had led distortions in some of their economic fundamentals, bringing them closer to those countries characterized as class C_2. However, their economic, political and financial structure generate favourable short and medium-term expectations concerning to their creditworthiness, which allow them to gain the highest rating for their sovereign debt. The third group comprises the PIGS countries, strongly punished by financial markets during the sovereign debt crisis episode. Portugal, Spain and Greece are negatively misclassified by some models. In this case, the differences between projected and real rating is due to the negative expectations on their future economic performance, especially those aspect related to the fiscal policy. As for Greece, the reliability of the data employed [68] may also be the cause of its better performance. On the other hand, Fitch model committed a positive error classifying Ireland. This error could be due to its higher fiscal deficit compared to those patterns in classes C_4 and C_5.

Finally, the case to be analysed is that of Cyprus which is assigned by the three models to class C_2. Thus, the S&P model misclassified negatively this pattern. It also seems to indicate that S&P model did not reflect the fact
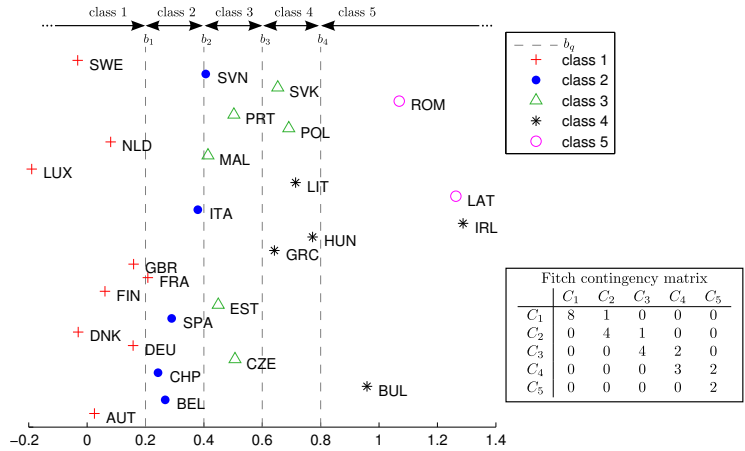
Figure 4: Predicted projection values and contingency matrix for the Fitch test set (year 2010) by using SVR-PCDOC.
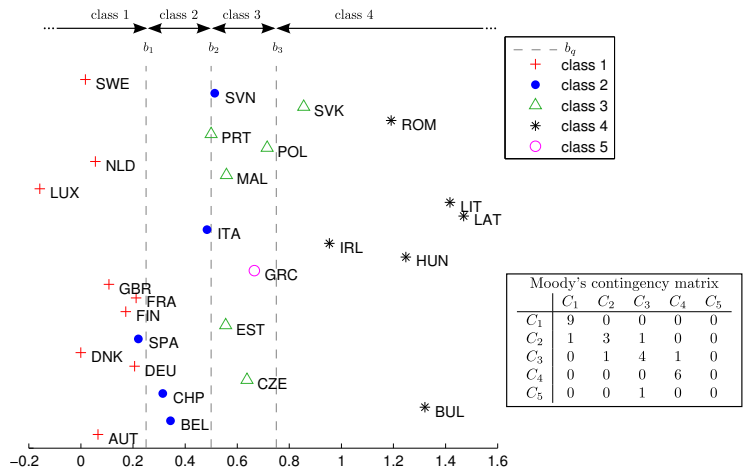
Fitch contingency matrix

	C_1	C_2	C_3	C_4	C_5
C_1	8	1	0	0	0
C_2	0	4	1	0	0
C_3	0	0	4	2	0
C_4	0	0	0	3	2
C_5	0	0	0	0	2

Figure 5: Predicted projection values and contingency matrix for the Moody's test set (year 2010) by using SVR-PCDOC.

Moody's contingency matrix

	C_1	C_2	C_3	C_4	C_5
C_1	9	0	0	0	0
C_2	1	3	1	0	0
C_3	0	1	4	1	0
C_4	0	0	0	6	0
C_5	0	0	1	0	0

that S&P tends to downgrade issuers when compared to the other two agencies [8]. Indeed, S&P downgraded Cyprus from C_2 to C_3 at the end of 2010, while Moody's and Fitch did it during the first half of the following year.

4.5. Analysis of the predicted projection

This section analyses the values predicted by the regressor in the latent space ($\mathcal{L}$) for the three datasets, which are generated by the SVR-PCDOC model. Due
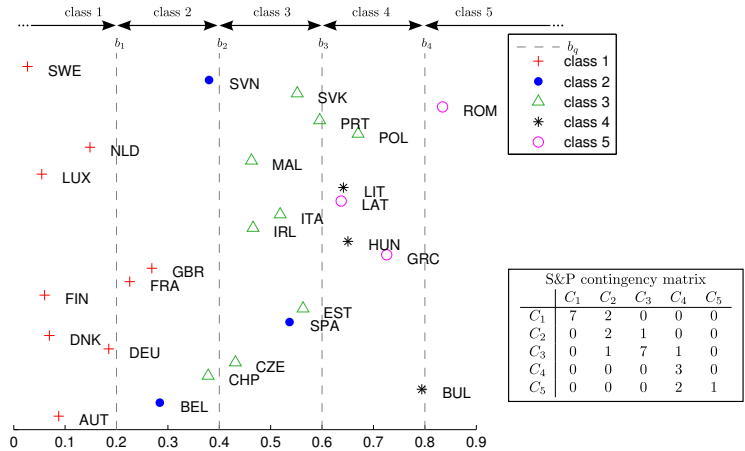
Figure 6: Predicted projection values and contingency matrix for the S&P test set (year 2010) by using SVR-PCDOC.

the way the PCD projection is built, and the quality of the regressor model – which is validated through its classification performance–, it makes sense to use these predictions to provide additional information to potential decision makers. We propose to use these predicted values as a one dimensional measure of the overall patterns rank.

Figures 4, 5 and 6 show the predicted one dimensional projection of the SVR-PCDOC model. This is, the value predicted by the SVR model trained with the PCD projection. Patterns of different classes are highlighted with different colours and symbols corresponding to the actual class. Thresholds for each class are plotted with dashed lines so that it can be seen whether the prediction matches the right class or not (a pattern is classified into a class depending on how it is relatively positioned with respect to the thresholds, see Eq. (1)). Note that for Moody's (Figure 5) there are only three thresholds since the training set contained patterns for only four classes.

For all the plots, the position of the patterns into the one dimensional variable space should be taken into account, so that patterns close to the thresholds are more likely to be misclassified, and it would be advisable to get an expert revision of the final classification to complete a more robust decision support system. This is the case of Great Britain and France rated with C_1 for the Fitch

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and Moody's datasets (see Figures 4 and 5). However, the predictions of the models are very close to the thresholds, and these predictions are quite consistent with the ones of the S&P model that definitely places Great Britain and France in C_2. On the other hand, there are some countries that are placed in the minimum values of the first class (C_1) interval (e.g. Luxembourg, Sweden, Finland, Denmark or Austria). According to the input variables, the trained model places this countries as "better positioned" on class one than all the other patterns.

Together with the plots, it is included the corresponding confusion or contingency matrix of the classification results. Observe that errors are aligned through the diagonal of the matrices. As mentioned in Subsection 3.1, this is one of the aims of the ordinal classification. The only exception is the case of Greece in Moody's dataset (see Figure 5), where this pattern of C_5 is placed on class C_3. However, we can argue that this case is not common since the model was trained by only considering four classes.

5. Conclusions

In this paper, we have applied the PCD projection as a suitable methodology for data classification and validation. The robustness of the classifier is demonstrated using four performance metrics for comparing the PCDOC classifier to nominal and ordinal classifiers in the three main CRA's datasets. In most cases, the errors committed by the three models implies the misclassification of patterns in only one upper or lower class rather than several ranks in the ordinal rating scale. It supposes an advantage for decision making process based on scenarios considering sovereign ratings.

On the other hand, the pattern by pattern analysis indicates that the set of explanatory variables has to be augmented with other qualitative variables for some countries. In this regard, the historical information about a country's economic performance could be completed with data on economic short and medium-term expectations, as a result projected rating would turn into forward-

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looking evaluation of the country's ability to honour its sovereign debt.

This study can be extended in two different ways to increase the accuracy of the proposed models, once data comparable across countries and for periods of less than one year are available. First, by using quarterly or monthly data that provide more detailed information on the economic and financial developments in the context of the high volatility and the repricing of risk in financial markets. Secondly, by including data about country banking sector's exposures to sovereign debt and housing bubble that finally crystallize on the government's balance sheet.

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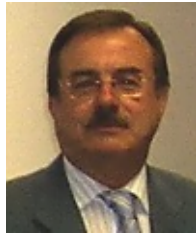
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